Chapter 7

Lifetime modeling and optimization

In this chapter, a membrane lifetime model is developed and experimentally validated. The lifetime model is based on the Weibull probability density function. The lifetime model can be used to determine an unambiguous characteristic membrane lifetime. Experimental results showed that membrane lifetime shortens if the average membrane fouling status increases. The lifetime modeling results are then used to determine the economic lifetime of membranes. Subsequently, the economic lifetime of a membrane is used to optimize membrane lifetime, i.e. minimizing the total costs. Based on the experimental results it can be concluded that the total costs are minimal if the average membrane fouling status is approximately $1.7 \times$ the membrane resistance.

7.1 Introduction

Recently attention has been directed towards optimization of filtration [1], backwashing [2] and chemical cleaning [3] of UF membranes. Intermediate term optimization, where membrane performance is optimized over multiple production cycles was also reported [4; 5]. Long term optimization of UF membranes, where membrane ageing/lifetime is incorporated, is another area of research interest.

In [6] the setup and execution of an experimental design, in which potential
ageing factors were evaluated by means of accelerated lifetime testing was discussed. These results showed that the membrane fouling resistance is a significant ageing factor, influencing lifetime.

![Figure 7.1: Optimization at membrane lifetime level.](image)

In this chapter we will develop and validate a statistical membrane lifetime model. From the model a clear definition of membrane lifetime can be derived. Subsequently, the model is used to evaluate the capital costs and the operational costs of an ultrafiltration membrane as function of the fouling resistance. This result may be useful in the development of a long term fouling control strategy.

### 7.2 Theory

#### 7.2.1 Lifetime modeling

Statistical analysis and modeling based on data collected by means of accelerated ageing tests has been done in different fields of science and engineering [7; 8; 9; 10; 11; 12]. An example of a commonly used lifetime model is the so-called Weibull distribution. Application to the lifetimes or durability of manufactured items such as ball bearings, automobile components and electrical insulation is very common. It is also used in biological and medical applications, for example, in studies on the time to the occurrence of tumors in human populations or in laboratory animals [13].
7.2. Theory

7.2.2 Definitions

Consider the case of a single continuous lifetime variable, $T$. Specifically, let $T$ be a nonnegative random variable representing the lifetimes of individuals in some population. All functions are defined over the interval $[0, \infty)$. Let $f(t)$ denote the probability density function (p.d.f.) of $T$ and let the (cumulative) distribution function (c.d.f.) be:

$$F(t) = \int_0^t f(x)dx$$  \hspace{1cm} (7.1)

The probability of an individual surviving to time $t$ is given by the survivor function or reliability function:

$$S(t) = \int_t^\infty f(x)dx$$  \hspace{1cm} (7.2)

The hazard function specifies the instantaneous rate of death or failure at time $t$:

$$h(t) = -\frac{d}{dt} \log [S(t)]$$  \hspace{1cm} (7.3)

7.2.3 The Weibull distribution

The Weibull distribution is perhaps the most widely used lifetime distribution model. The Weibull distribution has a hazard function of the form:

$$h(t) = \lambda \beta (\lambda t)^{\beta - 1}$$  \hspace{1cm} (7.4)

where $\lambda > 0$ and $\beta > 0$ are distribution model parameters. It includes the exponential distribution as the special case when $\beta = 1$. By Eq. 7.1 and 7.2, the p.d.f. and survivor functions of the distribution are:

$$f(t) = \lambda \beta (\lambda t)^{\beta - 1} \exp \left[-(\lambda t)^\beta\right]$$  \hspace{1cm} (7.5)

and:

$$S(t) = \exp \left[-(\lambda t)^\beta\right]$$  \hspace{1cm} (7.6)

The Weibull hazard function is a monotone increasing function when $\beta > 1$, decreasing when $\beta < 1$ and constant when $\beta = 1$. The model is flexible and has been found to provide a good description of many types of lifetime data. This property and the fact that the model gives simple expressions for the
p.d.f., survivor- and hazard functions partly account for its popularity. The Weibull distribution arises as an asymptotic extreme value distribution and in some instances this can be used to provide motivation for it as a model. The scale parameter $\alpha = \lambda^{-1}$ is often used instead of $\lambda$. In some areas, especially in engineering, $\alpha$ is termed the characteristic life of the distribution. The shape of the Weibull p.d.f. and hazard function depends only on $\beta$, which is sometimes called the shape parameter for the distribution. The effect of $\lambda$ is to change the scale of the horizontal axis and not the basic shape of the distribution. In figure 7.2 different Weibull models are shown for different values of $\beta$ and $\lambda$.

Figure 7.2: Examples of the Weibull probability density function, the cumulative density function, the survivor function and the hazard function for different values of $\lambda$ and $\beta$. Legend: $(-)$ $\lambda = 0.05 \beta = 3$; $(.)$ $\lambda = 0.05 \beta = 2$; $(- -)$ $\lambda = 0.05 \beta = 1$; $(.-)$ $\lambda = 0.03 \beta = 3$; $(.-)$ $\lambda = 0.02 \beta = 3$. 
7.3 Materials and methods

7.3.1 The pressure pulse unit

In fig. 7.3 and fig. 7.4 the PPU (Pressure Pulse Unit) is shown. The main part of the PPU is the membrane pump, which can pump sodium hypochlorite - or water - at a pressure of 0 to 3 bar through membrane modules with a frequency of approximately 20 to 30 pulses per minute (in normal operation a back pulse is applied 4 times per hour). Pulse tests are performed, reflecting a plant’s "worst case scenario", where valves open and close frequently, while generating fast pressure changes (from 0 to 2 bar in 2 to 3 seconds). Using two pressure restrictions, two sets of experiments can be simultaneously performed in threefold at two different pressures.

The used membrane modules were Norit-Xiga RX300 PSU hollow fiber UF modules with a membrane surface of 0.07 m². The Xiga fibers have a poly sulphone housing and PES/PVP flow distributors (fibers). Every test module contains 100 fibers with a length of 30 cm. Potting procedures and materials for clean and fouled fibers were the same.

Three series of pulse tests were performed with clean fibers, intermediately fouled fibers and severely fouled fibers. The average initial membrane resistance was determined to be $5.10^{11} \, (m^{-1})$, $1.25.10^{12} \, (m^{-1})$ and $2.10^{12} \, (m^{-1})$, respectively.

Severely fouled modules were assembled from fibers that were taken from a module that was operated in a pilot pant. Intermediately fouled modules
were obtained from a module operating in a full scale installation. The module from the pilot plant was operated over a six month period, and was only cleaned with sodium hydroxide. The module taken from the full scale installation was operated over a 7 year period, the cleaning history was not exactly known, but it is assumed that the module was cleaned frequently with sodium hydroxide and hydrochloric acid, while hypochlorite cleaning was not done extensively.

### 7.3.2 Procedure

At frequent intervals, the pulse tests were interrupted and membrane integrity was evaluated by means of permeability testing, pressure decay testing and bubble point testing. This last method was used to determine the number of defected fibers in a module as function of the number of applied back pulses. Defected fibers were detected and closed with special pins, before experimentation continued.
7.4 Results and discussion

7.4.1 Lifetime modeling

In figure 7.5 the experimental data and the fitted Weibull models are plotted. Table 7.1 shows the calculated model parameters. During accelerated ageing tests, fibers take a long time to fail. To collect data within acceptable experimentation time, it is necessary to terminate the experiment before all fibers have failed. Consequently, certain fiber lifetimes are censored.

Although censoring takes place, data can be interpreted and fitted correctly because a proper model structure has been chosen, the Weibull model. The Weibull model has properties that, for example, a linear approximation does not have, e.g. the surface below the curve should be equal to one (100% of the fibers have defected).

The results of table 7.1 show that the characteristic life ($\alpha$), as well as $t_{1/10}$ (time at which one tenth of the fibers defected) and $t_{1/100}$ (time at which one hundredth of the fibers defected) decreases when the fouling state of the membrane increase: membrane lifetime shortens when the membrane is increasingly fouled.

In figure 7.6 the cumulative density function, the survivor function and the hazard function are plotted for the specific experimental cases.
Table 7.1: Calculated model parameters and characteristic lifetimes.

<table>
<thead>
<tr>
<th>Condition</th>
<th>( R ) (( 1/m ))</th>
<th>( \lambda ) (( 1/\text{yrs} ))</th>
<th>( \beta ) (( 1/\text{yrs} ))</th>
<th>( \alpha ) (( \text{yrs} ))</th>
<th>( t_{1/10} ) (( \text{yrs} ))</th>
<th>( t_{1/100} ) (( \text{yrs} ))</th>
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<tr>
<td>Clean</td>
<td>0.50.10^{12}</td>
<td>0.026</td>
<td>3.0</td>
<td>38.5</td>
<td>18.1</td>
<td>8.3</td>
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<tr>
<td>Intermediate fouling</td>
<td>1.25.10^{12}</td>
<td>0.040</td>
<td>3.0</td>
<td>25.0</td>
<td>11.8</td>
<td>5.4</td>
</tr>
<tr>
<td>Severe fouling</td>
<td>2.00.10^{12}</td>
<td>0.050</td>
<td>3.0</td>
<td>20.0</td>
<td>9.5</td>
<td>4.3</td>
</tr>
</tbody>
</table>

### 7.4.2 Lifetime optimization

The cumulative density function reflects the total number of defected fibers in a module as a function of operating time. The costs for fiber reparation are proportional to the number of defected fibers. In figure 7.6, the upper left figure shows the reparation costs as a function of time in \( \text{EURO.m}^{-2} \). The reparation costs are calculated from the cumulative density function according to:

\[
C_{\text{REP}} = \frac{Nc}{A} \cdot S(t) \quad (7.7)
\]

Where \( N \) is the number of fibers present in a module, \( c \) are the costs for reparation of a single fiber and \( A \) is the membrane surface. Calculations were performed for a commercially available module with \( N = 10,000 \), \( c = 5\text{EURO} \) and \( A = 40m^2 \). In the upper left corner of figure 7.7 also a horizontal line is shown, expressing the costs for membrane replacement \( C_{\text{replace}} = 75\text{EURO.m}^{-2} \). The point were the horizontal line intersects with the curve, determines the economic lifetime \( t_L \) of a module by a simple hyperbolic expression (\( C_{\text{CAP}} = K/t_L \) where \( K = 375 \text{EURO.y.m}^{-2} \)). For increased fouling, the economic lifetime becomes shorter, as shown in the upper right corner of figure 7.7.

The capital costs (or investment costs) are correlated to the economic lifetime of a module. If the economic lifetime is short, the capital costs will be high, if the lifetime is long, the capital costs will be low, as shown in the lower left corner of figure 7.7.

By combining the results of the upper right figure and the lower left figure of figure 7.7 the relationship between the fouling resistance and the capital costs can be obtained, as shown in the lower right corner of figure 7.7.
Figure 7.6: The Weibull probability density function, the cumulative density function, the survivor function and the hazard function for different membrane fouling states. Legend: (–) $R = 0.5 \times 10^{12}(m^{-1})$; (.) $R = 1.25 \times 10^{12}(m^{-1})$; (- -) $R = 2 \times 10^{12}(m^{-1})$.

Capital costs increase as the membrane fouls more, but operational costs decrease if more fouling is allowed. In figure 7.8 the capital costs $C_{CAP}$ and an operational costs approximation $C_{OP}$ are plotted together with the total costs $C_{TOT}$.

Operational costs as function of the average membrane irreversible fouling state were calculated using a fouling model and cost function, based on energy requirements, material costs (Feedwater, wastewater, coagulant, cleaning chemicals, etc) and depreciation costs, over multiple chemical cleaning cycles. From figure 7.8 it can be seen that the total costs are minimal at a fouling level of around $1.7 \times$ the membrane resistance ($0.5 \times 10^{12}(m^{-1})$).
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Figure 7.7: Upper left: reparation costs as function of operating time for different fouling resistances. Legend: (–) $R = 0.5 \times 10^{12} (m^{-1})$; (.) $R = 1.25 \times 10^{12} (m^{-1})$; (- -) $R = 2 \times 10^{12} (m^{-1})$, upper right: resistance as function of economic lifetime, lower left: capital costs as function of economic life time and lower right: capital costs as function of fouling resistance.
Figure 7.8: Optimal fouling resistance calculated from the capital costs and operational costs.
7.5 Conclusions

A membrane lifetime model was developed and experimentally validated. The lifetime model is based on the Weibull probability density function. The lifetime model can be used to determine an unambiguous characteristic membrane lifetime. Experimental results showed that membrane lifetime shortens if the average membrane fouling status increases. The lifetime modeling results are then used to determine the economic lifetime of membranes. Subsequently, the economic lifetime of a membrane is used to optimize membrane lifetime, which means minimization of the total costs. Based on the experimental results presented, the total costs are minimal if the average fouling status is approximately $1.7 \times 10^{-12} (m^{-1})$. 


7.5. CONCLUSIONS

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit (trivial) [SI]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Characteristic lifetime</td>
<td>$(y)$ [s]</td>
</tr>
<tr>
<td>$A$</td>
<td>Membrane surface</td>
<td>$(m^2)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Shape parameter</td>
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<tr>
<td>$c$</td>
<td>Repair costs</td>
<td>(EURO)</td>
</tr>
<tr>
<td>$C_{CAP}$</td>
<td>Capital costs</td>
<td>(EURO.m$^{-2}$)</td>
</tr>
<tr>
<td>$C_{REP}$</td>
<td>Repair costs</td>
<td>(EURO.m$^{-2}$)</td>
</tr>
<tr>
<td>$C_{OP}$</td>
<td>Operational costs</td>
<td>(EURO.m$^{-2}$)</td>
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<tr>
<td>$f$</td>
<td>Probability density function</td>
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</tr>
<tr>
<td>$F$</td>
<td>Cumulative density function</td>
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<tr>
<td>$h$</td>
<td>Hazard function</td>
<td>(-)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Scale parameter</td>
<td>$(y^{-1})$ [s$^{-1}$]</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of fibers in a module</td>
<td>(-)</td>
</tr>
<tr>
<td>$R$</td>
<td>Membrane fouling resistance</td>
<td>$(m^{-1})$ [m$^{-1}$]</td>
</tr>
<tr>
<td>$S$</td>
<td>Survivor function</td>
<td>(-)</td>
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<td>$t$</td>
<td>Time</td>
<td>$(y)$ [s]</td>
</tr>
<tr>
<td>$t_{1/10}$</td>
<td>Time at which one tenth of the fibers defects</td>
<td>$(y)$ [s]</td>
</tr>
<tr>
<td>$t_{1/100}$</td>
<td>Time at which one hundredth of the fibers defects</td>
<td>$(y)$ [s]</td>
</tr>
<tr>
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<td>$(y)$ [s]</td>
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<tr>
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<td>Reference time</td>
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Bibliography


