Abstract—We consider the problem of detecting object contours in natural images. In many cases, local luminance changes turn out to be stronger in textured areas than on object contours. Therefore, local edge features, which only look at a small neighborhood of each pixel, cannot be reliable indicators of the presence of a contour, and some global analysis is needed. We introduce a new morphological operator, called adaptive pseudo-dilation (APD), which uses context dependent structuring elements in order to identify long curvilinear structure in the edge map. We show that grouping edge pixels as the connected components of the output of APD results in a good agreement with the gestalt law of good continuation. The novelty of this operator is that dilation is limited to the Voronoi cell of each edge pixel. An efficient implementation of APD is presented. The grouping algorithm is then embedded in a multi-threshold contour detector. At each threshold level, small groups of edges are removed, and contours are completed by means of a generalized reconstruction from markers. The use of different thresholds makes the algorithm much less sensitive to the values of the input parameters. Both qualitative and quantitative comparison with existing approaches prove the superiority of the proposed contour detector in terms of larger amount of suppressed texture and more effective detection of low-contrast contours.

Index Terms—Edge and boundary detection, Gestalt grouping, morphological analysis methods.

I. INTRODUCTION

Contour detection is important for many computer vision applications, such as shape discrimination and object recognition. It is essential to distinguish the concepts of edge and contour. We use the term edge to refer to any non-negligible local luminance change. The term contour will be used to refer to the boundary line that delimits (a part of) an object from its background.

Different techniques have been proposed for edge detection: examples are linear filtering [1]–[5], local orientation analysis [6], [7], fitting of analytical models to the image data [8]–[11], local energy [12]–[15], statistical analysis of the gradient field [16]–[19], anisotropic diffusion [20]–[22], complementary analysis of boundaries and regions [23]–[25], and biologically motivated surround suppression [26]–[29]. The main limitation of these operators is that they detect contours by only analysing a small neighborhood around each pixel. Conversely, the human visual system performs a contour integration process, so that long chains of collinear edges are grouped together and recognized as object contours. A large amount of research has been carried out in order to develop contour detection algorithms which include computational models of contour integration. The approaches that have been proposed for this purpose can be classified in two categories: 1) computation of a global saliency map, and 2) grouping edges according to gestalt laws.

Saliency maps are obtained by iteratively increasing the strength of those edges which have collinear edges in their immediate surrounding, and by reducing the strength of edges that are surrounded by random patterns. The best known algorithms are based on relaxation labelling [30]–[32], which works in a graph based framework, and tensor voting schemes [33], [34], which associate a vector or tensor field with each edge pixel. Concerning the latter, recent work [35]–[37] shows that schemes similar to tensor voting can successfully model the facilitation and contour integration processes that are performed in the front-end part of the human visual system. Special extension fields, called association fields in the psychophysical literature [38], are defined in order to take into account inhibition and facilitation. For an overview of the related biological background, we refer to [39] and [40]. In [41], an iterative scheme based on uniform networks is developed in order to compute a saliency map from local information about curvature and curvature changes.

On the other hand, Gestalt theory concerns the ability of human observers to group visual stimuli which share certain common characteristics. The best known grouping criteria are proximity (stimuli that are close to each other are grouped together), similarity (stimuli that have the same shape, color or size are grouped together), and common fate (objects that move at the same speed in the same direction are grouped together).

Whenever elementary stimuli or objects are grouped together, they form a new larger object, a gestalt. Such a larger object can be grouped together with other similar objects and the process can be repeated hierarchically until only a few large meaningful structures are identified. As a starting point of this contribution we use the ability of humans to group together local edge cues that are nearby and collinear, forming long and smooth lines (Fig. 1). In psychology, this phenomenon is generally referred to as gestalt law of good continuation (GLGC) [42]–[45].

Computational models of Gestalt laws can find applications in many computer vision tasks. For instance, Fig. 2 shows the result of a local edge detector [26] for a natural image. It effectively rejects texture while preserving most of the contour
be a point set in the plane and let \( \mathcal{E} \) be equal to the set of those points and the LSEs. The proposed approach to edge grouping is

\[
\text{APD}(b) \triangleq \bigcup_k R_k \cap V(r_k).
\]

Let \( b = \{r_1, \ldots, r_N\} \) be a point set in the plane and let \( V(r_k) \) be the Voronoi cell of \( r_k \). We also assume that a subset \( R_k \) of \( \mathbb{R}^2 \), which we call local structuring element (LSE), is associated with each point \( r_k \). With this notation, APD is defined as an operator that takes as input the point set \( b \) and the LSEs \( R_k, k = 1, \ldots, N \), and produces as output the union of all the intersections between each LSE and the Voronoi cell of the corresponding point

\[
\text{apd}(b) \triangleq \bigcup_k R_k \cap V(r_k).
\]

Let us assume that the Cartesian representation of the collection \( \mathcal{R} = \{R_1, \ldots, R_N\} \) is available in the form of a positive construction function \( \Delta_{\mathcal{R}}(r, r_i) \), defined such that \( \Delta_{\mathcal{R}}(r, r_i) \) is less than one if and only if \( r \) belongs to \( R_k \)

\[
r \in R_k \iff \Delta_{\mathcal{R}}(r, r_k) < 1.
\]

A formal proof of (3) is given in the Appendix A.

This result leads to a very efficient implementation of APD. Specifically, for each pixel \( r \), we compute its nearest neighbor \( q(r) \in b \) and add \( r \) to \( \text{apd}(b) \) if \( \Delta_{\mathcal{R}}[r, q(r)] < 1 \). In this process, the most demanding operation is the computation of the first nearest neighbor transform \( q(r) \) of the input image \( b \), which can be computed very efficiently. We remark that while the computation time of the Voronoi tessellation of \( M \) points in a continuous domain is \( O(M \log M) \), the computation of \( q(r) \) on a square lattice is linear with the size of the input image.

III. EDGE GROUPING

In this section, we use APD to group edges according to the GLGC. We first present the method with LSE of a fixed size (Section III-A) and then we describe a simple procedure to adaptively compute the size of the LSE for each different edge point (Section III-B).

A. Proposed Approach

Let \( b = \{r_1, \ldots, r_N\} \) be a set of edge points with local edge orientations \( \theta_1, \ldots, \theta_N \). The proposed approach to edge grouping is simple: first, we compute \( \mathcal{D}_b \equiv \text{apd}(b) \), by using
elongated LSEs which are oriented along the local edge direction $\theta_k$. Second, we perform a connected components analysis of $\mathcal{D}_b$, and we group together points of $b$ that belong to the same connected component of $\mathcal{D}_b$.

This process is illustrated by Fig. 3, where the LSEs are ellipses. Each ellipse, centered on point $r_k$, has the major axis oriented along the local edge direction $\theta_k$. The input point set [Fig. 3(a)] contains a long chain of collinear edges, with a high curvature point $B$ and a large gap between points $C$ and $D$. The Voronoi tessellation of the input point set is shown in Fig. 3(b) and the result $\mathcal{D}_b$ of APD is shown in Fig. 3(c).

Intersecting each ellipse with the corresponding Voronoi cell has several benefits. First, edges that are close to each other, but which are not collinear, are not grouped together [such edges $E$ and $F$ of Fig. 3(a)]. Classical dilation, or adaptive schemes which do not limit dilation to $V(r_k)$ [56], [57] would not be effective in this respect. Second, if no such intersection is performed, undesired spikes would be present on high curvature points, which may have a devastating effect on the grouping process (Fig. 4). As a result, very long ellipses can be used without negatively affecting $\mathcal{D}_b$ and, consequently, the grouping process; such long ellipses are necessary for grouping together collinear edges that are quite distant, like points $C$ and $D$ in Fig. 3(a).

**B. Automatic Adaptive Selection of the LSE Size**

We now describe a simple adaptive procedure that computes the size of the LSE associated with each edge point. For each connected component $C_k$ of the input binary image $b$, we assign a single value $2a_1^{(k)}$ to the length of the major axis of each ellipse, and a value $2a_2^{(k)}$ to the minor axis. Thus, all edges of the same connected component are dilated by ellipses of the same size.

The length $2a_2^{(k)}$ of the minor axis is computed as follows: let $V(C_k)$ be the Voronoi cell of $C_k$, defined as the union of all the Voronoi cells $V(r_i)$ of the points $r_i$ that belong to $C_k$.

$$V(C_k) \triangleq \bigcup_{r_i \in C_k} V(r_i).$$  \hfill (4)

Let $r \in C_k$ be an edge point and let $q(r)$ be the nearest neighbor of $r$ on the boundary of $V(C_k)$ (Fig. 5). We indicate by $\rho(r)$ the Euclidean distance between $r$ and $q(r)$, and by $\rho_H^{(k)}$ the harmonic mean of $\rho(r)$, for $r \in C_k$

$$\rho_H^{(k)} = \left[ \frac{1}{N_k} \sum_{r_i \in C_k} \frac{1}{\rho(r_i)} \right]^{-1}$$  \hfill (5)

where $N_k$ is the number of edge points in $C_k$. In this notation, the length $a_2^{(k)}$ of the minor semi-axis is computed as a monotonically increasing function $f$ of $\rho_H^{(k)}$, which satisfies the conditions $f(0) = 0$ and $f(\xi) \leq \xi$ for $\xi \geq 0$

$$a_2^{(k)} = f\left[\rho_H^{(k)}\right].$$  \hfill (6)

Since smaller values of $\rho(r)$ have higher influence on the harmonic mean, and being $f(\xi) < \xi$, the structuring element is always narrower than the Voronoi cell $V(C_k)$. In this way, the dilation orthogonally to the edge direction $\theta$ prevents that noncollinear components that run in parallel are grouped together (such as $C_1$ and $C_3$ in Fig. 6). On the other hand, $a_2^{(k)}$ is sufficiently large to compensate for slight misalignment between collinear components, such as $C_1$ and $C_2$ in Fig. 6.
possible choices for the function $f$ are $f(\xi) = \log(\xi + 1)$, or $f(\xi) = (\xi + a)^q - a$, with $a > 0$ and $q \in (0, 1)$.

Concerning the length of the major axis, we observe that if it exceeds the length of the corresponding Voronoi cell, the actual value of $a_1^{(k)}$ only has a small influence on the final output (Fig. 7). In our experiments, we found that the choice $a_1^{(k)} = \eta d_2^{(k)}$, with $\eta \geq 3$, is sufficiently accurate for this purpose. Specifically, with such a choice of $a_1^{(k)}$, almost all points that belong to long chains of collinear edge points are dilated with an ellipse that intersects the corresponding Voronoi cell; thus, they will be grouped together. On the other hand, almost all isolated edge points are dilated with ellipses which do not intersect the corresponding Voronoi cells and, consequently, they are not grouped together. This is in agreement with the GLGC.

In Fig. 8, an example of how this procedure works is presented. We observe that, due to the structure induced by the Voronoi tessellation, the LSE is large on isolated contours and small on texture, so that the former have much higher probability to be grouped together than the latter.

IV. AN APPLICATION: CONTOUR DETECTION

In this section, APD and the grouping algorithm introduced above are applied to the problem of contour detection in natural images. The proposed approach is depicted in Fig. 9.

Like many existing edge detectors, we first compute a local quantity $L(\mathbf{r})$ that measures the visual similarity of a local pattern to an ideal step edge. Examples of such quantities are the gradient magnitude [1], Gabor energy [26], phase congruency [14], template matching [18], and texture gradient [19]. In our experiments, $L(\mathbf{r})$ is a quantity called *contourness*, introduced in [29] and [58] with the purpose of suppressing texture, and successively improved in [59]–[61]. Thinning by nonmaxima suppression, which is also a local operation, is included in the computation of $L(\mathbf{r})$.

Then, $N_t$ binary images $b_1 \subset b_2 \subset \cdots \subset b_{N_t}$ are obtained by thresholding $L(\mathbf{r})$ at $N_t$ different levels $\tau_1 > \tau_2 > \cdots > \tau_{N_t}$

$$b_k = \{ \mathbf{r} \mid L(\mathbf{r}) > \tau_k \}, \quad k = 1, \ldots, N_t.$$  

Each threshold $\tau_k$ is computed such that a given percentage $p_k$ of the values of $L(\mathbf{r})$ is above $\tau_k$. Examples of such binary images are shown in Fig. 10. As the threshold decreases, the corresponding binary map contains more object contours, but more undesired responses as well.

In order to remove undesired edges, we apply a pruning step to each binary map $b_k$ (block “GLGC pruning” of Fig. 9). The idea is to group edges by means of the grouping algorithm presented in Section III and to remove those groups that contain less than $l_{\min}$ edges, where $l_{\min}$ is a threshold. Specifically, the pruning operator $\text{pru}(\cdot)$ is defined as

$$\text{pru}(b) \triangleq \{ \pi_k \in b \mid \text{card} \{ \pi_k \} \geq l_{\min} \}$$

where $\pi_j$ are the groups detected by our algorithm and $\text{card} \{ X \}$ indicates the number of elements of set $X$. We chose to set $l_{\min}$ proportional to the linear size of the image, $l_{\min} = \xi d$, where $d$ is the length of the diagonal of the input image and $\xi$ is a proportionality factor. In our experiments, we found that for $\xi = 0.03$ small groups $\pi_k$, for which $\text{card} \{ \pi_k \} < \xi d$, do not contain any significant information; therefore, they can be suppressed.

Fig. 11 shows the outputs $\overline{b_1}$, $\overline{b_2}$, and $\overline{b_3}$ of pruning for the binary images $b_1$, $b_2$ and $b_3$ respectively. Remarkably, a large amount of texture edges is removed while preserving most of the object contours; this is due to the fact that edge grouping makes pruning robust to contour fragmentation.

We observe that the pruned contour maps at higher thresholds contain a very little amount of texture, but several useful contours might be missing; such contours are still present in the pruned contour maps at lower thresholds. In order to recover them, we introduce a simple operator (block “APD Reconstruction”) that generalizes the well known reconstruction from markers (RFM). By definition, standard RFM takes in input two binary images $B_1$ and $B_2$, with $B_1 \subseteq B_2$, and returns the set of those connected components $\pi_i$ of $B_2$ which contain at least one point of $B_1$

$$\text{rfm}(B_1, B_2) \triangleq \{ \pi_i \in B_2 \mid \pi_i \cap B_1 \neq \emptyset \}.$$  

We propose to perform RFM based on the connectivity induced by APD. Specifically, our operator $R_{\text{APD}}(\cdot)$ is defined as follows:

$$R_{\text{APD}}(B_1, B_2) \triangleq \text{rfm}[\text{apd}(B_1), \text{apd}(B_2)] \cap B_2.$$  

In Fig. 12(a), we show the behavior of $R_{\text{APD}}(\cdot)$, which is applied to the results $\overline{b_k} = \text{pru}(b_k)$ and $\overline{b_{k+1}} = \text{pru}(b_{k+1})$ of pruning. As we see, contours in $\overline{b_{k+1}} = R_{\text{APD}}(\overline{b_k}, \overline{b_{k+1}})$ are as complete as in $\overline{b_{k+1}}$, while the undesired edges of $\overline{b_{k+1}}$ that
Fig. 8. Illustration of the proposed procedure for adaptively computing the size of the LSE. From left to right: Input binary map, the Voronoi tessellation of its connected components, and the output of APD. The LSE is large on isolated contours and small on texture, so that the former have much higher probability to be grouped together than the latter.

Fig. 9. Overall scheme of the proposed contour detector.

Fig. 10. Input image and the binary maps obtained by thresholding $L(r)$. The thresholds correspond to the values $p_1 = 0.13, p_2 = 0.35$, and $p_3 = 0.5$.

Fig. 11. Outputs $\tilde{b}_1, \tilde{b}_2,$ and $\tilde{b}_3$ of the block “GLGC pruning” applied to the binary maps $b_1, b_2,$ and $b_3$ of Fig. 10.
are not present in $\hat{b}_k$ are not present in $\hat{b}_{k+1}$ either. Performing RFM on the dilated images, instead of on $\hat{b}_k$ and $\hat{b}_{k+1}$ directly, makes the operator more robust to contour fragmentation; therefore, more contours can be recovered with respect to the standard RFM [Fig. 12(b)].

An important difference between classical RFM and $R_{APD}$ concerns their behaviors when they are applied in a cascade. Specifically, given three binary maps $b_1 \subseteq b_2 \subseteq b_3$, it is well known that a cascade of two RFMs applied on $b_1$, $b_2$, and $b_3$ is equivalent to a single RFM applied directly to $b_1$ and $b_3$

$$r_{\text{fin}}[r_{\text{fin}}(b_1,b_2),b_3] = r_{\text{fin}}(b_1,b_3).$$

However, (11) does not hold for $R_{APD}$, due to the fact that $b_1 \subseteq b_2$ does not imply $apd(b_1) \subseteq apd(b_2)$. Let us consider, as an example, the synthetic image of Fig. 13 and the binary maps $b_1 \subseteq b_2 \subseteq b_3$ obtained by thresholding $L(x)$. Fig. 14 compares a cascade of two $R_{APD}$ applied on $b_1$, $b_2$, and $b_3$ (top), and a single $R_{APD}$ applied on $b_1$ and $b_3$ directly (bottom). While in the former case contours are correctly detected, in the latter a considerable amount of undesired response is present, due to the very large LSE which are used to dilate $b_1$. Large LSE are useful to group sparse objects that typically occur at high thresholds. Therefore, the insertion of an intermediate layer that bridges the large LSE of $b_1$ and the crowded edges of $b_3$ proves to be beneficial.

V. EXPERIMENTAL RESULTS

We now present and comment the outputs of the proposed contour detector, tested with natural images, in comparison with other three existing algorithms: the standard Canny edge detector [1], the contour detector with surround inhibition (SI) proposed in [58], and the edge detector EDISON proposed in [18]. Canny edge detector computes the gradient of the input image $f(x)$, and applies nonmaxima suppression and hysteretic thresholding on the gradient magnitude [1]. SI improves the Canny edge detector by replacing the gradient magnitude with a quantity called contourness that takes into account the local surrounding context of each pixel [58]. EDISON introduces a two-dimensional feature vector in order to discriminate between contour pixels and undesired edge responses: the first component is the gradient magnitude and the second one is derived from a linear template matching analysis.

We tested the four algorithms on the 40 natural images of the RuG dataset [58] and the 300 images of the Berkley dataset [62]. For all test images, the results of our algorithm are obtained with the same values of the input parameters, which are

![Fig. 12. (a) Illustration of $R_{APD}(\cdot)$, applied to the images $\hat{b}_k = \text{pr}(b_k)$ and $\hat{b}_{k+1} = \text{pr}(b_{k+1})$. (b) Output of $r_{\text{fin}}(\hat{b}_1,\hat{b}_2)$, where the arrows mark contours that would be lost if RFM was applied without APD.](image)

![Fig. 13. Synthetic image and the binary maps $b_1 \subseteq b_2 \subseteq b_3$ obtained by thresholding $L(x)$.](image)
the following: the number of thresholds is $N_T = 3$; the values $\tau_k$ of the thresholds are computed for each image, such that a fixed fraction $p_k$ of pixels is above $\tau_k$ (in our experiment, we used the values $p_1 \approx 0.07$, $p_2 \approx 0.12$ and $p_3 \approx 0.17$). In Section V-B, we show that the performance of the proposed algorithm does not depend very much on the specific values of these thresholds. For the other three algorithms, the parameters have been optimized as suggested in the corresponding references. We present both a qualitative and quantitative comparison of the studied algorithms.

A. Qualitative Comparison

Some examples of the outputs of the considered algorithms are shown in Fig. 15(a). The results related to the entire datasets are available online.1 As we see, the outputs of the proposed algorithm (second row) contain the smallest amount of undesired edges and the most complete contours. For instance, in the first picture of Fig. 15(a) the basket has very low contrast contours, whereas the grass presents a very high-contrast texture. Nonetheless, the proposed algorithm succeeds very well in isolating contours and rejecting texture, while the other approaches fail. The second example (elephant) presents a quite low-contrast texture in the grass, but also low contrast region boundaries, such as the contour of the hill in the top-right corner. We see that in such cases the simple Canny edge detector (third row) can achieve a decent texture suppression as well, but at the expense of the loss of low contrast contours. On the opposite, the proposed approach and SI (fourth row) can suppress texture while preserving low-contrast contours. EDISON (fifth row) can preserve low-contrast contours, but is less effective in texture suppression. The third example (bear) exhibits textures that occupy low-frequency regions of the power spectrum, such as the plants in the background of the bear. Such texture is quite difficult to

1http://www.cs.rug.nl/~imaging/APD
remove. We see that the proposed algorithm can eliminate such undesired edge response, while other approaches are unable to do it. Other examples of images that present low-frequency texture are shown in Fig. 15(a) (second-last column), especially in plants in the background in the picture of the goat.

The last example in Fig. 15(a) shows the ability of the proposed method to extract fragmented contours by means of the GLGC-based edge grouping: the very low contrast diagonal line in the background, which separates two differently textured areas, is detected by our approach quite well, whereas with the other approaches such a line is either missing or masked by many other undesired edges. In general, we can conclude that the proposed operator outperforms the other studied approaches in terms of texture suppression, preservation of low-contrast contours and robustness to contour fragmentation.

B. Quantitative Comparison

We measured the dissimilarity between the output of each considered algorithm and a hand drawn desired output (ground truth). Well known performance indicators are recall $R$ and precision $P$, which relate the number of correctly detected contour pixels to the number of ground truth (GT) pixels and the number of contour pixels that are actually detected by an algorithm (DC), respectively

$$R = \frac{\text{card} \{ DC \cap GT \}}{\text{card} \{ GT \}}; \quad P = \frac{\text{card} \{ DC \cap GT \}}{\text{card} \{ DC \}}.$$  \hspace{1cm}(12)

Following [19], we evaluate the performance in terms of the so-called $F$-measure, defined as the harmonic mean of $P$ and $R$.

$$F = \frac{2PR}{P + R}.$$  \hspace{1cm}(13)

Fig. 16 shows the average values and the standard deviations of $F$, computed on both datasets for all studied algorithms. The average value of $F$ achieved with the proposed approach is about 20% higher than the values achieved by the other algorithms. In particular, the comparison with SI, which deploys the same local edge strength $J(x)$ that we use here, directly validates the superiority of the multithreshold algorithm presented in Section IV, which is based on APD and edge grouping, with respect to the classical hysteresis thresholding.

Fig. 17 compares the dependence on the thresholds of the performances achieved by the proposed approach and the Canny edge detector: for both algorithms, $F$ is plotted versus the fraction $p_1$ of pixels that are above the highest threshold. For both algorithms, the values of $p$ that correspond to the lower thresholds are chosen proportional value to $p_1$. Canny edge detector exhibits a peaked maximum for $p_1 = 0.014$ and decreases quickly as $p_1$ deviates from that value. In contrast, the proposed approach reaches the maximum values of $F$ for a higher value of $p_2 (p_2 \approx 0.057)$ and the performance decreases much more slowly as $p_1$ increases.

VI. DISCUSSION

Adaptive mathematical morphology extends classical morphological operators to structuring elements that are context-de-
The novelty of APD is that dilation is limited to the Voronoi cell of each edge point, which brings several advantages since long ray spatial relations between edge points are taken into account in dilation process. Moreover, APD can be computed efficiently by simply thresholding the quantity $\Delta_R[\mathbf{r}, q(\mathbf{r})]$ as in (3).

In previous literature, different approaches have been proposed for adaptive dilation. In [56], a nonlinear diffusion equation is designed in order to perform dilation only along a preferred direction. While APD only works in the binary case, operators based on nonlinear diffusion can be applied to gray-level images as well. However, they are computationally demanding due to their iterative nature and stopping rules are usually left to heuristics. On the other hand, multithreshold schemes, such as the one presented in Section IV, allow to use APD with grey-level images, too. A noniterative algorithm for adaptive dilation is proposed in [57], with applications to edge linking. It consists in replacing each edge point with an ellipse whose major axis is oriented along the local edge direction. However, it has problems in presence of high curvature points and with edges that are close to each other but which are not collinear (see, for instance, Figs. 3 and 4). These limitations are overcome by the proposed operator, by intersecting the LSE with the corresponding Voronoi cell. More recently, higher degree
of adaptivity has been achieved by morphological amoeba filters [64]. However, their computation time grows quadratically with respect to the linear size $l_{max}$ of the LSE, which could be quite large for edge grouping purposes, whereas the complexity of APD does not depend on $l_{max}$.

APD has some mathematical relationships with the classical structural dilation. Specifically, if the construction function is equal to $\Delta_{R}(r, r') = |r-r'|/R$, then APD reduces to a dilation with a disk of radius $R$ as structuring element. In general, an operator $D : X \mapsto X$, where $X$ is the space of all non-negative digital signals, is called dilation if the following requirements are fulfilled:

$$
\begin{align*}
D(\alpha x_1) &= \alpha D(x_1) \\
D[\sup(x_1, x_2)] &= \sup[D(x_1), D(x_2)]
\end{align*}
$$

(13)

with $x_1, x_2 \in X$ and $\alpha \in \mathbb{R}$. For the binary case, the first condition is replaced by $D(0) = 0$ and the supreme reduces to the union. It is easy to show that the proposed operator does not fulfill the second requirement (13). However, it can be easily shown that the only possible operator which fulfills the requirements (13) is the standard non-adaptive dilation (see Appendix B). Therefore, as soon as adaptivity is desired, the constraint of commutation with the supreme must be weakened in some way.

We use APD to group edges according to the GLGC: collinearity is taken into account by elongated LSE, while the neighborhood induced by the Voronoi tessellation effectively models the Gestalt law of proximity [65], [66]. In our experiments, we used ellipses as LSEs for reasons of elegance. However, any other elongated shape could be used as well.

The proposed approach for contour detection is a combination of two techniques: a grouping algorithm described in Section III, and the iterative multithreshold process presented in Section IV. Both of them are necessary to achieve good results. In fact, the grouping algorithm itself simply groups edges together, without touching the edge map. Texture suppression and contour completion are achieved by the pruning/reconstruction process, which is not part of the grouping algorithm. On the other hand, the multithreshold algorithm of Section IV would perform very poorly without edge grouping. In fact, in case of a strongly fragmented contour, a pruning process based on the number of pixels of each connected component would be totally unable to distinguish between fragmented object contours and spurious edges. An example is shown in Fig. 18.

Grouping edges based on their Voronoi tessellation has some duality with the algorithm proposed in [50], on which the concept of Constrained Delaunay triangulation is introduced. However, even when the input parameters are optimized for a specific dataset, a considerable amount of useful contours is missed, especially for what concerns internal details of the represented objects, such as the facial details in Fig. 19. In contrast, our method results in a much better contour preservation without increasing the amount of undesired edge response. The reason is that in [50] thresholding and grouping are performed in two separate steps, whereas we propose to group edges at each threshold and to combine together the results of different groupings.

The algorithm presented in Section IV is a postprocessing step, which operates on a local quantity $L(r)$ that is high in presence of edges. In our experiments, we use the local contourness proposed in [26], which can be computed by means of only two convolutions. On the other hand, higher performance can be achieved by means of local edge strengths based more sophisticated schemes. For instance, the average value of $F$ that is achieved with the approach proposed in [19] for the Berkeley dataset ($\bar{F} = 0.63$) is higher than the one that we achieve ($\bar{F} = 0.47$). However, the local edge strength $p_B(r)$ proposed in [19] has two drawbacks: first, it is computationally demanding, as it requires 13 convolutions with Gabor filters and several additional nonlinear machine learning operations. Second, the resulting contours are blurred (Fig. 20). As pointed out in [67], most of the procedures used for quantitative performance evaluation do not penalize sufficiently such undesired smoothing.

In order to measure the speed of the proposed contour detector, we run its Matlab implementation 1000 times for a grayscale image ($321 \times 481$ pixel) on a Pentium 4 CPU 3.00 GHz, under Windows XP. Table I details the time required for each step, averaged over the 1000 runs.

VII. SUMMARY AND CONCLUSIONS

The main contribution of this paper is a new morphological operator, called adaptive pseudo-dilation, that we use for edge grouping according to the GLGC. A simple and computationally efficient algorithm for APD is provided. The operator defined in (4) is a generalized Minkowski sum, where a context-dependently structuring element is intersected with the Voronoi cell of
is the structuring element. Since $r$ is the $d$-dimensional signals as soon as the indexes $r_k$, the following equality holds:

$$\Delta_{\mathcal{R}}(r, r_k) < 1 \iff \Delta_{\mathcal{R}}[r, q(r)] < 1, \quad \forall r \in V(r_k).$$  \hspace{1cm} (A1)

Or, equivalently, by the hypothesis

$$A \cap V(r_k) = R_k \cap V(r_k).$$ \hspace{1cm} (A2)

Let us take the union of both the sides, on all the points $r_k \in b$

$$\bigcup_k A \cap V(r_k) = \bigcup_k R_k \cap V(r_k).$$  \hspace{1cm} (A3)

With basic calculus, and the observation that the union of all the Voronoi cells $V(r_k)$ is the entire plane, we can show that the first member of (A3) is equal to $A$. This completes the proof.

### APPENDIX A

We formally prove that APD defined in (1) is equal to the set $A$ of those points for which $\Delta_{\mathcal{R}}[r, q(r)] < 1$. Specifically, we prove the following.

**Theorem:**

**Hypothesis:**

$$r \in A \iff \Delta_{\mathcal{R}}(r, r_k) < 1, \quad r \in \mathbb{R}^2, \quad r_k \in b$$

$$r \in A \iff \Delta_{\mathcal{R}}[r, q(r)] < 1$$

**Proof:**

By the definition of $q(r)$, we have $q(r) = r_k, \forall r \in V(r_k)$. Therefore, we have

$$\Delta_{\mathcal{R}}(r, r_k) < 1 \iff \Delta_{\mathcal{R}}[r, q(r)] < 1, \quad \forall r \in V(r_k).$$

**Thesis:**

$$A = \bigcup_k R_k \cap V(r_k).$$

**Proof:**

We used APD for contour detection in combination with a multithreshold algorithm that generalizes hysteresis thresholding. We tested the proposed approach on a large set of natural images. Both qualitative and quantitative comparison show the superiority of the proposed approach with respect to the other studied techniques in terms of texture suppression, preservation of low contrast contours, and insensitivity to the values of the input parameters.

### TABLE I

**TIME REQUIRED BY EACH STEP PERFORMED BY THE PROPOSED ALGORITHM**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local edge strength $L(r)$ (including thinning)</td>
<td>1.754</td>
</tr>
<tr>
<td>GLGC Pruning ($T_1 + T_2 + T_3$)</td>
<td>0.269</td>
</tr>
<tr>
<td>Adaptive size structuring element ($T_1$)</td>
<td>0.237</td>
</tr>
<tr>
<td>Adaptive pseudo-dilation ($T_2$)</td>
<td>0.097</td>
</tr>
<tr>
<td>Elimination of small groups ($T_3$)</td>
<td></td>
</tr>
<tr>
<td>APD Reconstruction</td>
<td>1.208</td>
</tr>
</tbody>
</table>

**APPENDIX B**

Let $x[k]$ be the $k$th sample of a positive digital signal $x$, $x[k] > 0$. We prove that under the hypothesis (13), any shift-invariant operator $\mathcal{D}$ must be expressed by a generalized convolution in the following form:

$$\{\mathcal{D}(x)[k] = \sup_s x[s, w[k - s]] \hspace{1cm} (B1)$$

where $w[k]$ is the structuring element.

**Proof:** Combining the two requirements (13), we obtain

$$\mathcal{D} \left( \sup_k \alpha_k x_k \right) = \sup_k \alpha_k \mathcal{D}(x_k).$$ \hspace{1cm} (B2)

We observe that, for any signal $x$, the following equality holds:

$$x = \sup_k x[k] \delta_k, \quad \delta_k[s] = \begin{cases} 1, & s = k \\ 0, & s \neq k. \end{cases}$$ \hspace{1cm} (B3)

Therefore, combining (B2) and (B3), we have

$$\mathcal{D}(x) = \mathcal{D} \left( \sup_k x[k] \delta_k \right) = \sup_k x[k] \mathcal{D}(\delta_k).$$ \hspace{1cm} (B4)

As decomposition (B3) is applicable to any signal, it holds for $\mathcal{D}(\delta_k)$ as well. Therefore, we can write $\mathcal{D}(\delta_k) = \sup \theta_{k,s} \delta_s$, where $\theta_{k,s}$ is the $s$th sample of $\mathcal{D}(\delta_k)$. Since $\mathcal{D}$ is shift-invariant, it must be $\theta_{k,s} = \theta[s - k]$. Therefore, (B4) becomes

$$\mathcal{D}(x) = \sup_{k,s} x[k] w[s - k] \delta_s$$ \hspace{1cm} (B5)

which is equivalent to (B1). This completes the proof. This proof holds for $n$-dimensional signals as soon as the indexes $k$ and $s$ are meant to be $n$-dimensional.

**REFERENCES**

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