String effective actions, dualities, and generating solutions
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Chapter 1

Introduction

The first part of this thesis deals with the construction of the gauge theory living on a single D-brane and supergravity theories that can arise as low-energy effective descriptions of string theory and M-theory. The latter are thought to be consistent theories of quantum gravity, which unify the four different forces and thus reconcile quantum field theory (QFT) and general relativity (GR). The second part of the thesis is concerned with deriving brane solutions of (super)gravity theories, which have turned out to play an essential role in strengthening our belief in dualities in the non-perturbative limit. To fully appreciate the emergence and the merit of string and M-theory; and the discovery of dualities, we will first sketch the historical development of particle and high-energy physics.

1.1 Historical Remarks and Motivation

The larger part of 20th century theoretical physics has been dominated by two major achievements which both brought about a radical change in physics: quantum mechanics and general relativity.

In the nineteen twenties and thirties quantum mechanics was formulated as the theory that describes the behavior of particles at (sub)-atomic scales, and is therefore the theory to be used if one is dealing with elementary particles. Based on experiments it was noticed that all particles in nature have a fundamental property called spin, the value of which divides them into two categories: bosons and fermions. The fermionic sector contains all matter and consists of three generations, each comprising two quarks and two leptons (an electron and a neutrino). The lightest of these three generations makes up for nearly all known matter.

Between 1905 and 1916 Einstein proposed his theory of relativity. He states that the laws of physics should be the same for all observers and must therefore be formu-
lated in an observer-independent way (covariantly). The theory of relativity consists of two parts: the theory of special relativity, which radically changed our notions of space and time and showed how these concepts are intricately connected, and the theory of general relativity (GR) which describes spacetime itself as a dynamical entity, the metric field. In GR gravity manifests itself through the curvature of spacetime, which in turn is caused by the presence of mass and energy.

A combination of special relativity and quantum mechanics finally led to the standard model (SM) around 1970, which quite successfully describes the interaction between the elementary particles. The standard model is a particular quantum field theory (QFT) of infinitely many possible ones. Here the concept of gauge symmetry plays an important role. By making symmetry transformations local, i.e. introducing coordinate dependent transformation parameters, spin one gauge bosons are introduced that mediate the force between particles. Actually the matter particles mentioned above interact by exchanging bosons: the electromagnetic, weak and strong force are described by the exchange of photons, W/Z intermediate vector bosons and gluons, respectively. The group of SM is \( \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \). The experimental confirmation of the SM is excellent up to \( 10^2 \) GeV, however, some problems remain. Firstly the Higgs sector which is responsible for giving masses to the other fundamental particles, has eluded discovery so far\(^1\). Secondly, there are compelling theoretical arguments to consider possible extensions: first of all the SM contains nineteen fine-tuned parameters\(^2\) that can not be predicted, and hence it is not a fundamental theory. Furthermore, it is difficult to explain the smallness of the Higgs mass (with \( m_H \leq 1 \text{ TeV}/c^2 \)), which goes under the name of the hierarchy problem. Also, the occurrence of three generations of matter particles has not been understood yet. It moreover turns out that the three running coupling constants that are associated with the SM gauge group become approximately equal at the enormously high energy of \( 10^{15} \) GeV. This suggests that at this energy the three forces become unified in a single ‘grand unified theory’ (GUT) based on a simple gauge group. Note that the SM does not contain the fourth fundamental force, gravity, since the strengths of the other three forces are much stronger than gravity.

Let us get back to GR. The experimental and theoretical successes of GR are as impressive as those of SM. For example, GR accounts for the bending of light by massive objects like our sun. GR also predicts the occurrence of spacetime singularities inside black holes\(^3\). It also plays a pivotal role in contemporary cosmology, where it explains for instance the observed cosmological redshift of the light of distant galaxies as a consequence of the expansion of the universe. So far GR is used as a classical field theory. An attempt to describe gravity by using similar quantization techniques

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\(^1\)This is one of the primary goals of the new LHC accelerator of CERN, which raises the experimental scale up to \( \sim 10^4 \) GeV.

\(^2\)For example the parameters which correspond to masses of elementary particles.

\(^3\)Black holes are objects that are so massive that they are hidden behind an event horizon, a surface from which even light can not escape (at least classically).
1.1 Historical Remarks and Motivation

as used for the SM failed. The theory suffers from infinities, which, contrary to SM (‘t Hooft and Veltman [1]), cannot be controlled. This can be seen from the fact that the gravitational coupling constant $\kappa = \frac{8\pi G}{c^4}$ is not dimensionless, and it is therefore unsuitable to be used for performing perturbative expansions. The scale at which quantum gravity becomes important is the Planck scale given by

$$l_{\text{Planck}} = \sqrt{\frac{8\pi G\hbar}{c^3}} \approx 4.1 \times 10^{-35} \text{ m}, \quad M_{\text{Planck}} = \left(\frac{h c^5}{8\pi G}\right)^{1/2} \approx 10^{18} \text{ GeV}, \quad (1.1.1)$$

with $h$ Planck’s constant. As one can see the Planck scale is very close to the GUT scale (10^{15} \text{ GeV}). This observation shows the need for a theory of “quantum gravity” that can handle all four fundamental forces simultaneously.

As a first attempt, physicists were thinking about a theoretical improvement of SM by introducing a different type of symmetry, called supersymmetry. This is a symmetry between bosons and fermions that predicts that for every boson in nature there exists a corresponding fermionic partner, and vice versa. The first motivation for using such a symmetry was to avoid the hierarchy problem; it has been shown that the Higgs mass is protected from quantum corrections by supersymmetry. However, supersymmetry transformations also introduce many new particles–sparticles–which have not been observed\(^4\). A partial success in unifying all fundamental forces was reached in 1976 by considering theories based on local supersymmetry. Such theories are called supergravities, extensions of GR theory that behaved better at high energies, namely the infinities partially canceled. The spin 2 gauge boson responsible for mediating the gravitational force is called the graviton. Its supersymmetric partner is the gravitino\(^5\).

String theory is the most promising proposal that can deal with quantum gravity. String theory replaces particles by the oscillation modes of relativistic strings\(^6\). Remarkably, the graviton and (non)abelian gauge fields are necessarily part of the spectrum. Thus string theory naturally unifies the gravitational interaction with Yang-Mills theory (nonabelian version of Maxwell theory). In addition, string theory provides a discrete but infinite tower of massive vibration modes. Their mass scale is of the order of the Planck mass. In supersymmetric versions of string theory (superstring theories), the graviton is at the massless level accompanied by the supergravity field content. Indeed, it was found that the low-energy limit of superstring theory is given by supergravity. There is an intuitive reason why superstring theory is free of infinities. These infinities usually appear at singular points, but a string moves in a spacetime tracing out a two dimensional surface. This fact exactly causes the

\(^4\)If supersymmetry exists, it must therefore be spontaneously broken, yielding super-particles of higher mass. It is strongly hoped that these will be discovered at LHC.\(^5\)In order to measure these particles energies would be needed that are way out of the range of our present accelerators.\(^6\)Note that string has a typical length $l_s$ of the order of the Planck length $l_p$. 
interactions not to occur at one single point, but to be spread out over a small area. It turns out that the perturbative string interactions are UV finite.

String theory needs besides supersymmetry six extra dimensions to be set up consistently. One can take this as a virtue rather than a vice. It has been known for a long time that higher-dimensional theories have a number of attractive features. In the 1920’s, Kaluza and Klein [2,3] tried to unify Einstein and Maxwell theories by embedding four-dimensional gravity and electromagnetism in five-dimensional space-time. By the same token, in string theory, we take the internal six dimensions to be very small and therefore invisible to the present-day experiments. This procedure is called Kaluza-Klein dimensional reduction.

Unfortunately, string theory has also its disadvantages. It is only defined perturbatively, namely scattering amplitudes are expressed as an infinite expansion in powers of the string coupling $g_s$, associated with the Feynman-diagrams of string theory. The main setback however became apparent where there seemed to be five different superstring theories, whereas we hoped to obtain one unique theory of quantum gravity. This means that perturbative string theories only provide part of the whole picture.

Fortunately, a lot of progress has been made on this point. The major step forward was the discovery of dualities, that are symmetry transformations that link the different string theories. They relate in some cases weak coupling regime to strong coupling regime so that perturbative calculations in the first theory provide non-perturbative information on the second theory (called S-duality). In addition, string theories on different backgrounds were found to be equivalent (called T-duality). An important role was played by the so-called “brane” solutions of string theory. These are solitonic objects that can be seen as higher-dimensional generalizations of strings. An important class of branes are Dirichlet branes, D-branes. These are special, since they arise on one hand as hyperplanes on which strings can end, and on the other hand as stationary solutions of (super)gravity theories. There is another class of brane solutions, S-branes (spacelike branes) which are time-dependent solutions of (super)gravities. The five apparently distinct theories and their brane solutions are related by a web of dualities. During the 1990’s, it became gradually understood that these five theories all represented different limits in the parameter space of a single eleven-dimensional underlying theory, called M-theory. The fundamental degrees of freedom of M-theory remain largely unknown. Rather than being a completed theory, M-theory remains very much work in progress.

Thus we have gained some insights and had a better understanding of perturbative and non-perturbative string theory. However, there are still many interesting open issues. First there is the lack of experimental evidence. Indeed, despite all of the

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7 There is no need for introducing an ultraviolet cut-off and the theory is consistent up to high energy scales and hence is fundamental.

8 Branes can also be considered as higher-dimensional generalizations of black holes.
promises string theory does not make a single hard verifiable prediction. Neither do rival theories of quantum gravity. It is possible to construct configurations in string theory that resemble to a high extent the SM, for instance by using intersecting D-branes. However, as of yet there is no way to single out these models as a preferred vacuum. In addition, since string theory leans heavily on supersymmetry, and supersymmetry is shared with many other theories, most notably the supersymmetric SM, the experimental discovery of supersymmetry would hardly be a full confirmation of string theory. Because of the extremely high energies involved, perhaps the future of experimental verification lies not in particle accelerators but in astrophysical and cosmological developments. Note that string theory has already passed an important test in partially solving a problem that arises when describing a typical general relativistic object, a black hole, in a quantum mechanical way: it succeeds in computing the semiclassically predicted entropy of a supersymmetric black hole by counting its microstates. Unfortunately, many tough nuts remain to be cracked in these domains such as the explanation of the observed small positive cosmological constant and the construction of string theory in time-dependent backgrounds (e.g. S-brane solutions).

However, the discussion so far left out that string theory is sometimes an incredibly powerful tool in other fields of physics and mathematics. In this small space we can only give a few examples. Most successfully there is the connection with gauge theories. It turns out that many properties of gauge theory have a geometric interpretation in terms of D-branes. Some time ago ’t Hooft argued that the large N limit of gauge theories [4] very much looked like a string theory. A first concrete realization of such a connection was the AdS/CFT duality\(^9\). Other examples are the incorporation of Montonen-Olive duality [6] of gauge theory in the larger S-duality of string theory and the recent advances in the non-perturbative calculation of the chiral sector of \(N = 1\) Super-Yang-Mills theory [7]. Nonetheless, many of these links are not established in a strict mathematical sense. Indeed, for instance the AdS/CFT correspondence and S-duality are in fact conjectures, but meanwhile an impressive amount of indirect evidence has been found.

The low-energy limit (field theory limit) of string theory remains an important tool to study the different phenomena in string theory. Many features of string and M-theory are also present in its low-energy limit, such as D-branes and dualities, and therefore it is interesting to study this effective description.

In this thesis we will study first the low-energy limit of string theory. In particular we will show how the strings manifest themselves as a gauge theory that lives on the D-brane. We will see how the corrections to the leading order of the Maxwell action provide interesting information about the ‘stringy’ aspects of D-brane physics. We will then try to constrain these corrections, using the electromagnetic duality symmetry. Also supergravity actions will be presented in this thesis as the low energy

\(^9\)This correspondence states that \(N = 4\) Super-Yang-Mills theory is dual to string theory on \(AdS^5 \times S^5\) [5].
Introduction

tree-level effective action of string theory for slowly varying curvature. Derivative corrections, in particular corrections of order \( \alpha' \) to heterotic string\(^{10} \), will be studied.

Back to brane solutions. The second goal of this thesis is to study branes that are solutions of (super)gravity theories. As we will see, the dimensions of the extended object form the worldvolume of the brane. The remaining spacetime dimensions form the transverse space. We distinguish between two types of branes: if time is part of the worldvolume the brane is called “timelike” \( p \)-brane. Here the \( p \) stands for the number of spatial worldvolume directions. The total number of dimensions of the worldvolume is \( p + 1 \). If time is not included in the worldvolume the brane is called “spacelike” \( S_p \)-brane. For such a brane the total number of dimensions is \( p + 1 \) which are all spatial. Thus in both cases \( p \) refers to a \( p + 1 \)-dimensional worldvolume.

Investigating brane solutions by directly solving the equations of motion that follow from the (super)gravity action is highly non-trivial. Instead we are going to look at brane solutions whose dynamics depend only on one parameter (particle-like solution). We will see that this parameter is one of the coordinates of the transverse space. This means that the worldvolume coordinates will not explicitly enter in the solutions. This implies that one can effectively dimensionally reduce the solution over the worldvolume\(^{11} \). This maps a \( p \)-brane to a \( (-1) \)-brane solution. If we reduce over an Euclidean torus, the resulting lower-dimensional theory is a Minkowskian theory and the corresponding solution is a \( S(-1) \)-brane. If the reduction is over a Minkowskian torus (having a timelike direction), the lower-dimensional theory lives in an Euclidean spacetime, and has a \( (-1) \)-brane (instanton) as a solution\(^{12} \).

The number of global symmetries becomes larger and larger as one goes down in dimensions. This can be used to simplify further our quest for brane solutions. The \( (-1) \)-brane solutions of the lower-dimensional theory are carried by the metric and the scalar fields. We will show that one can decouple the gravity field equations from the scalar field equations. As a result, one can solve for the metric and the scalar fields independently. The solution-generating technique will enable us to find the most general scalar field solutions.

\(^{10}\)Heterotic string is one of the five perturbative superstring theories mentioned before.

\(^{11}\)The reduction over the worldvolume of the brane gives rise to a massless lower dimensional theory, while the reduction over the transverse directions of the brane will generate a scalar potential in the lower dimensional theory. If the lower dimensional massive theory lives in a Minkowski spacetime, one then has two distinct solutions: time-dependent solutions (cosmology) and time-independent solutions (domain-walls).

\(^{12}\)In this analysis we only consider consistent reductions. This means that one can always undo the steps of the reduction in such a way that we are guaranteed that we also have a solution of the action we started with. Thus one might construct a higher-dimensional solution via uplifting (oxidation) a lower-dimensional solution.
1.2 Outline

We have structured the material into several chapters:

Chapter 2 opens with an overview of string theory. It glances over perturbative string theories first and then turns to non-perturbative effects with an emphasis on D-branes. The last part of the chapter introduces the low-energy effective action of a single D-brane (abelian). It gives an overview of past attempts and successes in constructing this effective action.

Chapter 3 can be considered as a natural extension of chapter two. It starts with an introduction of supergravity theory, outlining various approaches that have been pursued to construct the corresponding low-energy effective action and its derivative corrections. The second part summarizes paper [C], where the effective action of the heterotic string to order $\alpha'$ has been analyzed, establishing that the supersymmetric $R^2$ effective action, computed from the supersymmetrization of the Lorentz Chern-Simons term, is equivalent modulo field redefinitions to heterotic string effective actions obtained by different methods.

Chapter 4 treats the approach of [B] to constrain the derivative corrections of the 4-dimensional abelian Born-Infeld action from the requirement that those terms together with the Born-Infeld action should admit electromagnetic duality symmetry. In the rest of the chapter we review the properties of interacting field theories which are invariant under electromagnetic duality rotations (selfduality) which transform a vector field strength into its dual. The focus will be on introducing the relevant ingredients one needs for formulating a theory as a nonlinear realization of the duality group. We show that the invariance of the equations of motion requires that the Lagrangian changes in a particular way under duality. We use this property in the general construction of the supergravity Lagrangian.

In chapter 5 the concept of nonlinear $\sigma$-models is introduced. We exhibit how such models arise in Kaluza-Klein theories and extended supergravities. We restrict to reductions over tori that are relevant for studying brane solutions in (super)gravity.

Chapter 6 starts with reviewing brane solutions that appear in (super)gravity theories. In particular we spend some time on $p$- and $Sp$-brane solutions. Then we illustrate the power of nonlinear sigma-model techniques in finding brane solutions in a purely algebraic way. We look for time-dependent $Sp$-branes via reducing over their worldvolumes. By means of the solution-generating technique and the coset symmetry we will be able to construct the most general $S$-brane vacuum solution (pure Einstein-gravity solution) with deformed worldvolume. We also consider solutions for an Euclidean theory, i.e. instantons. We show that those solutions can be obtained from timelike dimensional reductions of ordinary Lorentzian (super)gravities. The focus will mainly be on finding the generating Euclidean brane solutions that can be seen as a generating geodesic on non-Riemannian moduli spaces.

Note that all papers that I co-authored are indicated by capital letters in contrast
to all other referred articles which are labeled by numbers.