Chapter 8

The plastic collapse stress of open-cell aluminium foam

This chapter concentrates on the compressive properties of the same open-cell foam that has been used in the previous chapters. The classical scaling laws and an extended scaling law will be compared to the results of the compression tests. The scaling laws give an estimate of the quality of the foam through the value of the proportionality constant, $C_E$, this value can be compared to theoretical predictions stated in section 8.1.

8.1 Introduction

Metal foam owes many of its special properties, such as a high stiffness/density ratio, high energy absorption in compression and high vibrational absorption, to its cellular structure. According to Gibson and Ashby [1], the stiffness and plastic collapse stress are determined by the relative density, $\rho_r$, according to:

\[
\frac{E^*}{E_s} = C_E \rho_r^2
\]  

(8.1)

\[
\frac{\sigma_{PCS}}{\sigma_{ys}} = C_y \rho_r^{3/2}
\]

(8.2)

where $E^*$ is the stiffness of the foam, $E_s$ that of the metal, $\sigma_{PCS}$ the plastic collapse stress of the foam, $\sigma_{ys}$ the yield stress of the solid metal and $C_E$, $C_y$ topology dependent constants. Eq. 8.1 and 8.2 are based on the Gibson and Ashby (G&A) unit cell and the stiffness, $E^*$, has been calculated by the elastic bending of slender beams perpendicular to the loading direction, without taking the uniaxial strain in the parallel beams into account. The plastic collapse stress of the foam has been
obtained by assuming plastic hinge formation at the nodes. The plastic collapse stress is attained when the moments at the hinges are equal to the plastic moment and elastic-perfect plastic behavior is assumed. Since metals usually display high ductility and strain hardening San Marchi et al. extended the G&A framework to include power-law hardening of the base material [2]. When power-law hardening of the base material is taken into account Eq. 8.2 evolves to:

\[
\frac{\sigma_{PCS}^*}{\sigma_{ys}} = C_E^{-2} \cdot \rho_r^{-2} \cdot \rho_r^{3+n_s}
\]  

(8.3)

where \( n_s \) is the strain hardening exponent of the base material.

The advantage of using Eqs. 8.1 and 8.3 is that only one topology-dependent parameter is needed, which gives Eq. 8.3 a predictive character when the proportionality constant of the topology, C_E, is obtained from Eq. 8.1 and \( n_s \) and \( \sigma_{ys} \) are derived from experiments on the base material. C_E in Eqs. 8.1 and 8.3 should go to unity as the relative density goes to 100%, but for lower relative densities, C_E can have a different value depending on the specific morphology of the cellular architecture. Experimental data on polymer open-cell foams suggest a value for C_E of ~1 [1]. Warren and Kraynik used randomly oriented tetrahedral elements to arrive at a relative density dependent C_E ranging from 0.67 for a relative density of 25% to 1.1 for relative densities going to zero [3]. Zhu et al. calculated a value for C_E of ~1 for tetrakaidecahedral cells [4] and San Marchi et al. derived a value of 1.33 for C_E using triangular beams in a tetrahedral arrangement [2]. Experimentally San Marchi et al. measured a C_E of 0.48 for replicated pure aluminium foam. A good fit through the stress at 2% nominal strain for those samples, using Eq. 8.3, \( C_E=0.48 \) and \( n=0.26 \), resulted in a yield stress of the base material of 21 MPa (Fig. 4 and 5 of Ref. [2]). This is equal to the yield stress of annealed pure aluminium, which is 15-20 MPa [5], verifying the validity of Eq. 8.3 for this kind of (replicated) foam.

The objective of the research reported in this chapter was to investigate whether the extended G&A scaling relation (Eq. 8.3) can predict the plastic collapse stress for Duocel open-cell foam by measuring the parameters in Eq. 8.3 independently. To do so, we varied the plastic properties of the base material by heat treatments and vary the structural morphology of the foam by exploiting the anisotropy in cell shape. All the parameters in Eq. 8.1 and 8.3 were determined for two heat treatments (AN and HT). For each heat treatment the long axis of the cells was oriented parallel to the loading direction, i.e. in the longitudinal direction (LD) or in the transverse direction (TD). The advantage of this is that all the samples are
related to each other by either the heat treatment or the orientation of the long axis of the cells (see Fig. 8.1). In addition, the heat treatment does not affect the stiffness of the base material nor the cellular morphology of the foam, which allows for a clean assessment of the role of $C_E$ in Eqs. 8.1 and 8.3. Experimental results will be presented and the accuracy of the extended G&A scaling laws will be critically compared to the classical G&A scaling laws.

8.2 Material and experimental procedure

8.2.1 Aluminium foam

The material used is ERG Duocel open-cell aluminium foam (20 PPI, alloy AA6101). The relative densities of the samples are grouped according to the following three ranges: 3-5% (low), 6-7% (medium) and 10-13% (high). The relative density of each sample was calculated by measuring the mass and dividing it by the volume and the mass of pure Al (2.7 g/cm$^3$). The samples were received in the annealed (AN) condition (3 hrs at 412°C, followed by slow cooling to 260°C and maintaining that temperature for 0.5 hr followed by slow cooling to RT). Some of the samples were subjected to an additional precipitation hardening heat treatment (HT) to increase the yield stress. First the samples were exposed to a solid solution heat treatment (527°C for 8 hrs) followed by quenching in RT water. After 16 hrs at RT the samples were artificially aged for 8 hrs at 177°C, followed by cooling inside the furnace.

The compression test samples were machined with electro-discharging to the correct size and shape, i.e. cubic (33$^3$ mm$^3$). The compression tests were performed using a MTS 810 servohydraulic testing machine with a 10 kN load cell and two stainless steel plates, which were lubricated by Teflon spray to reduce friction. The strain was calculated using the displacement of the plates and the rate of displacement was 2 mm/min (strain rate $10^{-3}$ s$^{-1}$). The plastic collapse stress was
calculated from the initial peak load before the plateau and during the compression tests the evolution of the stiffness was monitored by unloading the sample at different strains. Since the sides of the cubic compression samples are not perfectly parallel to the steel plates initially, the stiffness, obtained by unloading, at small strains is very low. With increasing strain the stiffness increases to a maximum and decreases again. The initial stiffness, $E_0$, is obtained by extrapolating the stiffness curve after the maximum back to zero strain [6].

The long axis of the cells was oriented longitudinally (LD) and transversely (TD) to the loading direction. The aspect ratio of the cells was measured using a PEAK magnifier with a 0.05 mm accuracy. Backlighting provided good measurability and the short and long axis was measured for 40 cells for five different samples (200 in total).

8.2.2 Base material

To measure the base material properties, tensile tests were performed on an Al alloy with a similar composition as the foam material. Flat solid Al alloy tensile samples were used according to the ASTM norm B 557M-02a. The thickness of the samples was 3 mm and the length of the reduced cross-section 32 mm. An MTS extensometer with a gauge length of 25 mm was used for measuring the displacements. DIC (digital image correlation) software was used for calculating the Poisson’s ratio in the solid aluminium tensile tests. Table 8.1 gives the composition of the foam and of the alloy used for the solid tensile test samples. The alloy used for the solid test samples is AA6063 and the composition is quite similar to the composition of the foam (see table 8.1). The composition of the foam was measured by an inductively coupled plasma atomic emission spectrometer (ICP-AES) and the composition of the aluminium alloy sample used was provided by the supplier.

Three solid test samples of each heat treatment were tested and Table 8.2 lists the averages of the yield stress ($\sigma_{ys}$, i.e. the 0.2% off-set stress), strain hardening exponent of the solid ($n_s$), UTS and the strain at UTS ($\varepsilon_f$). The strain hardening exponent of the solid, $n_s$, was obtained by fitting a power-law relation ($\sigma = K\varepsilon^n$) to the data points of the true stress-true strain curve, lying between the yield stress and 4% true strain (rest of the curve was removed and no translation was applied to the remaining part). The measured Young’s modulus was 66 GPa, being close to the literature value of 69 GPa [5]. Since the compositions of the two alloys used for the foam and the solid tensile test samples are quite similar, it is expected that the
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Table 8.1 (a) Al alloy composition (wt%) of the foam. One sample of each density was measured with ICP-AES (size of the samples >30 cells) (accuracy is in the promille range). Si concentration was measured with EDS.

<table>
<thead>
<tr>
<th>Density</th>
<th>Si</th>
<th>Mg</th>
<th>Fe</th>
<th>Cu</th>
<th>Zn</th>
<th>Mn</th>
<th>Ni</th>
<th>Cr</th>
<th>B</th>
<th>Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>0.45</td>
<td>0.34</td>
<td>0.14</td>
<td>0.02</td>
<td>0.007</td>
<td>0.007</td>
<td>0.004</td>
<td>0.003</td>
<td>0.003</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>6%</td>
<td>0.45</td>
<td>0.33</td>
<td>0.14</td>
<td>0.02</td>
<td>0.008</td>
<td>0.007</td>
<td>0.004</td>
<td>0.002</td>
<td>0.002</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>11%</td>
<td>0.45</td>
<td>0.47</td>
<td>0.18</td>
<td>0.02</td>
<td>0.008</td>
<td>0.006</td>
<td>0.004</td>
<td>0.003</td>
<td>0.007</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

(b) Al alloy composition (wt%) of the solid tensile test samples, as specified by the supplier.

<table>
<thead>
<tr>
<th>Element</th>
<th>Si</th>
<th>Mg</th>
<th>Fe</th>
<th>Mn</th>
<th>Zn</th>
<th>Ti</th>
<th>Cu</th>
<th>Cr</th>
<th>Ca</th>
<th>Na</th>
</tr>
</thead>
<tbody>
<tr>
<td>wt%</td>
<td>0.41</td>
<td>0.50</td>
<td>0.17</td>
<td>0.027</td>
<td>0.003</td>
<td>0.010</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table 8.2 Mechanical properties of the solid tensile test samples for the two heat treatments: the 0.2% off-set stress ($\sigma_{ys}$), strain hardening exponent ($n_s$), failure strain at the UTS ($\epsilon_f$) and the UTS. The strain hardening exponent, $n_s$, was obtained by fitting a power-law relation ($\sigma = K\epsilon^{n_s}$) to the data points of the true stress-true strain curve, lying between the yield stress and 4% true strain.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\sigma_{ys}$</th>
<th>$n_s$</th>
<th>$\epsilon_f$</th>
<th>UTS</th>
<th>Increase compared to yield stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>AN</td>
<td>49 ± 1</td>
<td>0.256 ± 0.002</td>
<td>20.3 ± 0.6</td>
<td>100 ± 1</td>
<td>104</td>
</tr>
<tr>
<td>HT</td>
<td>124 ± 1</td>
<td>0.152 ± 0.003</td>
<td>16.0 ± 0.3</td>
<td>193 ± 1</td>
<td>56</td>
</tr>
</tbody>
</table>

Mechanical properties are similar as well. To confirm this, micro-indentation measurements have been performed on the medium density foam samples and the solid tensile test samples for both heat treatments. Before the indentation measurements all samples were embedded and polished. The indents were made on the cross-section of strut nodes and the size of the indent (~ 100 µm) was much smaller than the strut node size (> 500 µm). The Vickers hardness, HV, of the AN foam samples was 30 ± 2 and for the annealed solid tensile test samples the HV was 34 ± 1. For the HT foam samples the HV was 71 ± 5 and for the HT solid tensile test samples HV was 67 ± 4.

8.3 Results

To see whether Eq. 8.3 correctly predicts the plastic collapse stress of the AN and HT samples in both orientations, the strain hardening exponent and the yield stress of the base material and $C_E$ have to be known.
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Fig. 8.2 shows the stiffness, $E_0$, of the AN and HT samples for both cell orientations as a function of the density. The Young’s modulus of the base material is taken to be 69 GPa, independent of the heat treatment, leading to an overall stiffness, $E_0$, that is also independent of the heat treatment, as can be observed in Fig. 8.2. $E_0$ does however depend on the orientation of the cell and a fit of Eq. 8.1 through the data points results in $C_E$=0.87 for the LD and $C_E$=0.45 for the TD direction.

Table 8.2 shows the mechanical properties of the Al alloy. The yield stress of the AN samples, $\sigma_{ys-AN}=49\pm1$ MPa, is equal to the value in literature of aluminium alloy 6063, which is 48-50 MPa [5]. The yield stress of the samples with the specific heat treatment, HT, is $\sigma_{ys-HT}=124\pm1$ MPa. The strain hardening exponent of the base material is $0.256 \pm 0.002$ for the AN samples and $0.152 \pm 0.003$ for the HT samples.

In contrast to the LD samples, the TD samples do not show an initial peak in the stress-strain curve for both heat treatments. Also a clear horizontal plateau was not always present. Therefore, the stress at 3.5% strain for the HT samples and 6% for the AN samples is used for the plastic collapse stress of the TD samples. These strain values correspond to the average strain at the plastic collapse stress of the LD samples. It was found that the stress values obtained for the TD samples were almost equal to the plateau stress or the stress at a minimum in the slope of the curve. For the LD samples, the stress at the initial peak was taken as the plastic collapse stress.

Fig. 8.3 shows the plastic collapse stress of the AN and HT samples as a function of the density for the (a) LD and (b) TD direction. The lines through the data points are predicted by Eq. 8.3 and the parameters are as stated above (see also the caption of Fig. 8.3). The predicted plastic collapse stress is slightly higher or lower for the HT and AN samples, respectively, predominantly in the medium and high density range for both orientations. The error, however, is small (around 15%), given the fact that the parameters used are measured independently.

An interesting aspect of the studied samples is that their material properties and cellular architecture are correlated as illustrated in Fig. 8.1. Although the prediction of Eq. 8.3 is already excellent, this connection allows to identify a further improvement of the predicted values. When, for example, $C_E$ in Fig. 8.3a is increased so that Eq. 8.3 predicts the values for the LD AN samples correctly, the predicted values for the LD HT have also increased, thereby increasing the error for those samples. This rules out an improvement of the topology-dependent pre-factor. However, when the yield stress of the base material is slightly altered, the fits for both orientations can be further improved. By increasing the yield stress of the AN
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Figure 8.2 Stiffness vs. the relative density on a log-log scale. The stiffness of the foam samples is independent of the heat treatment just as for the base material \((E_{s-AN/HT}=69\text{GPa})\) and all LD and TD samples are fitted by Eq. 8.1 resulting in a value of the proportionality constant \(C_{E,LD}=0.87\) for the LD samples and \(C_{E,TD}=0.45\) for the TD foam samples.

To see whether the plastic collapse stress of the samples can be described by the classical G&A relation, Eq. 8.2 is fitted to the data points to obtain a value for \(C_y\). The values for the base material yield stress are the independently-obtained values \(\sigma_{ys-AN}=49\ \text{MPa}\) and \(\sigma_{ys-HT}=124\ \text{MPa}\). Fig. 8.5 shows the fit of Eq. 8.2 as a function of the density. The obtained values for \(C_y\) are 0.67 for the LD samples and 0.43 for the TD samples. Also in this case, the fits correspond very well with the experimental data points. Clearly, also the classical plastic slope of 3/2 is in close accordance with the actual density dependence. Also here, the fit can be improved by a slight increase in \(\sigma_{ys-AN}\) to a value of \(\sigma_{ys-AN}=55\ \text{MPa}\), which is still a very reasonable value, because 55 MPa is the yield stress of various annealed Al alloys in the 6000 series with similar composition. Note that as in the case of Eq. 8.3 in Fig. 8.3, adjusting \(C_y\) will not improve the overall results.

### 8.4 Discussion

Note that the extended G&A relation in Eq. 8.3 has the inherent dependence that by increasing the hardening exponent of the base material (but keeping its yield
stress unaltered), the overall collapse stress decreases. This is due to the value of $C_E$ and the density being below unity. This dependence on hardening, however, is counter-intuitive.

As illustrated in Fig. 8.1, the samples are linked by either the heat treatment or
Figure 8.5 Plastic collapse stress of both heat treatments vs. the relative density for the (a) LD and (b) TD foam samples on a log-log scale. The solid lines represent the fits of Eq. 8.2 based on the following parameters: $\sigma_{ys-AN}=49$ MPa, $\sigma_{ys-HT}=124$ MPa. The resulting $C_y$ values give either a good fit for the AN- or HT foam samples. When $\sigma_{ys-HT}=124$ MPa is used the fits result in proportionality constants of $C_y$ values for the AN sample the annealed base material yield stress has to be increased to $\sigma_{ys-AN}=55$ MPa.

the cell orientation. The LD/TD ratio is obtained from the absolute values of $C_E$ for the different cell orientations and the HT/AN ratio from the yield stress of the Al alloy for different heat treatments. The yield stress is related to the hardness of the base material and the ratio between the Vickers hardness of the HT and AN foam samples is $70.6/30.4=2.32$. This is very close to the yield stress ratio $124\text{MPa}/49\text{MPa}=2.53$ of the independently obtained yield stresses as well the optimal ratio as used in Fig. 8.4: $106\text{MPa}/55\text{MPa}=1.93$. Also the optimal values for the classical G&A values of Fig. 8.5 are close: $124\text{MPa}/55\text{MPa}=2.25$.

The aspect ratio in the cell shape is $1.43\pm0.15$, which is in close accordance to the value of $1.42\pm0.12$ obtained by x-ray tomography in Ref. [7]. Within the classical G&A framework, where elastic-perfect plastic behavior is used, this results in a theoretical aspect ratio in the plastic collapse stress of $1.68$, which is close to the aspect ratio of $C_{y-LD}/C_{y-TD}=0.67/0.43=1.56$, obtained from Eq. 8.2 for both heat treatments. When $n_{s-AN}=0.256$ is used in Eq. 8.3 the aspect ratio in the plastic collapse stress between the two orientations is $0.87^{0.628}/0.45^{0.628}=1.51$. When $n_{s-HT}=0.152$ is used the aspect ratio is $0.87^{0.576}/0.45^{0.576}=1.46$. 

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8.5 Conclusion

The extended G&A scaling law (Eq. 8.3) has been effectively used to predict the plastic collapse stress as a function of the relative density for four sample variations, i.e. two heat treatments for two cell orientations. The classical G&A scaling law can be fitted to the data with reasonable accuracy as well, for the parameters used. Both can even be improved by changing the yield stress of the base material by less than 15%, which is still within the variance between the measured foam samples. The big advantage of the extended G&A scaling law (Eq. 8.3) is that it has no fitting parameters at all.

References