Production and trapping of Na isotopes for beta-decay studies
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A. Characterization of the Magneto-Optical Trap

In the following we derive relations that are essential to characterize the MOT operation.

A.1 Trapping Efficiency

We derive a quantitative description for loading a MOT with a beam of particles entering the trapping cell. The number of atoms in the cell volume $V$ is $N$. $N$ follows from a rate equation

$$\frac{dN}{dt} = R - L_s - L_e,$$

(A.1)

where $R$ is the input rate of incoming particles, $L_s$ is the loss rate due to the permanent sticking to the glass cell wall and $L_e$ is the loss rate due to escape of particles through the pumping ports of the cell. The loss rates are

$$L_s = \frac{1}{4} n A_s \bar{v} P_s,$$  

(A.2)

$$L_e = \frac{1}{4} n A_e \bar{v},$$  

(A.3)

where $n = N/V$ is the density of atoms, $\bar{v}$ is an mean velocity of the atoms inside the cell, $A_s$ and $A_e$ are the surface areas of the cell and the pumping ports. $P_s$ is the probability that an atom gets lost on the wall. The MOT capture rate for the particles inside the cell is

$$R_c = \frac{1}{4} n A_{\text{capture}} \int_0^{\bar{v}c} v f(v)dv \approx \frac{3}{4} n A_{\text{capture}} v_c P_1,$$

(A.4)

where $A_{\text{capture}}$ is the surface area of the cell which overlaps with the laser beams and $P_1$ is the probability for capturing a particle into a MOT in a single pass (Eq. 5.16).

The average number of passes for an atom to be captured is

$$N_{\text{pass}} \approx \frac{A_{\text{capture}}}{A_e + A_s P_s},$$

(A.5)
Appendix A: Characterization of the Magneto-Optical Trap

For a coated cell wall surface, where $A_s P_s \ll 1$, we have

$$N_{\text{pass}} \approx \frac{A_{\text{capture}}}{A_e}. \quad (A.6)$$

In equilibrium there is $dN/dt = 0$, such that $R = L_s + L_e$. The MOT capture efficiency is

$$\varepsilon_c = \frac{R_e}{R} = 3 \frac{v_c}{v} P_1 \frac{A_{\text{capture}}}{A_e + A_s P_s}. \quad (A.7)$$

### A.2 Time Dependence of the MOT Population

In the following we consider the number of atoms $N$ as function of time in a cell with volume $V$ when an amount $N_s$ is introduced suddenly, for example when after accumulation on the neutralizer the atoms are released:

$$\frac{dN}{dt} = -\frac{1}{4V} \bar{v}(A_s + A_e) + \frac{N_s}{\tau_s}. \quad (A.8)$$

The last term is introduced to describe the gain from particles $N_s$ on the surface that are released with a rate determined by the temperature dependent sticking time $\tau_s$. The number of atoms on the surface evolves as

$$\frac{dN_s}{dt} = -\frac{N_s}{\tau_s} + \frac{1}{4V} \bar{v} A_s. \quad (A.9)$$

With $a_i = A_i \bar{v} / 4V$ this can be written as

$$\frac{dN}{dt} = -a_s N - a_e N + \frac{N_s}{\tau_s} \quad \text{and} \quad (A.10)$$

$$\frac{dN_s}{dt} = -\frac{N_s}{\tau_s} + a_s N, \quad (A.11)$$

which can be solved in a general way. However, more useful is the behavior when parameters approach limits:

$$N = N_0 e^{-(a_s + a_e)t}, \quad \text{sticking limit: } \tau_s \to \infty. \quad (A.12)$$

$$N = N_0 \frac{\tau_s a_s e^{-(a_s + \tau_s)t} + 1}{1 + a_s \tau_s}, \quad \text{no pumping: } a_e \to 0. \quad (A.13)$$

$$N = N_0 e^{-a_e t}, \quad \text{non-stick surface: } \tau_s \to 0. \quad (A.14)$$
A.2. Time Dependence of the MOT Population

A.2.1 Pulsed Operation of the Neutralizer

We can evaluate the number of particles trapped in the MOT as function of time. The change in the number $N_{\text{MOT}}$ of atoms trapped is governed by

$$\frac{dN_{\text{MOT}}}{dt} = \alpha_c N - \gamma_{\text{loss}} N_{\text{MOT}} - \beta N_{\text{MOT}}^2.$$  \hspace{1cm} (A.15)

Here $\alpha_c$ is the capture rate, and $\gamma_{\text{loss}}$ accounts for collisional losses with the background gas. The last term arises from intra-MOT scattering, which we can neglect here. The capture rate of the MOT is

$$\alpha_c \approx \frac{3}{4} \frac{A_{\text{capture}}}{V} v_c P_1.$$  \hspace{1cm} (A.16)

The trap decay rate $\gamma_{\text{loss}}$ can be estimated as

$$\gamma_{\text{loss}} = \tilde{v} \sigma_b n_b,$$  \hspace{1cm} (A.17)

where $n_b = p/kT$ is the density of background particles determined by the residual pressure in the cell, and $\sigma_b$ is the cross section for collisions with the MOT atoms.

Eq. A.15 can be solved by inserting the solution for $N(t)$ given in section A.2. Here we will assume that $a_e \tau_a \ll 1$, the solution for $N(t)$ in Eq. A.14. We obtain

$$N_{\text{MOT}} = \frac{\alpha_c N_0}{\gamma_{\text{loss}} - a_e} \left( e^{-a_e t} - e^{-\gamma_{\text{loss}} t} \right).$$  \hspace{1cm} (A.18)

The MOT signal is maximal for

$$t = \frac{\ln \gamma_{\text{loss}} - \ln a_e}{\gamma_{\text{loss}} - a_e}.$$  \hspace{1cm} (A.19)

In practice the loss of atoms not yet trapped is much faster than the loss of trapped atoms, i.e. $a_e \gg \gamma_{\text{loss}}$ and

$$N_{\text{MOT}} \approx \frac{\alpha_c N_0}{a_e} e^{-\gamma_{\text{loss}} t}.$$  \hspace{1cm} (A.20)

We also note that $N_0 = R\Delta t$, where $R$ is the incoming beam current and $\Delta t$ the accumulation time.
A.2.2 Continuous Operation of the Neutralizer

Now we consider a pulsed ion beam entering a cell with a neutralizer kept at a fixed high temperature. When the beam is turned on, the number of particles in the volume increases as

\[ N(t) = \frac{R}{a_e} (1 - e^{-a_e t}). \]  
(A.21)

Here it has been assumed that \( a_s \gamma_s \ll 1 \). Inserting this dependence in Eq. A.15 one obtains

\[ N_{\text{on MOT}} = \frac{\alpha_e R}{a_e} \left( \frac{1}{\gamma_{\text{loss}}} + \frac{1}{a_e - \gamma_{\text{loss}}} e^{-a_e t} \right) - \frac{\alpha_e R}{\gamma_{\text{loss}} - a_e} e^{-\gamma_{\text{loss}} t}. \]  
(A.22)

\[ N_{\text{off MOT}} \approx \frac{\alpha_e R}{a_e \gamma_{\text{loss}}} e^{-\gamma_{\text{loss}} t}. \]  
(A.23)

where the approximations assume that \( a_e \gg \gamma_{\text{loss}} \). The last expression, Eq. A.24, uses the result of Eq. A.20. The loading and decay of the trap therefore resembles the charging and discharging of a capacitor.

A.2.3 Non-coated MOT Cell and Neutralizer Temperature Dependence

A number of measurements were made with a non-coated cell. This means that the surface of the cell does not play a role. The atoms are only coming directly from the neutralizer. This situation is also convenient for a discussion of the temperature dependence of the neutralizer.

Pulsed mode

When \( N_0 \) atoms are collected and the neutralizer temperature is rapidly increased, the emission rate from the neutralizer is \( \kappa N_0 \exp(-\kappa t) \), where \( \kappa \) is the strongly temperature dependent desorption rate introduced in Sec. 5.3.1. A differential equation analogous to Eq. A.15 governs this situation

\[ \frac{dN_{\text{MOT}}}{dt} = P_1^T \kappa N_0 e^{-\kappa t} - \gamma_{\text{loss}} N_{\text{MOT}}. \]  
(A.25)

\( P_1^T \) is the capture probability for atoms emitted from a surface at a temperature \( T \). The solution of this equation is

\[ N_{\text{MOT}} = \frac{P_1^T \kappa N_0}{\gamma_{\text{loss}} - \kappa} \left( e^{-\kappa t} - e^{-\gamma_{\text{loss}} t} \right). \]  
(A.26)
A.2. Time Dependence of the MOT Population

In the limit of a hot and a cold neutralizer we have

\[ N_{MOT} \approx P_T^T N_0 e^{-\gamma_{loss} t}, \text{ hot neutralizer } (\kappa >> \gamma_{loss}) \]  \hspace{1cm} (A.27)

\[ N_{MOT} \approx \frac{P_T^T \kappa N_0}{\gamma_{loss}} e^{-\kappa t}, \text{ cold neutralizer } (\kappa << \gamma_{loss} << t) \]  \hspace{1cm} (A.28)

Continuous Mode

When the neutralizer remains at constant temperature and the beam is switched on at a time \( t = 0 \), the emission rate is \( R(1 - e^{-\kappa t}) \) resulting in

\[ N_{MOT}(t) = \frac{P_T^T R}{\gamma_{loss}} \left\{ 1 - \frac{1}{1 - \kappa/\gamma_{loss}} e^{-\kappa t} - (1 - \frac{1}{1 - \kappa/\gamma_{loss}}) e^{-\gamma_{loss} t} \right\}. \]  \hspace{1cm} (A.29)

The relevant limiting behavior is then for the hot neutralizer

\[ N_{MOT} = \frac{P_T^T R}{\gamma_{loss}} (1 - e^{-\gamma_{loss} t}) \]  \hspace{1cm} (A.30)

and an initially linear dependence

\[ N_{MOT} = \frac{P_T^T R}{\gamma_{loss}} \kappa t \]  \hspace{1cm} (A.31)

emerges for the cold neutralizer.

Finally, we note that equations A.27 and A.30 are analogous to equations A.20 and A.23, respectively, except for a common factor.