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Published in:
Journal of Mathematical Physics

DOI:
10.1063/1.2989145

Loss and recovery of Gibbsianness for *XY* models in external fields

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(Received 3 June 2008; accepted 31 August 2008; published online 11 December 2008)

We consider planar rotors (*XY* spins) in $\mathbb{Z}^d$, starting from an initial Gibbs measure and evolving with infinite-temperature stochastic (diffusive) dynamics. At intermediate times, if the system starts at low temperature, Gibbsianness can be lost. Due to the influence of the external initial field, Gibbsianness can be recovered after large finite times. We prove some results supporting this picture. © *2008 American Institute of Physics.* [DOI: 10.1063/1.2989145]

I. INTRODUCTION

Time evolution of spin systems with different initial Gibbs measures and different dynamics shows various interesting features. In particular, in the transient regime, the structure of the evolved measure can have various properties, which may change in time. For example, in Refs. 7, 9, 14, 15, and 3 the question was investigated whether the time-evolved measure is Gibbsian or not. Results about conservation, loss, and recovery of the Gibbs property could be obtained. Ising spin systems were considered in Ref. 7 and different types of unbounded spin systems in Refs. 3 and 15. In Refs. 9 and 14 compact continuous spin systems are investigated. In more physical terms, the question is whether one can or cannot associate an effective temperature (=inverse interaction norm) with the system when it is in this nonequilibrium situation.[19]

Variations in both the initial and the dynamical temperature [the temperature of the Gibbs measure(s) to which the system will converge, which is a property of the dynamics] have influence on the existence (or the absence) of the quasilocality property of the time-evolved measure of the system. This quasilocality property is a necessary (and almost sufficient) condition to have Gibbsianness.[8,13,16]

In Ref. 9 we showed that the time-evolved measure for planar rotors stays Gibbsian for either short times, starting at arbitrary temperature and with arbitrary-temperature dynamics, or for high- or infinite-temperature dynamics starting from a high- or infinite-temperature initial measure for all times. Furthermore the absence of the quasilocality property is shown for intermediate times for systems starting in a low-temperature regime with zero external field and evolving under infinite-temperature dynamics. The fact that there exist intermediate times where Gibbsianness is lost for *XY* spins even in two dimensions is remarkable because those systems do not have a first-order phase transition due to the Mermin–Wagner theorem. However, it turns out that conditionings can induce one. To establish the occurrence of such conditional first-order transitions is a major step in the proof that a certain measure is not Gibbsian. Similar short-time results for more general compact spins can be found in Ref. 14.

These results about compact continuous spins can be seen as intermediate between those for discrete Ising spins and the results for unbounded continuous spins. Conservation, loss and recovery results can be found in Ref. 7 for Ising spins and conservation for short times and loss for

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larger times for unbounded spins in Ref. 15. Conservation for short times for more general dynamics (e.g., Kawasaki) for discrete spins was proven in Ref. 17 and for unbounded spins with bounded interactions in Ref. 3.

This paper is a continuation of Ref. 9. As in that paper, we consider *XY*-spins living on a lattice sites on $\mathbb{Z}^d$ and evolving with time. The initial Gibbs measure is a nearest neighbor ferromagnet, but now in a positive external field. So we start in the regime where there is a unique Gibbs measure. The system is evolving under infinite-temperature dynamics. We expect, that just as in the Ising case, whatever the initial field strength, we have after the short times when the measure is always Gibbsian, if the initial temperature is low, that a transition toward a non-Gibbsian regime occurs, and that after another, longer time, the measure becomes Gibbs again. We can prove a couple of results which go some way in confirming this picture.

We prove that when the initial field is small, and d is at least 3, there exists a time interval, depending on the initial field, during which the time-evolved measure is non-Gibbsian. We present a partial result, indicating why we expect the same phenomenon to happen in two dimensions. Furthermore, we argue that the presence of an external field is responsible for the re-entrance into the Gibbsian regime for larger times, independently of the initial temperature. We can prove this for the situation in which the original field is strong enough.

II. FRAMEWORK AND RESULT

Let us introduce some definitions and notations. The state space of one continuous spin is the circle, $\mathbb{S}^1$. We identify the circle with the interval $[0,2\pi)$, where 0 and 2π are considered to be the same points. Thus the configuration space Ω of all spins is isomorphic to $[0,2\pi)^{\mathbb{Z}^d}$. We endow Ω with the product topology and natural product probability measure $d\nu_0(x)=\otimes_{i\in\mathbb{Z}^d}d\nu_0(x_i)$. In our case we take $d\nu_0(x_i)=(1/2\pi)dx_i$. An interaction φ is a collection of $\mathcal{F}_\Lambda$-measurable functions φ_Λ from $([0,2\pi))^\Lambda$ to $\mathbb{R}$, where $\Lambda\subset\mathbb{Z}^d$ is finite. $\mathcal{F}_\Lambda$ is the σ-algebra generated by the canonical projection on $[0,2\pi)^\Lambda$.

The interaction φ is said to be of finite range if there exists a $r>0$ such that $\mathrm{diam}(\Lambda)>r$ implies $\varphi_\Lambda\equiv 0$ and it is called absolutely summable if for all i, $\Sigma_{\Lambda\ni i}\|\varphi_\Lambda\|_\infty<\infty$.

We call ν a Gibbs measure associated with a reference measure ν_0 and interaction φ if the series $H_\Lambda^\varphi=\Sigma_{\Lambda'\cap\Lambda\neq\varnothing}\varphi_{\Lambda'}$ converges (φ is absolutely summable) and ν satisfies the Dobrushin-Lanford-Ruelle (DLR) equations for all i:

$$d\nu_\beta(x_i|x_j,j\neq i)=\frac{1}{Z_i}\exp(-\beta H_i^\varphi(x))d\nu_0(x_i), \tag{1}$$

where $Z_i=\int_0^{2\pi}\exp(-\beta H_i^\varphi(x))d\nu_0(x)$ is the partition function and β proportional to the inverse temperature. The set of all Gibbs measures associated with φ and ν_0 is denoted by $\mathcal{G}(\beta,\varphi,\nu_0)$.

Now, instead of working with Gibbs measures on $[0,2\pi)^{\mathbb{Z}^d}$ we will first investigate Gibbs measures as space-time measures Q^{ν_β} on the path space $\tilde{\Omega}=C(\mathbb{R}_+,[0,2\pi))^{\mathbb{Z}^d}$. In Ref. 4 Deuschel introduced and described infinite-dimensional diffusions as Gibbs measures on the path space $C([0,1])^{\mathbb{Z}^d}$ when the initial distribution is Gibbsian. This approach was later generalized in Ref. 2 which showed that there exists a one-to-one correspondence between the set of initial Gibbs measures and the set of path-space measures Q^{ν_β}.

We consider the process $X=(X_i(t))_{t\geq 0,i\in\mathbb{Z}^d}$ defined by the following system of stochastic differential equations:

$$\begin{aligned} dX_i(t) &= dB_i^{\odot}(t), \quad i\in\mathbb{Z}^d, t>0,\\ X(0) &\sim \nu_\beta, \quad t=0, \end{aligned} \tag{2}$$

for $\nu_\beta\in\mathcal{G}(\beta,\tilde{\varphi},\nu_0)$ and the initial interaction $\tilde{\varphi}$ given by

$$\tilde{\varphi}_\Lambda(x) = -J \sum_{i,j\in\Lambda: i\sim j} \cos(x_i - x_j) - h\sum_{i\in\Lambda} \cos(x_i), \tag{3}$$

J,h some non-negative constants and $d\nu_0(x)=(1/2\pi)dx$. $\tilde{H}$ denotes the initial Hamiltonian associated with $\tilde{\varphi}$ and $(B_i^{\odot}(t))_{i,t}$ is independent Brownian motion moving on a circle with transition kernel given (via the Poisson summation formula)

$$p_t^{\odot}(x_i,y_i) = 1 + 2\cdot \sum_{n\geq 1} e^{-n^2 t}\cos(n\cdot(x_i - y_i))$$

for each $i\in\mathbb{Z}^d$, just as we used in Ref. 9. Note also that the eigenvalues of the Laplacian on the circle, which is the generator of the process, are given by $\{n^2, n\geq 1\}$, see also Ref. 20. We remark that the normalization factor $1/2\pi$ is absorbed into the single-site measure ν_0.

Obviously $\tilde{\varphi}$ is of finite range and absolutely summable, so the associated measure ν_β given by (1) is Gibbs.

For the failure of Gibbsianness we will use the necessary and sufficient condition of finding a point of essential discontinuity of (every version of) the conditional probabilities of ν_β, i.e., a so-called bad configuration. It is defined as follows.

Definition 2.1: *A configuration ζ is called bad for a probability measure μ if there exists an $\varepsilon>0$ and $i\in\mathbb{Z}^d$ such that for all Λ there exists $\Gamma\supset\Lambda$ and configurations ξ, η such that*

$$|\mu_\Gamma(X_i|\zeta_{\Lambda\setminus\{i\}}\eta_{\Gamma\setminus\Lambda}) - \mu_\Gamma(X_i|\zeta_{\Lambda\setminus\{i\}}\xi_{\Gamma\setminus\Lambda})| > \varepsilon. \tag{4}$$

The measure at time t can be viewed as the restriction of the two-layer system, considered at times 0 and t simultaneously, to the second layer. In order to prove Gibbsianness or non-Gibbsianness we need to study the joint Hamiltonian for a fixed value y at time t.

The time-evolved measure is Gibbsian if for every fixed configuration y the joint measure has no phase transition in a strong sense (e.g., via Dobrushin uniqueness or via cluster expansion/analyticity arguments). In that case, an absolutely summable interaction can be found for which the evolved measure is a Gibbs measure. On the other side the measure is non-Gibbsian if there exists a configuration y which induces a phase transition for the conditioned double-layer measure at time 0 which can be detected via the choice of boundary conditions. In that case no such interaction can be found, see, for example, Ref. 10.

The results we want to prove are the following.

Theorem 2.1: *Let Q^{ν_β} be the law of the solution X of the planar rotor system (2) in $\mathbb{Z}^d$, $\nu_\beta \in\mathcal{G}(\beta,\tilde{\varphi},\nu_0)$ and $\tilde{\varphi}$ given by (3), with β the inverse temperature, J some non-negative constant and $h>0$ the external field, and d at least 3. Then, for β large enough and h small enough, there is a time interval $(t_0(h,\beta),t_1(h,\beta))$ such that for all $t_0(h,\beta)<t<t_1(h,\beta)$ the time-evolved measure $\nu_\beta^t=Q^{\nu_\beta}\circ X(t)^{-1}$ is not Gibbs, i.e., there exists no absolute summable interaction φ^t such that $\nu_\beta^t\in\mathcal{G}(\beta,\varphi^t,\nu_0)$.*

Theorem 2.2: *For any h chosen such that βh is large enough, compared to β, there exists a time $t_2(h)$, such that for all $t\geq t_2(h)$ the time-evolved measure is Gibbs, $\nu_\beta^t\in\mathcal{G}(\beta,\varphi^t,\nu_0)$.*

Proof of Theorem 2.1: We consider the double-layer system, describing the system at times 0 and t. We can rewrite the transition kernel in Hamiltonian form, and we will call the Hamiltonian for the two-layer system the dynamical Hamiltonian (as it contains the dynamical kernel). It is formally given by

$$-\mathbf{H}_\beta^t(x,y) = -\beta\tilde{H}(x) + \sum_{i\in\mathbb{Z}^2}\log(p_t^{\odot}(x_i,y_i)),$$

where $x,y\in[0,2\pi)^{\mathbb{Z}^d}$, $p_t^{\odot}(x_i,y_i)$ is the transition kernel on the circle and $\tilde{H}(x)$ is formally given by

$$-\tilde{H}(x) = J\sum_{i\sim k}\cos(x_i - x_k) + h\sum_i \cos(x_i).$$

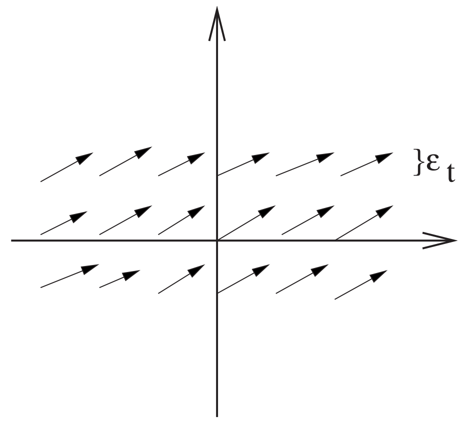

FIG. 1. Rightpointing ground state, direction $(\pi/2-\varepsilon_t)_i$.

First we want to prove that there exists a time interval where Gibbsianness is lost. For this we have to find a *bad configuration* such that the conditioned double-layer system has a phase transition at time 0, which implies (4) for the time-evolved measure. We expect this to be possible for each strength of the external field and in each dimension at least 2. At present we can perform the program only for weak fields and for dimension at least 3. We also show a partial result, at least indicating how a conditioning also in $d=2$ can induce a phase transition.

Thus, given $h>0$, we immediately see that the spins from the initial system prefer to follow the field and point upward (take the value $x_i=0$ at each site i). To compensate for that, we will condition the system on the configuration where all spins point downward (at time t), i.e., $y^{\text{spec}} \coloneqq (\pi)_{i\in\mathbb{Z}^d}$. Thus the spin configuration in which all spins point in the direction opposite to the initial field will be our bad configuration. We expect that then the minimal configuration of $-\mathbf{H}^t_\beta(x,y^{\text{spec}})$, so the ground states of the conditioned system at time 0 will need to compromise between the original field and the dynamical (conditioning) term. In the ground state(s) either all spins will point to the right (see Fig. 1), possibly with a small correction ε_t, $(\pi/2-\varepsilon_t)_{i\in\mathbb{Z}^2}$, or to the left (see Fig. 2), $(3\pi/2+\varepsilon_t)_{i\in\mathbb{Z}^2}$, also with a small correction. ε_t is a function depending on t. Finally these two symmetry-related ground states will yield a phase transition of the "spin-flop" type, also at low temperatures. It is important to observe that for this intuition to work, it is essential that the rotation symmetry of the zero-field situation will not be restored due to the appearance of higher-order terms from the expansion of the transition kernel, as we will indicate below.

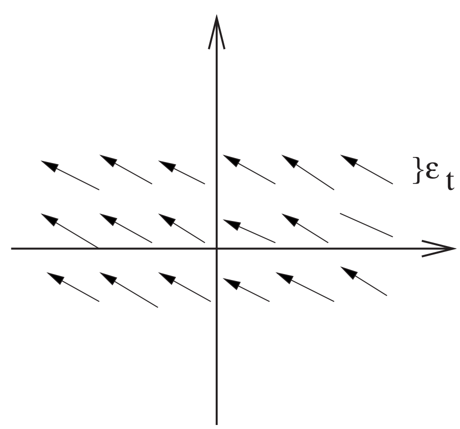

FIG. 2. Leftpointing ground state, direction $(3\pi/2+\varepsilon_t)_i$.

We perform a little analysis for the logarithm of the transition kernel $p_t^{\odot}$. Let $y^{\text{spec}} := (\pi)_{i \in \mathbb{Z}^d}$. We want to focus on the first three terms coming from the expansion of the logarithm.

$$\log\left(1 + 2\sum_{n\geq 1} e^{-n^2 t}\cos(n(x_i - \pi))\right) = -2e^{-t}\cos(x_i) - 2e^{-2t}\cos^2(x_i) - \frac{8}{3}e^{-3t}\cos^3(x_i) + R_t(x_i),$$

where

$$R_t(x_i) := \left[\sum_{n\geq 1}\frac{(-1)^{n+1}}{n}\left(2\sum_{k\geq 1} e^{-k^2 t}\cos(k(x_i - \pi))\right)^n\right]\mathbb{1}_{\{n\neq 1,2,3\}\cup\{k\neq 1\}}$$

is of order $\mathcal{O}_i(e^{-4t})$, for details, see Appendix. We define $h_t = e^{-t}$. Note that given βh, there is a time interval where the effect of the initial field is essentially compensated by the field induced by the dynamics (containing the h_t). For large times the initial field term dominates all the others and the system is expected to exhibit a ground (or Gibbs) state following this field. For intermediate times the other terms are important, too. If we consider a small initial field, it is enough to consider the second and third order terms which we indicated above. Those terms create, however, the discrete left-right symmetry for the ground states which will now prefer to point either to the right or to the left.

For the moment we forget about the rest term $R_t(x_i)$ and investigate the restricted Hamiltonian $-\mathbf{H}^t_{\text{res3}}(x, y^{\text{spec}})$ which is formally equal to

$$\beta J\sum_{i\sim k}\cos(x_i - x_k) + \beta h\sum_i \cos(x_i) + \sum_i\left(-2h_t\cos(x_i) - 2h_t^2\cos^2(x_i) - \frac{8}{3}h_t^3\cos^3(x_i)\right). \tag{5}$$

To be more precise, the external field including the inverse temperature βh will be chosen small enough, and then the inverse temperature β large enough. We want first to find the ground states of the restricted Hamiltonian $\mathbf{H}^t_{\text{res3}}(x, y^{\text{spec}})$ which are points $x = (x_i)_{i\in\mathbb{Z}^d}$. It is fairly immediate to see that in the ground states all spins point in the same direction, so we then only need to minimize the single-site energy terms. The first-order term more or less compensates the external field, and the second-order term is maximal when $\cos^2(x_i)$ is minimal, thus when one has the value $\pi/2$ or $3\pi/2$. The higher-order terms will only minimally change this picture.

We can define a function ε_t depending on t such that asymptotically $\beta h = h_t + \varepsilon_t$ yields the following unique maxima $(\pi/2 - \varepsilon_t, \pi/2 - \varepsilon_t)$ and $(3\pi/2 + \varepsilon_t, 3\pi/2 + \varepsilon_t)$. The function ε_t is a correction of the ground states pointing to the left or right. We present a schematic illustration of the two ground states,

$$(3\pi/2 + \varepsilon_t)_i \quad (\pi/2 - \varepsilon_t)_i.$$

Hence for every arbitrarily chosen small external field h, we find a time interval depending on h, such that we obtain two reflection symmetric ground states of all spins pointing either (almost) to the right $(\pi/2 - \varepsilon_t)_{i\in\mathbb{Z}^d}$ or all spins pointing (almost) to the left $(3\pi/2 + \varepsilon_t)_{i\in\mathbb{Z}^d}$. The rest term $R_t(x_i)$ does not change this behavior since it is suppressed by the first terms and is of order $\mathcal{O}_i(e^{-4t})$. Moreover, it respects the left-right symmetry.

We will first, as a partial argument, show that the interaction

$$J\sum_{i\sim k}\cos(x_i - x_k) + h\sum_i \cos(x_i) + \sum_i\left(-2h_t\cos(x_i) - 2h_t^2\cos^2(x_i) - \frac{8}{3}h_t^3\cos^3(x_i)\right) \tag{6}$$

has a low-temperature transition in $d \geq 2$.

To show this we notice that we are in a similar situation as in Ref. 9. The conditioning of the double-layer system for the XY spins created left-right symmetric ground states.

Now we want to apply a percolation argument for low-energy clusters to prove that such that spontaneous symmetry breaking occurs. The arguments follow essentially Ref. 9 and are based on

Ref. 12. The potential corresponding to the Hamiltonian (5) is clearly a C-potential, that is, a potential which is nonzero only on subsets of the unit cube.[12] It is of finite range, translation invariant, and symmetric under reflections.

Including the rest term (which is a translation-invariant single-site term) does not change this. *A fortiori* the associated measure is reflection positive and we can again use the same arguments as in Ref. 9 to deduce that for β large enough, there is long-range order. This argument indicates how conditioning might induce a phase transition.

However, to get back to our original problem, that is, to prove the non-Gibbsianness of the evolved state, we need an argument which holds for values of not only of h but also of βh which are small uniformly in temperature. Then only we can deduce that there exists a time interval $(t_0(\beta,h),t_1(\beta,h))$ such that $|\mathcal{G}_\beta(\mathbf{H}^t_\beta(\cdot,y^{\text{spec}}),\nu_0)|\geq 2$.

To obtain this, for $d=3$, we can invoke a proof using infrared bounds (see e.g., Refs. 11, 13, and 1). Note that the infrared bound proof, although primarily developed for proving continuous symmetry breaking, also applies to models with discrete symmetry breaking as we have here. In fact, we may include the rest term without any problem here, as the symmetry properties of the complete dynamical Hamiltonian are the same as that of our restricted one, and adding single-site terms does not spoil the reflection positivity. From this an initial temperature interval is established, where Gibbsianness is lost after appropriate times.

Indeed, the infrared bound provides a lower bound on the two-point function which holds uniformly in the single-site measure (which in our case varies only slightly anyway, as long as the field and the compensating term due to the kernel are small enough). This shows that a phase transition occurs at sufficiently low temperatures, as for decreasing temperatures the periodic boundary condition state converges to the symmetric mixture of the right- and left-pointing ground-state configurations.

Comment: One might expect that, by judiciously looking for other points of discontinuity, the time interval of proven non-Gibbsianness might be extended, hopefully also to $d=2$; however, qualitatively this does not change the picture. In fact, there are various configurations where one might expect that conditioning on them will induce a first-order transition. For example, the *XY* model in at least two dimensions in a weak random field which is plus or minus with equal probability is expected to have such transitions.[21] The same situation should occur for various appropriately chosen (in particular, random) choices of configuration where spins point only up or down. In a somewhat similar vein, if the original field is not so weak, and thus also higher terms are non-negligible, we expect that qualitatively not much changes, and there will again be an intermediate-time regime of non-Gibbsianness at sufficiently low temperatures.

About the proof of Theorem 2.2: Let us now turn to the second statement. Here the initial temperature does not affect the argument. The intuitive idea, as mentioned before, is as follows: After a long time, the term due to the conditioning becomes much weaker than the initial external field—however, weak it is—uniformly in the conditioning, and thus the system should behave in the same way as a plane rotor in a homogeneous external field and have no phase transition. However, the higher-order terms which were helpful for proving the non-Gibbsianness now prevent us using the ferromagneticity of the interaction. Indeed, we cannot use correlation inequalities of Fortuin-Kasteleijn-Ginibre (FKG) type, and we will have to try analyticity methods.

In fact, we expect that the statement should be true for each strength of the initial field. Indeed, once the time is large enough, the dynamical single-site term should be dominated by the initial field, and, just as in that case, one should have no phase transition.[5,6,18] However, to conclude that we can consider the dynamical single-site term as a small perturbation, in which the free energy and the Gibbs measure are analytic, although eminently plausible, does not seem to follow from Dunlop's Yang–Lee theorem.

For high fields, we can either invoke cluster expansion techniques, showing that the system is completely analytic, or Dobrushin uniqueness statements. Precisely such claims were developed for proving Gibbsianness of evolved measures at short times in Ref. 9 and in Ref. 14. A direct application of those proofs also provides our theorem, which is for long times.

III. CONCLUSION

In this paper we extended the results from Ref. 9 and show some results on loss and recovery of Gibbsianness for XY spin systems in an external field. Giving a low-temperature initial Gibbs measure in a weak field and evolving with infinite-temperature dynamics, we find a time interval where Gibbsianness is lost. Moreover at large times and strong initial fields, the evolved measure is a Gibbs measure, independently of the initial temperature.

Generalizations are possible to include, for example, more general finite-range translation-invariant ferromagnetic interactions $\tilde{\varphi}$. We conjecture, but at this point cannot prove, that both the loss and recovery statements actually hold for arbitrary strengths of the initial field.

ACKNOWLEDGMENTS

We thank Christof Külske, Alex Opoku, Roberto Fernández, Cristian Spitoni, and especially Frank Redig for helpful discussions. We thank Roberto Fernández for a careful reading of the manuscript. We thank Francois Dunlop for a useful correspondence.

APPENDIX

The logarithm of the transition kernel is given by

$$\log\left(1+2\sum_{n\geq 1} e^{-n^2t}\cos(n(x_i-\pi))\right)=\sum_{k\geq 1}\frac{(-1)^{k+1}}{k}\left(2\sum_{n\geq 1} e^{-n^2t}\cos(n(x_i-\pi))\right)^k. \tag{A1}$$

Since the first term of the series of $p_t^{\odot}$ is dominating we can write

$$2\sum_{n\geq 1} e^{-n^2t}\cos(n(x_i-\pi))=-2e^{-t}\cos(x_i)+\mathrm{Rest}_t(x_i).$$

The rest term $\mathrm{Rest}_t(x_i)$ is smaller than $2e^{-4t}$ uniformly in x_i. Then we can bound

$$2\sum_{n\geq 1} e^{-n^2t}\cos(n(x_i-\pi))\leq -2e^{-t}\cos(x_i)+2e^{-4t}.$$

Furthermore we write (A1) as

$$(-2e^{-t}\cos(x_i)+\mathcal{O}(e^{-4t}))-\frac{1}{2}(-2e^{-t}\cos(x_i)+\mathcal{O}(e^{-4t}))^2+\frac{1}{3}(-2e^{-t}\cos(x_i)+\mathcal{O}(e^{-4t}))^3$$
$$+\sum_{k\geq 4}\frac{(-1)^{k+1}}{k}(-2e^{-t}\cos(x_i)+\mathcal{O}(e^{-4t}))^k$$

and afterwards bound it by

$$-2e^{-t}\cos(x_i)+\mathcal{O}(e^{-4t})-2e^{-2t}\cos^2(x_i)+\mathcal{O}(e^{-5t})-\tfrac{8}{3}e^{-3t}\cos^3(x_i)+\mathcal{O}(e^{-6t})+\mathcal{O}(e^{-4t}),$$

thus (A1) is then bounded by

$$-2e^{-t}\cos(x_i)-2e^{-2t}\cos^2(x_i)-\tfrac{8}{3}e^{-3t}\cos^3(x_i)+\mathcal{O}(e^{-4t}).$$

Altogether we consider the leading terms of the series (A1), $-2e^{-t}\cos(x_i)-2e^{-2t}\cos^2(x_i)-\frac{8}{3}e^{-3t}\cos^3(x_i)$, separately and bound the rest uniformly in x_i for every i by const $\times\, e^{-4t}$ for large t.

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