Solving large linear systems in an implicit thermohaline ocean model

de Niet, Arie Christiaan

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
2007

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):

Copyright
Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

Take-down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): http://www.rug.nl/research/portal. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.
Chapter 1

Introduction

1.1 The challenge

In February 2007 the Intergovernmental Panel on Climate Change (IPCC) came out with a report [59] containing new scientific data on the effect of greenhouse gases on the earth’s climate. Its main messages are that the global warming of the earth is undoubted, and that, with high probability, it is caused largely by human activity. Scientists predict that within a few decades the Arctic sea might be ice free during summers. The floating ice at the North Pole however is negligible compared to the enormous icecap on Greenland. The massive release of fresh water stored in this icecap can have dramatic impact on the earth’s climate.

For example the mild climate in North-Western Europe is caused by the Gulf Stream that carries warm relatively salt water along the surface from the Gulf of Mexico across the North Atlantic to Europe. On its way to the north the flow cools down and sinks. It returns in southern direction along the bottom. There is an ongoing debate about the stability of the Gulf Stream. What happens if the flow is distorted by a bulk of fresh water from melting ice on Greenland? Could the Gulf Stream collapse? If the answer is yes, it would eventually weaken the warming due to the emission of greenhouse gases in North-Western Europe.

The Gulf Stream is part of the so-called conveyor belt depicted in Figure 1.1: cooling down and sinking in the North Atlantic and upwelling and heating in the Pacific and Indian ocean. This global ocean circulation largely determines the earth’s climate. It is therefore very important to study how it reacts to large perturbations. The main way to do this is to construct a computer model that mimics the physics of the global ocean circulation.

The global ocean circulation is a complex physical phenomenon. The flow is driven by (i) the wind, (ii) differences in temperature due to varying surface heating by the Sun, and (iii) differences in salinity due to evaporation (e.g. near the equator), precipitation, inflow from rivers and melting ice. The flow becomes even more complex due to the presence of the Coriolis force, which is caused by the rotation of the earth. Furthermore we have continents spread over the domain and a bottom topography with steep slopes.

We would like to compute this complex three dimensional flow. But more than that: we
would like to predict how the flow reacts to large perturbations.

At the Institute of Marine and Atmospheric research Utrecht (IMAU), the computer model THCM has been developed. The model computes three dimensional ocean flows, which involves the flow velocity in three directions, pressure, temperature and salinity. The relations between all these quantities are given by nonlinear partial differential equations and these have been approximated on a grid covering the ocean. The resolution of the grid immediately influences the accuracy of the simulation. With THCM one is able to compute the effect of perturbations.

We note that modern computers only allow to work with simplified models containing a number of parameters related to physics that cannot be represented in the simplification. The values of these parameters are only known approximately, hence we also have to study the sensitivity of the flow to these parameters. We will describe the model and the mathematics involved in the variation of parameters (in which we include perturbations) and the computation of the flow in the next chapter.

At the start of the research project described in this thesis, THCM already existed, however several numerical problems limited the applicability of the model. A calculational run of the global ocean circulation required simply too much computer memory and time, which limited the resolution that can be used. Therefore we have to speed-up the numerics. The challenge for this thesis is to develop new numerical techniques, such that it is possible to do large scale computations on steady states of the global ocean circulation with sufficient accuracy.

More about the numerics later on in this introduction, first we will describe the background of the research project in more detail.
1.2 Reconstruction of the time-mean absolute velocity field of the global ocean circulation

The global ocean flow is time dependent. It has all kinds of periodicity, because the flow is influenced by the position of the earth with respect to the Moon and the Sun, which gives daily, monthly and seasonal variation. There are phenomena with an even longer period, like for example El Niño, which occurs approximately every 4 years. Nevertheless in the main part of this thesis we will consider the global ocean circulation as a steady flow, as happened in many studies in the past.

The aim of the research project is the reconstruction of the time-mean absolute velocity field of the global ocean circulation. To achieve this, we formulated a number of subgoals:

1. improve the physics in the model,
2. tune the model using observations,
3. speed-up the numerics.

To start with (1), the model used primitive mixing schemes for salt and temperature. Advanced mixing schemes available in the literature needed to be incorporated in THCM.

Subgoal (2) requires more explanation. We want to use satellite data for the sea surface height (SSH) and the geoid to tune the model.

The geoid is an isosurface of the gravity field of the earth, i.e. on the geoid the gravitational acceleration is constant. If there would be no flow at all the sea surface would coincide with the geoid. The geoid is non trivial, because the earth is not a perfect sphere and the gravity field is influenced by the presence of ridges and trenches at the bottom of the ocean. The difference between SSH and geoid contains information about the flow at the top layer of the ocean.

As the differences are small (in the Gulf Stream the deviation is in the order of magnitude of one meter), we need accurate data for the SSH and the geoid. For the SSH there is accurate time-mean data available. The measurement of the gravity field is much more difficult. The errors of the available data (from the GRACE mission) are in the same order of magnitude as the SSH itself. So the current geoid data are worthless for our purpose. Fortunately the GOCE mission [21] will launch a satellite that measures the geoid up to an accuracy of a few centimeters. The launch is scheduled at the end of 2007.

Given an accurate geoid and the SSH data we can construct the flow at the surface. Via a technique called data-assimilation we can tune the ocean model, such that the model gives a surface flow that is close to the observations. This allows us to reduce the model errors.

Of the three subgoals we mentioned, (1) and (2) were carried out by Terwisscha van Scheltinga, a PhD student in the same project at the IMAU. He developed the advanced mixing schemes for salinity and temperature, which we will describe in Section 2.2.2 in the next chapter. The new mixing schemes strengthen the coupling between temperature and salinity, which makes the solution of the equations more difficult and subgoal (3) even more important.
Subgoal (2) required the development of a data-assimilation algorithm for the ocean flows [76]. The algorithm uses a number of successive model runs, which further emphasizes the importance of (3), the speed-up of the numerics.

1.3 Towards a new solver for the ocean model

The focus in this thesis will be on subgoal (3). The computation of thermohaline ocean flows and the variation of parameters therein require the solution of large linear systems involving the Jacobian matrix. The systems are hard to solve because of their size, which is in the order of a million unknowns, and the complexity of the underlying physics.

For many years the black-box method MRILU [10] (we will describe it in more detail in Chapter 3) was used to solve the large linear systems in THCM. Even though MRILU was able to solve the equations, where most alternative methods simply fail, it had serious drawbacks. The solution of the systems required much memory and computation time, which made it to the bottleneck of the model runs. Due to limitations in the linear system solver, it was impossible to increase the resolution, which is necessary for higher accuracy.

Hence we set the following target for this thesis: to build a new solver dedicated for THCM that

(3a) gives a considerable speed-up of the computations,
(3b) allows for parallelization,
(3c) combines with data-assimilation.

Condition (3b) is important, because if we have a solver than can be parallelized, we can run
the model on one of the many available supercomputers, which not only have an enormous
amount of memory, but also many processors which can work together to solve the problem in
reasonable time.

We add condition (3c) because we want to use the solver and the ocean model in combi-
nation with data-assimilation. This gives some extra constraints on the solver. We will discuss
them in the next chapter as well.

Of the three conditions the first is the most important. If we won’t be able to satisfy con-
dition (3a), the other two are of no interest anymore, which explains why the main part of the
thesis is about fast system solvers.

In general there are two ways to solve large linear systems: via direct or via iterative meth-
ods. Direct methods try to compute a straightforward accurate solution of the equations, while
the iterative methods construct a sequence of approximations for the solution until a certain
desired accuracy has been reached. Direct methods usually require a huge amount of memory
and computing time, while iterative methods are cheaper, but less robust in the sense that in
many cases there is no guarantee that the method converges to the solution in reasonable time.
Our systems are too large to apply direct solvers, but most iterative methods simply fail. In an
overview paper on the history of iterative methods [65] it was suggested that improvements in
the solution of complex partial differential equations, that are hard to solve by iterative meth-
ods, could come from a combination of both iterative and direct techniques. Hence the primary
direction of search was the incorporation of direct techniques in MRILU.

In a large part of the thesis we will not study the ocean, but the saddle point problem, which
also occurs in many other flows. This problem is somewhat simpler than the ocean problem,
but likewise it is hard to solve by iterative methods. The detailed study of the solution of the
saddle point problem helped in the design of a new solver for the ocean problem.

1.4 Outline of the thesis

The outline of the thesis is as follows. Chapter 2 describes the ocean model and the mathematics
for parameter variations and determination of stability. In the final section of that chapter we
will describe the systems that need to be solved and give some characteristics. The saddle point
problem and its origin is described in Section 2.5. Chapter 3 is an introduction to the solution
of large linear systems and the techniques involved in direct and iterative methods.

In the three chapters that follow, we study solution methods for more general large linear
systems and the word ”ocean” will be rare. In Chapter 4 we propose a new direct method for
the solution of saddle point problems. We are able to prove that the method is numerically
stable. It appears to perform better than other existing methods. In Chapter 5 we incorporate
techniques from direct methods in incomplete LU factorizations in the hope to be able to improve MRILU for coupled partial differential equations. We succeeded to combine both worlds; the results for Laplace equations are good, but for the saddle point problem they are disappointing. Chapter 6 deals with solvers for the saddle point problem, that exploit the structure in the matrix. We introduce two alternative solvers with are proven to have some nice mathematical properties. Numerical results show that these alternative solvers perform better than methods from literature.

Then we return to the solution of large linear systems in THCM. In Chapter 7 we propose a new solver for the systems. We will consider the performance of the new approach in Chapter 8 on a number of test cases, amongst which we have a global ocean circulation problem. The thesis finishes with Chapter 9 where we will summarize the results and draw conclusions.