Handling losses in translog profit models

J.W.B. Bos\textsuperscript{a,1,2}, M. Koetter\textsuperscript{b,c,1}

\textsuperscript{a}Utrecht School of Economics, Utrecht University, Vredenburg 138, 3511 BG, Utrecht, the Netherlands
\textsuperscript{b}University of Groningen, Faculty of Economics, P.O. Box 800, 9700 AV Groningen, the Netherlands
\textsuperscript{c}Deutsche Bundesbank, Economic Research Center, P.O. Box 10 06 02, 60006 Frankfurt a. M., Germany

Abstract

In this paper, we compare standard approaches used to handle losses in logarithmic stochastic profit frontier models with a simple novel approach. We discuss discriminatory power, rank stability and precision of profit efficiency scores. Our new method enhances rank stability and discriminatory power, and improves the precision of profit efficiency scores.

Key words: profit efficiency, stochastic frontier analysis, truncation and censoring

JEL classification: G21, C24

1 Introduction

Profit models play an important role when we assess the determinants of firm profitability or when we benchmark firms’ success at maximizing profits.

\textsuperscript{1} We thank David Humphrey and Clemens Kool for encouraging us to write this paper, and Allen Berger, Jaap Bikker, Barbara Meller and Laura Spierdijk for helpful comments. We gratefully acknowledge support from the Bundesbank. All errors are our own.

\textsuperscript{2} Corresponding author. Vredenburg 138, 3611BG Utrecht, the Netherlands. Tel. +31-30-253-9682; fax +31-30-253-7373. E-mail addresses: j.bos@econ.uu.nl (Jaap Bos), m.koetter@rug.nl, michael.koetter@bundesbank.de (Michael Koetter)

\textit{Paper submitted to Applied Economics (September 2006)}
When estimating profit models, we employ (semi-)flexible functional forms like the translog. This is problematic if firms incur losses in our sample, since the logarithm of non-positive numbers is not defined. Hence, we face an important inconsistency between our theoretical model and our empirical specification.

In this paper, we compare standard methods used to handle these losses with a novel method.\(^3\) We compare both methods by estimating stochastic profit frontiers, where we can compare profit efficiency (PE). When we study profit efficiency scores, we are concerned both with the ability of specifications to discriminate between profit making and loss incurring firms, as well as their capability of achieving rank stability. Our paper is structured as follows. First, we explain the two most frequently used specifications to handle negative profits as well as our suggested alternative. Then, we introduce our data. Next, we test whether our specification improves the discriminatory power and rank stability of our model. Finally, we conclude.

2 Methodology

Theoretically, firms maximize profits by choosing in- and output quantities at prevailing prices subject to a production technology constraint. Most banking studies employ a modified model by Humphrey and Pulley (1997), that allows for price setting power in output markets.\(^4\) To implement either of the two models empirically, most studies follow Hasenkamp’s (1976) early suggestion and use sufficiently flexible functional forms with regard to curvature. The translog functional form proved particularly suited for bank efficiency studies to facilitate convergence when maximizing the likelihood function (Berger and Mester, 1997).

We use a true fixed-effect model with time-variant efficiency to estimate the following specification of an alternative profit model:\(^5\)

\[
\ln \pi_{kt}(w, y, z) = a_k + \sum_{i=1}^I a_i \ln m_{i kt} + \frac{1}{2} \sum_{i=1}^I \sum_{j=1}^J a_{ij} \ln m_{i kt} \ln m_{j kt} + \varepsilon_{kt}. \tag{1}
\]

\(^3\) For an overview and a solution to this problem for a non-parametric (DEA) model, see Färe et al. (2004).

\(^4\) The alternative profit model specifies an additional constraint: the pricing opportunity set. Banks choose prices for given output quantities subject to this and the technology constraint.

\(^5\) In the true fixed effect model, inefficiency scores are i.i.d. and no particular pattern of evolution of inefficiency is specified (see Greene (2002)).
Here $m$ consists of outputs $y$, input prices $w$, a control variable $z$ (equity), and a time trend $t$ that captures technological change. We assume that $\varepsilon_{kt}$ consists of a noise component $\nu_{kt}$, and an inefficiency component $u_{kt}$, where $\varepsilon_{kt} = \nu_{kt} - u_{kt}$. Here, $\nu_{kt}$ is normally distributed, i.i.d. with $\nu_{kt} \sim N(0, \sigma^2_{\nu})$. The inefficiency term $u_{kt}$ is drawn from a non-negative half-normal distribution and i.i.d. with $u_{kt} \sim |N(0, \sigma^2_{u})|$. Point estimates of $PE$ are obtained as the expected value of $u_{kt}$ given $\varepsilon_{kt}$ (Jondrow et. al, 1982).

The problem that we address in this paper arises because we assume in our theoretical model that $\pi \in \mathbb{R}$, whereas in our empirical specification $\ln \pi$ is not defined if $\pi \in \mathbb{R}_{-}$, where $\pi = [0, -\infty)$. In the literature, we find two solutions listed as (i) and (ii) in Table 1.

First, we can truncate $\pi$ and estimate our model only for those firms where $\pi \in \mathbb{R}_{+}^{*}$, since $\pi$ is then $(0, \infty)$. In our view, this approach suffers from two shortcomings: it prohibits us from obtaining efficiency scores for loss incurring firms, and not adjusting for truncation leads to biased results (for ordinary least squares (OLS) estimators, see Greene, 2003 (chapter 20)). Second, we can rescale $\pi$, to ensure that $\pi \in \mathbb{R}_{+}^{*}$ for all firms, for example by adding the maximum loss observed in the sample plus a small number (usually one) to each $\pi$. This appears to be the most popular solution in the literature (cf. Berger and Mester, 1997, Vander Vennet, 2002, Maudos et. al, 2002). However, we cannot control for the effect that this manipulation may have on our error term structure. This is particularly problematic in stochastic frontier analysis, where we are interested in the composition of total error, rather than coefficient estimates or marginal effects.

Summing up, these approaches either (i) result in a loss of crucial observa-

\footnote{We impose the consistency conditions of Bauer et al. (1998). Whereas the standard profit (and cost) function is the dual to the output distance function that characterizes production technology (i.e. the transformation function), the alternative profit function is the dual to the output distance function and the pricing opportunity set $g(p, y, w)$ (cf. Kumbhakar and Lovell (2003)). The latter “captures the producer’s ability to transform exogenous $(y, w)$ into endogenous product prices $p$” (p. 213), Kumbhakar and Lovell (2003, p. 213) write that: “it is reasonable to assume that [the alternative profit function] is nondecreasing in the elements of $y$ and nonincreasing in the elements of $w$.” Summing up, whereas imposing homogeneity of degree one on both outputs and input prices is indeed needlessly restrictive, our restrictions on input prices do not violate the approach suggested by Kumbhakar and Lovell (2003).}
Table 1: Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>LHS adjustment (π)</th>
<th>RHS adjustment (NPI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[i] Truncated</td>
<td>exclude</td>
<td>π</td>
</tr>
</tbody>
</table>
| [ii] Rescaled          | π + |min (π^-)| +1 |π + |min (π^-)| +1 |-
| [iii] Indicator        | 1                  | π                   | |π^-| |1 |

LHS (RHS) = Lefthand (Righthand) side.

...or (ii) they neglect the available information about the truncated part of the distribution of the dependent variable $\ln \pi$. We therefore propose an alternative solution, that is in fact similar to censoring and attempts to make use of the available information on the censored part of $\pi$. We also left-censor $\pi$, but assign a value of one to those banks with $\pi < 0$. We aim to include all information available on the censored part of $\pi$ and to this end specify an additional independent variable $NPI$ (for Negative Profit Indicator). Consequently, we define $NPI$ to be equal to one for observations where $\pi < 0$ and equal to the absolute value of $\pi$ for a loss incurring bank.\(^7\) Table 1 summarizes the resulting three specifications, including our “Indicator” approach.\(^8\)

3 Data

To estimate our alternative profit frontier, we use balance sheet and profit and loss account data for all German banks that reported to the Deutsche Bundesbank between 1993 and 2004.

We follow the intermediation approach and report our descriptive statistics for profits, input prices, output quantities, and equity in Table 2. In our sample, around 2% of observations (658) exhibit losses.\(^9\)

\(^7\) We expect and find a negative coefficient for this variable.

\(^8\) As a caveat we point out that we do not aim to combine ML functions derived for (OLS) limited dependent regressions with ML functions derived for SFA with a composed error term. In our view, this would certainly be the econometrically most correct way to tackle the problem of losses in PE research. To our knowledge no such efforts have been undertaken in the econometric literature and we deem the issue out of the scope of our paper.

\(^9\) Although our approach can also be used for firms with zero profits, there are no such firms in our data set.
### Table 2: Descriptive statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\pi \in \mathbb{R}_+^*$</th>
<th>$\pi \in \mathbb{R}_-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$ 1)</td>
<td>10.6 (67.4)</td>
<td>-6.9 (41.9)</td>
</tr>
<tr>
<td>Interbank loans 1)</td>
<td>377.2 (4,364.7)</td>
<td>648.7 (7,249.6)</td>
</tr>
<tr>
<td>Customer loans 1)</td>
<td>753.0 (6,724.4)</td>
<td>967.8 (13,683.0)</td>
</tr>
<tr>
<td>Securities 1)</td>
<td>357.0 (3,635.0)</td>
<td>783.0 (10,789.3)</td>
</tr>
<tr>
<td>Price of fixed assets 2)</td>
<td>21.8 (454.3)</td>
<td>91.6 (963.9)</td>
</tr>
<tr>
<td>Price of labor 3)</td>
<td>51.2 (152.7)</td>
<td>64.0 (36.3)</td>
</tr>
<tr>
<td>Price of borrowed funds 2)</td>
<td>3.9 (25.5)</td>
<td>5.7 (24.1)</td>
</tr>
<tr>
<td>Equity 1)</td>
<td>57.8 (498.9)</td>
<td>87.3 (809.7)</td>
</tr>
<tr>
<td>$N$ Observations</td>
<td>33,533</td>
<td>658</td>
</tr>
</tbody>
</table>

Means (standard deviations); 1) In millions of Euros; 2) In percentages; 3) In thousands of Euros; $w_1$ is depreciation and other expenditures on fixed assets/ fixed assets; $w_2$ is personnel expenses/ number of full-time equivalent employees; $w_3$ is interest expenses/ total borrowed funds.

### 4 Results

We start by comparing the efficiency distributions from all specifications. Figure 1 shows kernel density plots. We observe that the rescaled specification yields a distribution of $PE$ scores that exhibits the lowest standard deviation and is located the closest to full efficiency. Since the maximum loss in the sample equals 989 million Euros, the impact of rescaling the dependent variable for all banks (averaging 10.2 million Euros) appears to be substantial.\(^{10}\)

However, it is important to note that we have no baseline, ‘correct’ specification. Put differently, we have to accept the fact that $PE$ scores cannot be validated when comparing our specifications and drawing conclusions. In our comparisons, we build on two premises which we expect to hold in a ‘good’ specification:

(a) We expect the efficiency levels of firms with positive profits to be on average higher than those of firms with negative profits.
(b) We expect the relative efficiency ranking for firms with positive profits to be insensitive to the inclusion of firms with negative profits.

\(^{10}\) In fact, the high density may largely be due to the inability of unadjusted output quantity and input price variables to explain these profits and, more importantly, discriminate between production plan choices of banks.
In sum, we aim at a specification that can both discriminate between firms making a profit and firms incurring a loss and has stable efficiency ranks. Clearly, specification (i) is of little direct use to us, as it has no information on loss incurring firms.

**Figure 1: Kernel density of mean PE per specification**

Table 3 therefore lists our comparative statistics for specifications (ii) and (iii). In both specifications, outliers can influence the overall distribution of efficiency. In fact, we observe that mean efficiency is lower for our indicator approach than for the rescaled specification. To see whether outliers explain this difference, we also present bootstrapped results in table 4. More importantly, we observe that mean PE scores are always higher for profit making firms than for loss incurring firms. These differences are statistically significant, both with \((IST_1)\) and without \((IST_2)\) assuming equal variances. Hence both specifications appear to have sufficient discriminatory power.

Our second requirement concerns the ability of specifications to rank profit making firms’ efficiency in a stable manner. As several studies have shown, the ability of stochastic frontier models to yield stable ranks is very important (e.g. Bauer et. al, 1998). From the plots in Table 3, we observe that ranks change significantly with the rescaled specification.\(^{11}\) Many banks are

\(^{11}\) We calculate ranks for banks with \(\pi \in \mathbb{R}^+\) only as our prime interest is the stability of ranks across specifications. Note that the scatterplots are for a comparison vis-à-vis the truncated specification. Also, note that applying truncation to our Indicator approach results in the truncated specification.
Table 3: Comparative statistics for non-truncated specifications

<table>
<thead>
<tr>
<th></th>
<th>(ii) Rescaled</th>
<th>(iii) Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi \in \mathbb{R}^+$</td>
<td>0.755 (0.062)</td>
<td>0.326 (0.102)</td>
</tr>
<tr>
<td>$\pi \in \mathbb{R}^-$</td>
<td>0.670 (0.141)</td>
<td>0.175 (0.162)</td>
</tr>
<tr>
<td>$IST_1$</td>
<td>33.16***</td>
<td>37.19***</td>
</tr>
<tr>
<td>$IST_2$</td>
<td>15.35***</td>
<td>23.74***</td>
</tr>
</tbody>
</table>

$\rho = 0.4698^{***}$, $KW = 375.5^{***}$

N = 33,533 ($\pi^+$), 658 ($\pi^-$), IST = Independent samples test, with (1) and without (2) equal variance assumption; $\rho$ = Spearman rank correlations; KW = Kruskal Wallis chi-squared; *** denote significant at the 1% level. Piecewise correlation between truncated and indicator ranks is 0.971 and significant at the 1% level.

ranked markedly different by the truncated and rescaled specification, respectively. In contrast, our indicator specification ranks much more consistently. The Spearman rank order correlation $\rho$ with the truncated specification is 0.971 and significantly different from zero. Finally, we also perform Kruskall Wallis rank tests, which confirm that profit-making and loss-incurring banks are ranked significantly different from each other. Note, that our indicator specification exhibits substantially higher chi-squared values compared to the rescaled specification.

In addition to testing whether estimated efficiency distributions are identical, we also conduct a bootstrap analysis along the lines of Atkinson and Wilson (1995) to obtain standard errors of mean efficiency estimates (cf. Koetter (2006)). Thereby we can test the precision of $PE$ estimates obtained with the three alternative approaches to handle negative profits, respectively. We follow their suggestion and draw $j = 1, \ldots, J$ bootstrap samples with replacement of the original size $N$, i.e. 34,191 observations, where $J \simeq 1000$. For each draw $j$, we estimate mean $PE_j^*$ for the three approaches, respectively. We denote

12 For model 1, the truncation approach, we obviously only sample 33,533 observations.
the mean statistic obtained with the original sample as $PE_{\text{obs}}$ and calculate standard errors.\footnote{13} 

Table 4: Bootstrapped standard errors of mean PE

<table>
<thead>
<tr>
<th>Specification</th>
<th>$PE_{\text{obs}}$</th>
<th>$\overline{PE}^*$</th>
<th>$SE$</th>
<th>$LB$</th>
<th>$UB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truncated</td>
<td>39.3</td>
<td>38.1</td>
<td>0.24</td>
<td>38.9</td>
<td>39.8</td>
</tr>
<tr>
<td>Rescaled</td>
<td>75.3</td>
<td>81.1</td>
<td>4.65</td>
<td>66.2</td>
<td>84.4</td>
</tr>
<tr>
<td>Indicator</td>
<td>32.3</td>
<td>32.3</td>
<td>0.27</td>
<td>31.8</td>
<td>32.8</td>
</tr>
</tbody>
</table>

Notes: Bootstrapping results for 1,000 repetitions of full resampling with replacement; \footnote{13} Mean PE with original full sample; \footnote{13} Average mean efficiency after bootstrapping; \footnote{13} Standard errors; \footnote{13} Lower bound; \footnote{13} Upper bound.

In Table 4 we report bootstrapped standard errors and according confidence intervals at the five percent level for all profit models, respectively.\footnote{14} Bootstrapped standard errors are largest for the rescaled model. In contrast, the precision of efficiency estimates obtained from our indicator approach closely resembles that obtained for the case when loss-making banks are excluded from the sample. Hence, with our approach we gain discriminatory power without a loss of precision and possible outliers do not bias (mean) efficiency.

5 Conclusion

In this paper, we compare a novel approach to handling losses in translog profit models with specifications that rely on truncation (i) or rescaling (ii) of the dependent variable. We study the effect on stochastic frontier profit efficiency scores. The latter specifications either do not yield any efficiency scores for loss incurring firms (i), or yield efficiency scores which are discriminatory but not stable (ii). Censoring is shown to greatly improve the rank stability of efficiency scores. In addition, our indicator specification improves the discriminatory power of our translog profit model. Finally, bootstrapped standard errors show that the preciseness of the indicator approach is substantially higher than for the certainty of efficiency estimates obtained after scaling up all data.

\footnote{13} $SE = \left\{ \frac{1}{J-1} \sum_{j=1}^{J} (PE_j^* - \overline{PE}^*)^2 \right\}^{1/2},$ where $\overline{PE}^* = \frac{1}{J} \sum_{j=1}^{J} CE_j^*$.

\footnote{14} Confidence intervals are $[PE_{\text{obs}} - t_{1-\alpha/2,k-1}SE, PE_{\text{obs}} + t_{1-\alpha/2,k-1}SE]$; where $t_{1-\alpha/2,k-1}$ is the $(1 - \alpha/2)^{th}$ quantile of the t-distribution with $k - 1$ degrees of freedom.
References


