Multichannel EEG Visualization
Caat, Michael ten

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
2008

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):

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Chapter 3

Data-Driven Visualization of Multichannel EEG Coherence with Functional Units

Abstract

Synchronous electrical activity in different brain regions is generally assumed to imply functional relationships between these regions. A measure for this synchrony is electroencephalography (EEG) coherence, computed between pairs of signals as a function of frequency. A typical data-driven visualization of electroencephalography (EEG) coherence is a graph layout, with vertices representing electrodes and edges representing significant coherences between electrode signals. A drawback of this layout is its visual clutter for multichannel EEG. To reduce clutter, we define a functional unit (FU) as a data-driven region of interest (ROI). An FU is a spatially connected set of electrodes recording pairwise significantly coherent signals, represented in the coherence graph by a spatially connected clique. We present three methods to detect FUs. One is a maximal clique based (MCB) method (time complexity $O(3^{n/3})$, with $n$ the number of vertices). Another is a more efficient watershed based (WB) method (time complexity $O(n^2 \log n)$). To reduce the potential over-segmentation of the WB method, the improved watershed based (IW B) method (time complexity $O(n^2 \log n)$) merges basins representing FUs during the segmentation if they are spatially connected and if their union is a clique. The WB and IWB method both are up to a factor of 100,000 times faster than the MCB method for a typical multichannel setting with 128 EEG channels, thus making interactive visualization of multichannel EEG coherence possible. Results show that, considering the MCB method as the gold standard, the difference between IWB and MCB FU maps is smaller than between WB and MCB FU maps. We also introduce two novel group maps for data-driven group analysis as extensions of the IWB method. First, the group mean coherence map preserves dominant features from a collection of individual FU maps. Second, the group FU size map visualizes the average FU size per electrode across a collection of individual FU maps. Finally, we employ an extensive case study to evaluate the IWB FU map and the two new group maps for data-driven group analysis. Results, in accordance with conventional findings, indicate differences in EEG coherence between younger and older adults. However, they also suggest that an initial selection of hypothesis-driven ROIs could be extended with additional data-driven ROIs.
3.1 Introduction

Electroencephalography (EEG) is a method to measure the electrical activity of the brain using electrodes attached to the scalp at multiple locations. Synchronous electrical activity in different brain regions is generally assumed to imply functional relationships between these regions. A measure for this synchrony is EEG coherence, calculated between pairs of electrode signals as a function of frequency (Halliday et al. 1995, Maurits et al. 2006).

Related studies of functional brain connectivity use other noninvasive neuroimaging techniques, including magnetoencephalography (MEG) (Bosboom et al. 2006, Chen et al. 2003, Srinivasan et al. 1999) and functional magnetic resonance imaging (fMRI) (Achard et al. 2006, Cordes et al. 2002, Salvador et al. 2005b, Salvador et al. 2005a). A typical visualization of EEG, MEG, and fMRI coherence, is a two-dimensional graph layout. Vertices represent electrodes, superconducting quantum interference devices (SQUIDS), or fMRI regions of interest (ROIs), respectively. Edges represent significant coherences between electrode signals, SQUID signals, or fMRI-ROI time series, respectively. Vertices are commonly visualized as dots and edges as lines. For multichannel EEG (e.g., (Kamiński et al. 1997, Stein et al. 1999)), MEG (e.g., (Chen et al. 2003, Srinivasan et al. 1999)), and fMRI (e.g., (Achard et al. 2006, Salvador et al. 2005b)), this layout may suffer from a large number of overlapping edges, resulting in a cluttered visualization.

In the case of EEG, the reorganization of vertex positions (Fruchterman and Reingold 1991) to reduce clutter is not appropriate, because the electrodes have meaningful positions. Other solutions reorganize edges or vary visual attributes of the edges (Wong et al. 2003, Herman et al. 2000), but do not reduce the number of edges. Several methods divide EEG electrodes (Sarnthein et al. 1998, Gladwin et al. 2006), MEG SQUIDS (Bosboom et al. 2006), or fMRI voxels (Salvador et al. 2005a) into disjoint hypothesis-driven ROIs and study coherences within or between ROIs. Other methods set out ROIs representing EEG electrodes (Kamiński et al. 1997, Franaszczuk et al. 1994), MEG SQUIDS (Srinivasan et al. 1999), or fMRI-ROIs (Achard et al. 2006) along rows and columns, thus obtaining a square contingency table. By arranging ROIs along rows and columns of a matrix, the spatial relations are lost.

Visualization of multichannel EEG (at least 64 electrodes) is not always managed well (ten Caat et al. 2005, ten Caat et al. 2007c, ten Caat et al. 2007d). Researchers often employ a hypothesis-driven definition of certain ROIs in which all electrodes are assumed to record similar signals because of volume conduction effects (Lachaux et al. 1999). As an alternative for the hypothesis-driven approach, we introduce three methods to detect data-driven ROIs referred to as functional units (FUs) (ten Caat et al. 2007d). An FU is represented in the coherence graph by a spatially connected clique. A clique is a vertex set in which every two-element subset is connected by an edge. A clique \( C \) is maximal when it is not contained in any larger clique (‘larger’ meaning having more vertices). Within one FU, each pair of vertices represents two significantly coherent electrode signals. In any group of vertices other than a clique, there are two vertices representing two electrode signals which are not significantly coherent. Because larger ROIs are assumed to correspond to stronger source signals, larger FUs are considered to be more interesting. Therefore, we focus on maximal cliques, with vertex sets as large as possible.
Our first FU detection method is a maximal clique based (MCB) method (ten Caat et al. 2007d). Our second method is a watershed based (WB) method that detects spatially connected cliques in a greedy way (ten Caat et al. 2007e). However, it suffers from potential over-segmentation problems. A third method is an improved watershed based (IWB) method for FU detection. It merges FUs if they are spatially connected and if their union is a clique, thus reducing over-segmentation obtained with the WB method. A functional unit map shows the FU distribution for individual datasets. Each FU is a collection of Voronoi cells with identical gray value, with different gray values for adjacent FUs. FUs are connected by a line if the average coherence between FUs is significant.

In addition to individual dataset analysis, we introduce two new group maps for data-driven group analysis of multichannel EEG coherence as extensions of the IWB method. They serve as a data-driven alternative for the common hypothesis-driven selection of coherences for group analysis (Maurits et al. 2006, Gladwin et al. 2006, Knyazeva et al. 2006). First, the group mean coherence map preserves dominant features from a collection of individual FU maps. Second, the group FU size map visualizes the average FU size per electrode across a collection of individual FU maps. Results are reported for an extensive case study.

### 3.2 EEG Coherence

EEG can be recorded using currently up to 512 electrodes, labeled uniquely by a combination of letters and digits (e.g., F3, Cz, P4, as in Fig. 3.1 right). A conductive gel is applied between skin and electrodes to reduce impedance. The electrical potential is measured at all electrodes simultaneously. The measured signals are amplified, resulting in one recording channel for every electrode. If there are many electrodes, the term ‘multichannel’ or ‘high-density’ EEG is used. As a result of volume conduction (Lachaux et al. 1999), multiple electrodes can record a signal from a single source in the brain. Therefore, nearby electrodes usually record similar signals. Because sources of activity at different locations may be synchronous, electrodes far apart can also record similar signals. A measure for this synchrony is coherence, calculated between pairs of signals as a function of frequency. The coherence \( c_\lambda \) as a function of frequency \( \lambda \) for two continuous time signals \( x \) and \( y \) is defined as the absolute square of the cross-spectrum \( f_{xy} \) normalized by the autospectra \( f_{xx} \) and \( f_{yy} \) (Halliday et al. 1995), having values in the interval \([0, 1]\):

\[
c_\lambda(x, y) = \frac{|f_{xy}(\lambda)|^2}{f_{xx}(\lambda)f_{yy}(\lambda)}
\]

The cross-spectrum and auto-spectrum can be interpreted as covariance and variance as a function of frequency, respectively. An event-related potential (ERP) is an EEG recording of the brain response to a sensory stimulus. To calculate the coherence for an event-related potential (ERP) with \( L \) repetitive stimuli, the EEG data can be segmented into \( L \) segments, each containing one brain response. A significance threshold \( \phi \) for the estimated coherence is then given by (Halliday et al. 1995)

\[
\phi = 1 - p^{1/(L-1)},
\]

where \( p \) is a probability value associated with a confidence level \( \alpha \) (\( p = 1 - \alpha \)). For an overview of other common linear (and nonlinear) measures of synchrony, see (Pereda et al. 2005).
3.3 Related Work

We discuss visualizations of functional brain connectivity obtained using the noninvasive neuroimaging techniques EEG, MEG, and fMRI. MEG commonly uses up to 512 SQUIDs to measure magnetic fields induced by electrical brain activity. Similar to EEG coherence, MEG coherence is calculated between pairs of SQUID signals. fMRI measures time series of changes in cerebral blood oxygenation levels in the brain. Often, fMRI researchers compute coherence (or other similarity) values between mean time series for different ROIs which are commonly single voxels or connected sets of voxels (Cordes et al. 2002).

Although a comparison of the results obtained with different neuroimaging methods should be made carefully (Horwitz 2003), the common underlying data representation for the different types of connectivity is a graph. Therefore, we restrict the comparison for the different neuroimaging techniques to their graph visualizations and focus on hypothesis-driven and data-driven aspects. First, we consider EEG and MEG with typically up to 512 vertices, whose spatial relations can be represented by a planar graph. Later, we consider fMRI with vertices commonly representing thousands of voxels (Cordes et al. 2002). The overview also includes general graph drawing solutions.
3.3.1 EEG and MEG

EEG and MEG coherence graphs have vertices representing electrodes and SQUIDS, respectively. Most of the visualizations of EEG are applicable to MEG, and vice versa. For a two-dimensional visualization of the vertices, often planar projections are used of the three-dimensional electrode or SQUID locations on the surface of a head, usually mapping vertices to a top view of a head (e.g., Fig. 3.1 right), or sometimes to two separate side views of the left and right hemisphere (Stein et al. 1999, Sarnthein et al. 1998). Such visualizations may suffer from a large number of overlapping edges representing significant coherences, resulting in a cluttered visualization for multichannel EEG (e.g., (Kamiński et al. 1997, Stein et al. 1999); Fig. 3.1 left) or MEG (e.g., (Chen et al. 2003, Srinivasan et al. 1999, Tononi and Edelman 1998)). Existing solutions for the reduction of clutter involve an adapted visualization of the vertices and the edges.

The layout of the vertices can be changed, e.g., by a force-directed placement (Fruchterman and Reingold 1991). However, for EEG applications we prefer to maintain the spatial relationship between the vertices representing electrodes, because electrodes have meaningful positions. A different method uses an area dependent visualization of vertices of variable size (Archambault et al. 2006), but also does not preserve vertex positions. Other solutions vary (combinations of) visual attributes of vertices and edges, e.g., transparency (Wong et al. 2003), color (Chen et al. 2003, Srinivasan et al. 1999, Achard et al. 2006, Salvador et al. 2005b), saturation (Herman et al. 2000), line width (Salvador et al. 2005a, Herman et al. 2000), and line style (Salvador et al. 2005a). Nevertheless, the presence of many overlapping edges may still obscure other visualization elements, or the superposition of differently colored lines might result in an undesired mix of colors. Also the layout of the edges can be manipulated, e.g., by interactively curving away edges from the focus of attention (Wong et al. 2003). This has the undesirable side-effect that, in an already crowded field of view, the area which is out of focus will be even more crowded. Moreover, to get a complete overview of the graph, every vertex (out of up to 512 vertices for EEG coherence) has to be selected individually. Alternatively, elements (such as edges) can be left out selectively (Chiricota et al. 2003). Nevertheless, cluttered visualizations are even obtained for restrictions to the top 5% coherences for only 66 MEG SQUIDS (Chen et al. 2003), or the top 10% for 119 EEG electrodes (Fig. 3.1 left).

Existing analyses of multichannel EEG or MEG are hypothesis-driven. One method chooses a regularly distributed subset of electrodes (Maurits et al. 2006), ignoring the majority of the electrode signals. An MEG method divides channels into disjoint hypothesis-driven ROIs and maps the average coherence within a ROI to a color (Bosboom et al. 2006), ignoring coherences between ROIs. A similar EEG method divides electrodes into four disjoint ROIs and studies anterior-posterior connections between those ROIs (Sarnthein et al. 1998). Another EEG method divides (the majority of the available) electrodes into disjoint hypothesis-driven ROIs and studies coherences between these ROIs across datasets (Gladwin et al. 2006), but it does not simultaneously visualize which electrodes are part of which ROI. However, the main disadvantage of all these methods is the hypothesis-driven selection of the number and the positions of the ROIs instead of a data-driven selection.

An existing EEG approach which is data-driven sets out up to 21 electrodes along both the
3.3 Related Work

The result is a square contingency table showing coherence values for all possible electrode pairs. Each table entry is a square in which coherence is displayed between the two corresponding electrode signals as a function of frequency. By arranging the electrodes along the rows and the columns of the matrix, the spatial relations are lost. As a result, consecutive entries in the table do not need to imply coherence between pairs of signals recorded at adjacent electrodes on the scalp. Similarly, a square contingency table is created for 78 MEG SQUIDS sorted into four hypothesis-driven ROIs (Srinivasan et al. 1999) (left/right, anterior/posterior). Each table entry is square with the coherence of the corresponding signals mapped to a color. A different data-driven EEG approach first localizes dipoles corresponding to maximally independent components in the data, and then calculates and visualizes coherence between dipole activities (Delorme et al. 2002, Makeig et al. 2004, De Vico Fallani et al. 2007). However, dipole source solutions are not unique (Srinivasan 1999).

Another approach is restricted to local EEG coherence, which is defined as the coherence between two spatially neighboring electrodes (Rappelsberger and Petsche 1988, Schack et al. 1999). It requires additional methods to study coherences between electrodes which are not direct spatial neighbors. Another visualization creates a map of topographic submaps (Nolte et al. 2004), with one subplot for each electrode visualizing the coherence between itself and every other electrode. It does not explicitly visualize coherence between electrodes by connecting lines. As a consequence, every topographic subplot (out of up to 512 subplots) needs to be studied separately to obtain a complete overview. Another drawback is that local coherences dominate the visualization (Nolte et al. 2004). A subselection of two topographic submaps out of 128 is made by Knyazeva et al. (2006), without providing a complete overview of all coherences.

3.3.2 fMRI

For fMRI coherence, usually a limited number of so-called seed (or reference) voxels is selected on the basis of prior anatomical or functional information. However, the anatomy may be abnormal, and the choice of seed points may affect the results (Cordes et al. 2002). Nonetheless, an individual seed point or a spatially connected set of voxels including a seed point is considered as a ROI having a (mean) time series. Vertices represent ROIs and can be visualized three-dimensionally (Worsley et al. 2005) or two-dimensionally. A two-dimensional visualization uses, e.g., a planar projection of three-dimensional ROI positions or an approximation of functional distances by graphical distances using metric multidimensional scaling (Salvador et al. 2005a). An edge represents a significant similarity between two ROI time series. The visualization of edges as lines may lead to clutter (Achard et al. 2006, Salvador et al. 2005b, Salvador et al. 2005a, Worsley et al. 2005).

Filtering edges may still lead to cluttered visualizations (Achard et al. 2006). Other visualizations set out ROIs along the rows and columns, thus obtaining a square contingency table. Each table entry is a square with a similarity value between the two corresponding signals mapped to a color (Srinivasan et al. 1999, Achard et al. 2006). Existing data-driven graph clustering algorithms include hierarchical cluster analysis (Cordes et al. 2002) and independent component analysis (ICA) (Delorme et al. 2002, Makeig et al. 2004, van de Ven et al. 2004). The result of
hierarchical cluster analysis can be visualized as a dendrogram (Salvador et al. 2005a), showing the ROIs as leaves of a binary tree, thus losing the spatial relations between the ROIs. Also, ROIs can be visualized as colored volumes of interest (van de Ven et al. 2004) which may occlude each other. For the same reason, we do not favor three-dimensional EEG visualizations. Alternatively, ROIs can be visualized on anatomical slices (Cordes et al. 2002, Salvador et al. 2005a, Sun et al. 2004). However, a large number of two-dimensional slices is required to obtain a complete overview of a three-dimensional volume. Sometimes, instead of an explicit visualization of the connection between ROIs (e.g., with a line), all ROIs in one cluster are colored identically, with different colors and/or separate slices for different clusters (Salvador et al. 2005a).

3.3.3 Conclusion

The overview of related work has concentrated on the requirements we posed on an EEG coherence visualization: it should be (1) data-driven, (2) preserve electrode locations, (3) minimize visual clutter, and (4) present an overview. Many of the discussed methods still suffer from visual clutter or relocate vertices and edges and therefore do not meet requirement (2) or (3). On the other hand, existing methods which do meet requirements (2,3) are hypothesis-driven, thus failing to meet our requirement (1). In summary, the method proposed in this chapter combines a number of features which no single technique currently provides.

3.4 Data Representation

3.4.1 Experimental Setup

Here, brain responses from two groups of five younger (34 ± 10 years, mean ± standard deviation) and five older (62 ± 8 years) adults are studied, which were recorded using an EEG cap with 119 scalp electrodes. During a so-called P300 experiment, each participant was instructed to count target tones of 2000 Hz (probability 0.15), alternated with standard tones of 1000 Hz (probability 0.85) which were to be ignored. After the experiment, the participant had to report the number of perceived target tones. For each dataset, brain responses to 20 target tones were recorded in $L = 20$ segments of 1 s. EEG coherence is influenced by the choice of reference. We chose to use an average reference, which is a close-to-optimal approximation to a reference-free recording in the case of 128 electrodes (Maurits et al. 2006, Nunez et al. 1997). A procedure from Neurospec was adopted to compute the coherence (www.neurospec.org). We first averaged over segments and then over adjacent spectral lines in predefined frequency bands. Frequencies between 1 and 30 Hz are typically studied clinically. We calculated the average coherence within five EEG frequency bands (1-3, 4-7, 8-12, 13-20, and 21-30 Hz), because EEG synchrony varies with frequency (Maurits et al. 2006, Nunez et al. 1997). For 119 electrodes, in total 7021 coherence values were computed per frequency band. If the conductive gel accidentally connected two adjacent electrodes, very high coherences were measured. Coherences higher than 0.99 were therefore ignored. Typically, this threshold value eliminates approximately 0.01% of the coherences. Note also that using Eqn. [3.1] for determining signif-
icance levels is a coarse approximation, since it does not take the number of spectral lines per band into account. However, this approximation only overestimates the significance level, and does not influence the visualization method itself.

3.4.2 EEG Coherence Graph

The data are represented by a coherence graph with vertices representing electrodes. Coherences above the significance threshold (Eqn. 3.1) are represented by edges, coherences below the threshold are ignored. To determine spatial relationships between electrodes, a Voronoi diagram is employed which partitions the plane into regions of points with the same nearest vertex (Voronoi 1908). For EEG data, the vertex set equals the set of electrode positions (Fig. 3.1 right). The vertices are referred to as (Voronoi) centers, the region boundaries as (Voronoi) polygons. The area enclosed by a polygon is called a (Voronoi) cell. We call two cells Voronoi neighbors if they have a boundary in common. A collection of cells $C$ is called Voronoi-connected if for a pair $\phi_0, \phi_n \in C$ there is a sequence $\phi_0, \phi_1, \ldots, \phi_n$ of cells in $C$ with each pair $\phi_{i-1}, \phi_i$ consisting of Voronoi neighbors. Cells, vertices, and electrodes are interchangeable for the use with the terms “Voronoi neighbor” and “Voronoi-connected”.

3.5 FU Detection

Whereas there are many unsupervised graph clustering methods, e.g., hierarchical clustering and ICA (see Section 3.3), our choice is motivated by the type of cluster we desire. As a result of volume conduction (Lachaux et al. 1999), multiple electrodes can record a signal from a single source. Consequently, a spatially connected set of electrodes recording similar signals is considered as a data-driven ROI (a cluster). Such a ROI is referred to as functional unit (FU) and is represented in the EEG coherence graph by a clique consisting of a set of spatially connected vertices.

Recall that larger ROIs are assumed to correspond to stronger source signals and are considered to be more interesting. Therefore, our first method for FU detection is primarily based on the detection of maximal cliques (Bron and Kerbosch 1973, Tomita et al. 2006). We adapt this method to detect spatially connected sets of vertices (ten Caat et al. 2007d). Our second method for FU detection is based on watersheds, an efficient method for detecting spatially connected segments (Roerdink and Meijster 2000). We adapt this method to detected cliques in a greedy way (ten Caat et al. 2007e). However, it does not avoid the oversegmentation problem well-known for watersheds. A third method, also based on watersheds, reduces over-segmentation.

3.5.1 Maximal Clique Based (MCB) Method

Maximal Cliques

Bron and Kerbosch (B&K) (Bron and Kerbosch 1973) developed a method to detect all maximal cliques in a graph. It first branches the problem, and bounds unsuccessful branches. Its recursive
procedure maintains three dynamic vertex sets:

- the set \( \text{compsub} \) contains an increasing or decreasing clique;
- the set \( \text{candidates} \) contains vertices that are connected to all vertices in \( \text{compsub} \) and that can be added to \( \text{compsub} \);
- the set \( \text{not} \) contains vertices that are connected to all vertices in \( \text{compsub} \) and that were added to \( \text{compsub} \) previously.

At each call of the procedure, the first vertex \( v \) from the set \( \text{candidates} \) is selected, and is added to \( \text{compsub} \) and removed from \( \text{candidates} \). Next, \( \text{newcandidates} \) is the intersection of \( \text{candidates} \) and the neighborhood of \( v \). Similarly, \( \text{newnot} \) is the intersection of \( \text{not} \) and the neighborhood of \( v \). If both \( \text{newcandidates} \) and \( \text{newnot} \) are empty, then \( \text{compsub} \) is a maximal clique. This procedure is repeated recursively with local sets \( \text{newcandidates} \) and \( \text{newnot} \), until the candidate set is empty. In case the procedure is not repeated with \( \text{newcandidates} \) and \( \text{newnot} \), the vertex most recently added to \( \text{compsub} \) (vertex \( v \)) is removed from \( \text{not} \) and this branch is bounded.

An alternative selection of vertex \( v \) is more efficient if there is a large number of overlapping cliques (Bron and Kerbosch 1973). From the set \( \text{candidates} \), the vertex \( v^* \) is selected that has the largest number of connections with the other vertices in \( \text{candidates} \). If there are more such vertices, then one of these is randomly selected. Further, it is assured that \( v^* \) is not connected to the vertex just added to \( \text{not} \). The worst-case time complexity for detecting all maximal cliques is \( O(3^{n/3}) \), with \( n \) the number of vertices, because \( 3^{n/3} \) is the highest number of cliques (Tomita et al. 2006). In practice, performance of maximal clique detection strongly depends on graph structure (Wood 1997).

**Voronoi-Connected Maximal Cliques**

We extend the B&K method such that it only detects maximal cliques consisting of Voronoi-connected vertices. The three dynamic vertex sets are maintained, but the set \( \text{candidates} \) is split into a set \( \text{currentcand} \) and a set \( \text{complcand} \).

- The set \( \text{currentcand} \) contains the candidates that are Voronoi neighbor of at least one element in \( \text{compsub} \); only these can be added to \( \text{compsub} \) at the current step.
- The set \( \text{complcand} \) is the complement of \( \text{currentcand} \) in \( \text{candidates} \).

At each call, the element from \( \text{currentcand} \) which has the largest number of connections with the other candidates (\( \text{currentcand} \cup \text{complcand} \)) is added to \( \text{compsub} \). Let this element be \( v' \). The set \( \text{newcurrentcand} \) is the intersection of \( \text{currentcand} \) and the neighborhood of \( v' \) (in the coherence graph), united with the Voronoi-neighbors of \( v' \) in \( \text{complcand} \). The set \( \text{newcomplcand} \) is the intersection of \( \text{complcand} \) and the neighborhood of \( v' \) (in the coherence graph), minus the Voronoi-neighbors of \( v' \) in \( \text{complcand} \). The set \( \text{(new)not} \) is maintained as before. This is repeated
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until \textit{newcurrentcand} is empty. If \textit{newnot} is also empty, then \textit{compsub} is a Voronoi-connected maximal clique.

Fig. 3.2 illustrates maximal clique detection with the B&K algorithm (A and B) and Voronoi-connected maximal clique detection with the MCB method (C), for a graph with the adjacency matrix shown in Table 3.1. The first B&K iteration has an empty \textit{not} set (A). One of the later recursive iterations of the B&K method returns to the initial situation with all vertices in the \textit{candidates} set (not shown), puts the selected vertex labeled \( c \) in the not set (B1), and selects the vertex with the highest degree in the \textit{candidates} set (B2). Whereas the B&K method detects maximal cliques which can consist of more than one spatial component (A5), the MCB method detects spatially connected cliques instead (C4). (For the MCB method, the use of the \textit{not} set is the same as for the B&K method and is therefore not explicitly illustrated.)

The following detailed description contains (row, column) references to Fig. 3.2. C5. Vertex positions. Vertices are spatial neighbors if they are 4-connected (e.g., the spatial neighbors of vertex \( d \) are vertices \( a, e, \) and \( g \)). A. Iteration of B&K maximal clique detection with empty \textit{not} set. It starts with all nine vertices in the set candidates (not illustrated). A1. Then the vertex \( c \) with the highest degree (following Table 3.1) is first added to \textit{compsub}; its adjacent vertices are in \textit{candidates}. (Vertices not part of any set are shown as a black dot.) A2-A4. At every next step, the vertex with the highest degree in \textit{candidates}, let us say \( v \), is added to \textit{compsub}. (In the case of A3 and A4, multiple vertices have the highest degree and vertices \( g \) and \( f \) are selected randomly, respectively.) Further, vertices not adjacent to \( v \) are removed from \textit{candidates} (the removed vertices are denoted by \( \Gamma^c(v) \)). This continues until \textit{candidates} is empty. At A2, \( v = b, \; \Gamma^c(v) = \{d\} \); at A3, \( v = g, \; \Gamma^c(v) = \{e\} \); at A4, \( v = f, \; \Gamma^c(v) = \{h\} \); at A5, \( v = i, \; \Gamma^c(v) = \emptyset \). Now, \textit{compsub} = \{\( b, c, f, g, i \)\} is a maximal clique, because \textit{candidates} = \( \emptyset \) (and \textit{not} = \( \emptyset \)). B. A later iteration for B&K maximal clique detection returns to the situation preceding A1 with all vertices in the candidates set, and puts the first selected vertex \( c \) into the \textit{not} set. B1. Vertex \( c \) which was previously selected first (see A1) is now in the \textit{compsub}. B2-B4. Similar to A2-A4. B5. Similar to but different from A5, this leads to a situation with \textit{candidates} = \{\( b, f, g, i \)\}, and \textit{not} = \{\( c \)\}. This implies that the maximal clique \{\( b, c, f, g, i \)\} has been found before. C. MCB Voronoi-connected maximal clique detection with same starting point as A (with \textit{not} = \( \emptyset \)). C1. The vertex \( c \) with the highest degree is first added to \textit{compsub}; its adjacent vertices (see Table 3.1) are in \textit{currentcand} if they are a spatial neighbor ((\( \{b, f\} \)), or otherwise in \textit{complcand}. C2-C4. At every next step, the element from \textit{currentcand} which has the largest number of connections with the other candidates (\textit{currentcand} \( \cup \textit{complcand} \)) is added to \textit{compsub}. The spatial neighbors of \( v' \) in \textit{complcand} (denoted by \( \Lambda(v') \)) are moved from \textit{complcand} to \textit{currentcand}. Further, vertices not adjacent to \( v' \) are removed from both \textit{currentcand} and \textit{complcand} (the removed vertices we denote by \( \Gamma^c(v') \)). This continues until \textit{currentcand} is empty. At C2, \( v' = b, \; \Lambda(v') = \{e\}, \; \Gamma^c(v') = \{d\} \); at C3, \( v' = f, \; \Lambda(v') = \{i\}, \; \Gamma^c(v') = \{h\} \); at C4, \( v' = i, \; \Lambda(v') = \emptyset, \; \Gamma^c(v') = \{e\} \). C4. \textit{compsub} = \{\( b, c, f, i \)\} is a spatially connected maximal clique, because \textit{currentcand} = \( \emptyset \) (and \textit{not} = \( \emptyset \)). Remaining vertices in \textit{complcand} are in the adjacency list of all vertices in \textit{compsub} but are not a spatial neighbor of any vertex in \textit{compsub}. 
Table 3.1. Adjacency matrix for vertices a through i in Fig. 3.2 (1 (0) means (not) connected). Diagonal entries are zero, meaning that vertices are not self-connected.

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Figure 3.2. Illustration of maximal clique detection with the B&K algorithm for an iteration with an empty not set (A) and an iteration with a non-empty not set (B), and Voronoi-connected maximal clique detection with the MCB method (C), for a graph with adjacency matrix as in Table 3.1. For explanation, see text.

FU Labeling

Every vertex can be part of multiple (Voronoi-connected) maximal cliques. To assign a unique label to every vertex, a quantity total strength $S$ for an undirected (sub)graph $G = (V, E)$ is defined as the sum of all edge values (ten Caat et al. 2007d):

$$S(G) = \sum_{i,j} \{c(v_i, v_j) \mid v_i, v_j \in V : j > i\}. \quad (3.2)$$

This value is not normalized for the size of $E$. Consequently, if two graphs have an equal average coherence, the graph with more vertices has a higher total strength. Next, all cliques are queued in decreasing order by their total strength. Then the following labeling procedure is repeated, until there are no more cliques or until all vertices are labeled. The first clique is removed from
the queue, and all its vertices are assigned a unique label and are removed from the other cliques. If necessary, the changed cliques are separated into Voronoi-connected components. For all changed cliques, the total strength is recomputed before they are put in the appropriate position in the sorted queue. After completion of the labeling procedure, every set of identically labeled vertices is an FU.

### 3.5.2 Watershed Based (WB) Method

As an alternative to the MCB method, we present a greedy method approximating maximal cliques on the basis of the watershed transform (Roerdink and Meijster 2000). In the usual watershed algorithm, a subset of all local minima is selected as markers. Markers are labeled and are associated with basins. Basins contain vertices with the same label as the corresponding marker and are extended as follows, using the watershed implementation based on ordered queues (Beucher and Meyer 1993). The first vertex \( v \) is removed from a queue of vertices sorted in decreasing order of priority. Every unlabeled neighbor \( v' \) of \( v \) receives the same label as \( v \) and is put into the queue with a priority depending on the value of \( v' \), but not higher than the priority of \( v \). This continues until the queue is empty.

Now we modify the usual watershed transform in order to obtain spatially connected sets of electrodes, where all electrodes in a given set have recorded mutually significantly coherent signals. This modification concerns two steps in the watershed transform: (i) choice of markers; (ii) use of an edge queue instead of a vertex queue. We explain these two points in more detail.

First, we define a marker as an electrode recording a signal that is locally maximally coherent with signals of its spatially neighboring electrodes. Because the EEG coherence graph has edge values instead of vertex values, we first assign a coherence value to each vertex by computing the average of the edge values between this vertex and all its Voronoi neighbors. Then, we select all vertices which are local maxima as markers to be associated with basins, because those vertices are locally maximally similar to their spatially neighboring vertices. Note that we choose all local maxima as markers, instead of a small subset as is usually done when the watershed algorithm is applied to digital images. In our case the over-segmentation problem is less severe, because the number of electrodes is an order of magnitude smaller than the number of pixels in an image. If the number of basins (i.e., clusters) found is still too large, we can suppress basins below a certain size in a post-processing step.

The second point concerns the type of queue we use. Whereas the usual queue-based implementation of the watershed transform applied to digital images uses a vertex queue sorted in increasing order of value (Beucher and Meyer 1993), we use an edge queue sorted in decreasing order of coherence value. (The vertex values are only used for defining the markers.) In case the coherence graph has multiple identical edge values (which did not occur for our datasets), an ordered queue consisting of queues with identically valued elements can be used, as for digital images which usually contain multiple identically valued vertices (Beucher and Meyer 1993).

The WB method for greedy Voronoi-connected clique detection maintains the following dynamic vertex sets.
• $bsn_i$ contains a sorted list of the vertices in basin $i$.
• $L(v)$ contains the basin label of vertex $v$.
• $adjCohBsn_i$ contains a list of vertices (sorted by vertex number) which are adjacent to each of the vertices in $bsn_i$ in the coherence graph.
• $queue$ contains edges in decreasing order. When vertex $v$ receives a label, an edge $e = (v, v')$ is added to $queue$ for each unlabeled Voronoi neighbor $v'$ of $v$, provided that the corresponding edge value exceeds the significance threshold (Eqn. 3.1).

(Step 1) The edge queue is initialized with edges (corresponding with a significant coherence) between markers and their Voronoi neighbors. The first edge $(v, v')$ in this queue corresponds to the highest similarity (coherence) between any vertex $v'$ outside and a Voronoi neighboring vertex $v$ inside a basin. Therefore, vertex $v'$ is the first candidate to be added to a basin.

(Step 2) The main procedure consists of the following steps. Remove the first edge, say $e = (v, v')$ from $queue$. In case vertex $v'$ was also labeled between the insertion and removal of $e = (v, v')$, nothing is done and the procedure continues with a new edge. Otherwise ($v'$ is unlabeled), there are two cases. (i) In case $v' \in adjCohBsn_{L(v)}$ (ln. 19), $v'$ receives label $L(v)$ and (ii) $adjCohBsn_{L(v)}$ is replaced by its intersection with the neighborhood of $v'$ in the coherence graph (ln. 21); (iii) $v'$ is added to $bsn_{L(v)}$ (ln. 22); (iv) $queue$ is extended with the edges between $v'$ and its Voronoi-neighbors (ln. 23-27), provided that corresponding edge values exceed the significance threshold. In the other case, if $v' \notin adjCohBsn_{L(v)}$, $v'$ is not labeled (yet). This procedure is repeated until $queue$ is empty. Each basin then corresponds to an FU.

The time complexity of the WB method is $O(n^2 \log n)$, with $n$ the number of vertices (ten Caat et al. 2007e), which can be seen as follows. Step 1 consists of creating a sorted edge queue, so has complexity $n \log n$, because the order of the number of edges between Voronoi neighbors is the same as the order of the total number of edges in a planar graph (which is $O(n)$). In step 2, the following steps are executed $O(n)$ times with sorted vertex sets of at most $n$ vertices: (i) binary search for the presence of a vertex in a vertex set ($O(\log n)$); (ii) binary search for the insertion of a vertex into a vertex set ($O(\log n)$); (iii) intersecting two vertex sets ($O(n)$); (iv) insertion of at most $n$ edges into the sorted queue($O(n \log n)$). Step (iv) has a higher time complexity than (i)-(iii). Therefore, the time complexity for step 2 ($O(n^2 \log n)$) is higher than for step 1 ($O(n \log n)$), which makes the total time complexity of the WB method equal to $O(n^2 \log n)$.

### 3.5.3 Improved Watershed Based (IWB) Method

Over-segmentation is a potential problem of the WB method. To reduce over-segmentation, two spatially neighboring FUs are merged if their union is a clique in the coherence graph. To obtain the improved watershed based (IWB) algorithm (Alg. 1) we insert lines 11-15 and lines 29-42 in the pseudocode of the WB algorithm (see also (ten Caat et al. 2007e)). In words, the difference between the WB and IWB method is the following. In case vertex $v'$ was labeled between the insertion and removal of $e = (v, v')$, nothing is done if the label of $v'$ is equal to the label of $v$. Otherwise ($L(v') \neq L(v)$), see line 29, the following steps are executed consecutively
Figure 3.3. IWB FU map (EEG frequency band 1-3 Hz, dataset young 5). Top view, nose on top. Left: A circle with a cross inside indicates the geographic center of all Voronoi centers belonging to one FU and has the same gray value. The geographic center can be located in a cell not belonging to the corresponding FU. Middle: The same FU map, but only with FUs larger than 5 cells. White Voronoi cells are part of smaller FUs. Right: Lines connect FU centers if the inter-FU coherence exceeds the significance threshold (Eqn. [3.7]). The color of the line depends on the inter-FU coherence (see color bar, with minimum corresponding to the coherence threshold $\phi \approx 0.22$ for $p = .01$). Lines are drawn in the order from low to high inter-FU coherence values.

(for notation purposes, define $\psi$ as $L(v')$): (i) check if all vertices in $bsn_{L(v)}$ are in $adjCohBsn_{\psi}$, and vice versa (line 32). (ii) Replace $bsn_{L(v)}$ by the union of itself with $bsn_{\psi}$, because their union is a spatially connected clique in the coherence graph (line 33); (iii) all vertices in $bsn_{\psi}$ receive the label $L(v)$ (lines 34-36); (iv) $adjCohBsn_{L(v)}$ is replaced by the intersection of itself with $adjCohBsn_{\psi}$ (line 37); (v) $bsn_{\psi}$ and $adjCohBsn_{\psi}$ are made empty (line 38).

In the algorithm, the operation $insertEdgeSort(e(v,v'),queue)$ inserts edge $e(v,v')$ into the appropriate position in an edge queue $queue$ which is decreasingly sorted by edge value; similarly, $insertVSort(v,vqueue)$ inserts vertex $v$ into the appropriate position in vertex queue $vqueue$ which is decreasingly sorted by vertex number; $dequeue(queue)$ returns and removes the first edge from an edge queue $queue$; $intersect(.,.)$ gives the intersection of two sorted vertex sets; $merge(.,.)$ gives the union of two sorted vertex sets (without duplicates); $setInSet(V,V')$ returns ‘true’ if the sorted vertex set $V$ is a subset of the sorted vertex set $V'$, and ‘false’ if not. The size of a vertex set is denoted by $|.|$.

One adaptation further improves the average performance in practice. A matrix $bsnMat$ is created with the basins set out along the rows and the columns, and is initialized with only ones (lines 11-15). If two spatially neighboring basins $b_i$ and $b_j$ together are not a clique, then $bsnMat(b_i,b_j)$ and $bsnMat(b_j,b_i)$ are set to zero (line 40). In that case, basins $b_i$ and $b_j$ cannot be merged later either, and lines 31-41 are skipped the next time that $b_i$ and $b_j$ are candidates to be merged.

The difference between the WB and the IWB method affects the time complexity as follows. (i) line 32: the check to see if one sorted list is part of another has time complexity $O(n)$. Each of the next steps also has time complexity $O(n)$ for sorted lists of vertices of at most length
3.6 FU Visualization

3.6.1 FU Map for Individual Dataset Analysis

FU Map Coloring

An FU map visualizes each FU as a set of Voronoi cells with identical gray value, with different gray values for adjacent FUs. The problem of coloring the FUs corresponds to the coloring of a plane graph, assigning different colors to adjacent vertices. Humans can detect one color among a total of about five to seven different colors rapidly and accurately (Healey 1996), whereas there can be more than five FUs. However, for any plane graph, four colors are sufficient (Robertson et al. 1996).

To find a four-coloring of the FUs, the FUs are sorted by their number of neighboring FUs, from high to low. From a set of four available colors, each FU is assigned (one by one) a color different from its neighbors. If there are already four different colors among its neighbors, there is an impasse. To solve the impasse, we make use of a c-d Kempe chain, which is a connected component of a colored graph with vertices colored c or d. Interchanging the two colors in a Kempe chain is referred to as Kempe chaining (Morgenstern and Shapiro 1991). This is executed randomly with neighbors of the impasse FU, until the impasse is solved. If this does not terminate within a certain number of attempts, then the FUs are sorted randomly before restarting the coloring procedure.

Instead of four different colors, we use four different gray levels (Fig. 3.3, left, middle). Because larger FUs are considered to be more interesting, only FUs larger than 5 cells are considered. White Voronoi cells are part of smaller FUs.

FU Map Connections

Given the FUs, we define the inter-FU coherence $c'$ at frequency $\lambda$ between two functional units $W_1$ and $W_2$ as the sum of the coherence values between one vertex in $W_1$ and the other vertex in $W_2$, scaled by the total number of edges between $W_1$ and $W_2$ (ten Caat et al. 2007d):

$$c'_{\lambda}(W_1, W_2) = \frac{\sum_{i,j} \{c_{\lambda}(v_i, v_j) \mid v_i \in W_1, v_j \in W_2\}}{|W_1| \cdot |W_2|}.$$

(3.3)

Here, $|W_i|$ indicates the number of vertices in $W_i$. Note that coherences between any pair of vertices are taken into account, to normalize for the size of the FUs.
Algorithm 1 WB pseudocode with adaptations (ln. 11–15, 29–42) to obtain the IWB method.

**INPUT:** \( V \) is the vertex set; \( \text{marker}(i) = \text{marker~i} \); \( c(v, v') = \text{coherence}(v, v') = c(v', v) \);
\( \text{adjCoh}_v = \{ v' \in V \mid c(v, v') \geq \phi \} \); \( \phi = \text{sign. threshold} \);
\( \text{adjVor}_v = \{ v' \in V \mid v' \in \text{Vor.-neighbors}_v \& v' \in \text{adjCoh}_v \} \);
\( \{ \text{adjCoh}_v, \text{adjVor}_v \} \) are both sorted by vertex number

**OUTPUT:** \( \text{bsn}_i \) is basin \( i \) (i.e., an FU) sorted by vertex number

**INITIAL**: 
1. queue \( \leftarrow \emptyset \) \{queue of edges\}
2. **for all** \( v \in V \)** do
3. \( L(v) \leftarrow 0 \) \{label of vertex \( v \)\}
4. **end for**
5. **for** \( i = 1 \) to \( |\text{marker}| \)** do
6. \( \text{bsn}_i \leftarrow \text{marker}(i) \); \( v \leftarrow \text{marker}(i) \); \( L(v) \leftarrow i \); \( \text{adjCohBsn}_{L(v)} \leftarrow \text{adjCoh}_v \)
7. **for all** \( v' \in \text{adjVor}_v \)** do
8. \( \text{insertEdgeSort}(e(v, v'), \text{queue}) \)
9. **end for**
10. **end for**
11. **for** \( i = 1 \) to \( |\text{marker}| \)** do
12. **for** \( j = 1 \) to \( |\text{marker}| \)** do
13. \( \text{bsnMat}(i, j) \leftarrow 1 \) \{IWB modification\}
14. **end for**
15. **end for**

**MAIN**: 
16. **while** queue \( \neq \emptyset \)** do
17. \( e(v, v') \leftarrow \text{dequeue}(\text{queue}) \)
18. **if** \( L(v') = 0 \)** then
19. **if** \( v' \in \text{adjCohBsn}_{L(v)} \)** then
20. \( L(v') \leftarrow L(v) \)
21. \( \text{adjCohBsn}_{L(v)} \leftarrow \text{intersect}(\text{adjCohBsn}_{L(v)}, \text{adjCoh}_v) \)
22. \( \text{bsn}_{L(v)} \leftarrow \text{insertVSort}(v', \text{bsn}_{L(v)}) \)
23. **for all** \( v'' \in \text{adjVor}_{v'} \)** do
24. **if** \( L(v') = 0 \)** then
25. \( \text{insertEdgeSort}(e(v', v''), \text{queue}) \)
26. **end if**
27. **end for**
28. **end if**
29. **else**
30. **if** \( (L(v') \neq L(v)) \) \& \( \text{bsnMat}(L(v'), L(v)) \neq 0 \)** then
31. \( \psi \leftarrow L(v') \)
32. **if** setInSet(\( \text{bsn}_{L(v)}, \text{adjCohBsn}_{\psi} \)) \& setInSet(\( \text{bsn}_{\psi}, \text{adjCohBsn}_{L(v)} \)) then
33. \( \text{bsn}_{L(v)} \leftarrow \text{merge}(\text{bsn}_{L(v)}, \text{bsn}_{\psi}) \)
34. **for all** \( w' \in \text{bsn}_{\psi} \)** do
35. \( L(w') \leftarrow L(v) \)
36. **end for**
37. \( \text{adjCohBsn}_{L(v)} \leftarrow \text{intersect}(\text{adjCohBsn}_{L(v)}, \text{adjCohBsn}_{\psi}) \)
38. \( \text{bsn}_{\psi} = \emptyset \); \( \text{adjCohBsn}_{\psi} = \emptyset \)
39. **else**
40. \( \text{bsnMat}(L(v), \psi) \leftarrow 0 \); \( \text{bsnMat}(\psi, L(v)) \leftarrow 0 \)
41. **end if**
42. **end if**
43. **end if**
44. **end while**
A line is drawn between FU centers if the corresponding inter-FU coherence exceeds a threshold (Fig 3.3 right). We consistently choose this threshold to be equal to the significance threshold (Eqn. 3.1), as we already used this threshold to determine the initial graph.

### 3.6.2 Data-Driven Group Analysis

FU maps differ from individual to individual, making group analysis difficult. Therefore, we develop a data-driven method for group coherence analysis which detects common features in a collection of individual FU maps. Group coherence analyses are commonly based on group means of coherences of interest. We show how our data-driven ROIs, i.e., the FUs, lead to a data-driven selection of coherences of interest.

**Group Mean Coherence Map**

We define a *group mean coherence graph* as the graph containing the mean coherence for every electrode pair computed across a group, with vertices representing electrodes and edges containing coherence values. To obtain a data-driven coherence visualization for a group, the group mean coherence graph is thresholded, maintaining only the edges with a value exceeding the coherence threshold (Eqn. 3.1). Next, an FU map is created for the group mean coherence graph, referred to as a *group mean coherence map*.

**Group FU Size Map**

A group FU size map visualizes the average FU size for every electrode across a group, based on the FU maps for every individual dataset. The average FU size \( s \) of an electrode \( v \) is computed as

\[
s(v) = \frac{\sum_{all \; datasets} \{ |W| \mid v \in W \}}{#datasets}. \tag{3.4}
\]

with \( W \) the FU containing \( v \) in every FU map. The value \( s \) for an electrode is mapped to the gray value of its corresponding Voronoi cell, similar to a (gray scale) topographic map (ten Caat et al. 2007c). Lighter gray is used for higher average FU sizes, as higher values commonly correspond to lighter gray in gray scale images.

Consequently, a light Voronoi cell indicates that the corresponding electrode is on average part of large FUs.

### 3.7 Results

Throughout this section, we use \( p = 0.01 \). The corresponding coherence threshold is \( \phi \approx 0.22 \) (Eqn. 3.1).
3.7.1 FU Map

For a comparison of FU maps obtained with the three different FU detection methods, see Fig. 3.4. FU maps for the five datasets in each group and each of the five frequency bands are shown in Fig. 3.5 to 3.8 for the MCB and IWB method.

FU detection with the (non-optimized) MCB method was faster for smaller FU sizes, taking approximately 1 s for datasets with small FUs, and up to 2 h for the dataset with the largest FU. FU detection with the (non-optimized) WB method took around 0.04 ± 0.02 s (max. 0.14 s) and with the (non-optimized) IWB method around 0.05 ± 0.04 s (max. 0.25 s). Consequently, the WB and IWB methods are up to a factor of 100,000 times faster than the MCB method for this typical multichannel EEG setting with 128 channels.

Because the MCB method is assumed to obtain the most interesting FUs corresponding to the strongest source signals (Section 3.5), it is here considered as the gold standard. We compared the WB and the IWB method with the MCB method, and made an illustrative selection of seven (out of fifty) cases for a detailed discussion (Fig. 3.4). The selection includes those settings (a combination of participant and frequency band) which result in the largest difference between the MCB, WB, and IWB method. The order of the seven illustrations is chosen such that it facilitates the discussion.

i. The one anterior FU detected by the MCB method is represented by two (smaller) spatially connected anterior FUs by the WB method, whereas the IWB method merges two anterior FUs. Because the WB and IWB methods both follow a greedy approach, the anterior FUs do not correspond exactly to the anterior FU of the MCB FU map. Because the IWB method merges FUs during segmentation (and not afterwards, such as with hierarchical watersheds (Schultz et al. 2007)), the vertices in the large anterior FU of the IWB FU map do not correspond exactly to the vertices that are part of the smaller anterior FUs obtained by the WB method.

ii. Although multiple anterior FUs are obtained with the WB method, they are smaller than the minimum size and are therefore not shown, whereas the IWB method merges smaller FUs into an anterior FU identical to the anterior FU found with the MCB method.

iii. This is one of the occurrences of the maximal absolute difference in the number of FUs between the MCB (6 FUs) and IWB method (3 FUs). Nevertheless, the connection between an anterior and a posterior region which is visible in the MCB FU map is preserved in the IWB FU map.

iv. This is one of the occurrences of the maximal absolute difference in the number of FUs between the MCB (5 FUs) and WB method (10 FUs). Whereas the WB method shows visually cluttered edges, the IWB method gives a better overview more similar to the MCB method.

v. The significance threshold used is apparently too low, as one very large FU is found with the MCB method and two very large FUs are found with the IWB method. The WB
Figure 3.4. Illustration of FU maps (top view, nose on top) obtained with the three FU detection methods for seven (i-vii) selected datasets and frequency bands. **Left:** MCB method (see also Fig. 3.5 and Fig. 3.6). **Middle:** WB method with over-segmentation. **Right:** IWB method with over-segmentation reduction (see also Fig. 3.7 and Fig. 3.8). Datasets: (i) young 1, 4-7Hz; (ii) young 1, 8-12Hz; (iii) young 3, 4-7Hz; (iv) young 5, 4-7Hz; (v) old 2, 1-3Hz; (vi) old 4, 1-3Hz; (vii) old 4, 8-12Hz. For every dataset, the IWB FU map shows a number of FUs and a number of inter-FU connections closer to the MCB FU maps than the WB FU maps.
3.7 Results

method, however, results in 6 FUs completely connected by 15 lines and does not (directly) make clear that the used threshold is too low.

vi. Both FUs found with the MCB and the IWB method are identical. The WB method has more FUs in the same region instead.

vii. The large anterior FUs found with the MCB and the IWB method are identical. The WB method has multiple FUs in the same region instead.

In all cases, the number of FUs and their size and locations are highly similar for the MCB FU maps and the corresponding IWB FU maps (Fig. 3.5–3.8). The absolute difference in the number of FUs between the WB and the MCB method is on average 1.8 with a maximum difference of five FUs (four occurrences). The same difference between the IWB and the MCB is clearly smaller: 0.9 with a maximum of three FUs difference (two occurrences). Regarding the connections between FUs, those found with the MCB method are generally also found in the corresponding IWB FU maps. In particular, connections between a middle anterior and a middle posterior FU are present in the MCB FU map if and only if they are present in the corresponding IWB FU map, with one exception: for dataset old 5, 21-30 Hz, the inter-FU coherence is just above the threshold for the IWB method, contrary to the MCB method. For datasets old 2 and the frequencies 1-3 Hz, the connection between anterior and posterior regions is explicit in the IWB FU map (Fig. 3.8) and implicit in the MCB FU map (Fig. 3.6: the fact that one large FU consists of nearly all vertices implies that most anterior and most posterior vertices are completely connected).

3.7.2 Group Analysis

Group mean coherence maps (Fig. 3.9) and group FU size maps (Fig. 3.10) were obtained as extensions of the IWB FU detection method. They are shown for the two groups of younger and older adults and the five frequency bands.

Individual FU Maps versus Group Mean Coherence Maps

The largest FUs for individual datasets of younger adults (Fig. 3.5, 3.7) are mostly located anteriorly and posteriorly in the middle. This feature is also preserved in the corresponding group mean coherence maps (Fig. 3.9 left column). FU maps for older adults (Fig. 3.6, 3.8) usually show more lateral FUs (at the sides of the head), which are preserved in the corresponding group mean coherence maps (Fig. 3.9 right). For both younger and older adults, the number of FUs usually does not change much across frequency bands in the individual dataset FU maps (Fig. 3.5–3.8, compare rows), as well as in the group mean coherence maps (Fig. 3.9, compare rows). In four out of five frequency bands, inter-FU connections between a middle anterior and middle posterior FU are present in the group mean coherence map (Fig. 3.9) if they are present in the majority of the individual FU maps in the corresponding frequency bands (Figs. 3.5–3.8). The only exception is the 8-12 Hz band, with anterior-posterior connections just above the threshold for a majority of three (out of five) younger adults, and with anterior-posterior connections above the
Figure 3.5. Standard MCB FU maps, younger adults. FU maps ($|FU| > 5$, $p = 0.01$) for five younger adults for five frequency bands (1-3, 4-7, 8-12, 13-20, 21-30Hz). Each FU is visualized as a set of Voronoi cells with identical gray value, with different gray values for adjacent FUs. White Voronoi cells are part of FUs with $|FU| \leq 5$. A line connects FUs if the inter-FU coherence exceeds the significance threshold, with its color depending on the value (see color bar, bottom right, with minimum corresponding to the coherence threshold $\phi \approx 0.22$ for $p = 0.01$; the color bar is the same for all FU maps). Above each group mean coherence map, the number of FUs and the number of connecting lines between FUs are displayed.
3.7 Results

<table>
<thead>
<tr>
<th>OLD</th>
<th>Freq. (Hz)</th>
<th>1-3</th>
<th>4-7</th>
<th>8-12</th>
<th>13-20</th>
<th>21-30</th>
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</table>

Table 3.6. Standard MCB FU maps, older adults. Same parameters as in Fig. 3.5.

threshold for a minority of two (out of five) older adults (with one relatively high value). Thus, generally the common features from the individual FU maps are preserved well in the group mean coherence maps.

**Group Mean Coherence Map: Comparison Between Groups**

For all frequencies (1-30Hz), the number of FUs is lower for younger than for older adults in the corresponding frequency band (Fig. 3.9, compare left with right column). This probably cor-
responds to earlier findings (Maurits et al. 2006), indicating more, especially interhemispheric, coherence for older than for younger adults. Similarly, the number of white cells (corresponding to electrodes not part of any sufficiently large FU) is larger for younger than for older adults in every frequency band, again confirming the presence of more coherence for older than for younger adults (Maurits et al. 2006).

For lower frequencies, there is a connecting line between an anterior and a posterior FU in most group mean coherence maps for younger adults (Fig. 3.8 1-7 Hz) and older adults
### 3.7 Results

#### 3.7.1 Novel IWB FU maps, older adults

**Older Adults**

<table>
<thead>
<tr>
<th>Freq. (Hz)</th>
<th>1-3</th>
<th>4-7</th>
<th>8-12</th>
<th>13-20</th>
<th>21-30</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>4 FU(s); 2 sign. conn(s).</td>
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<td>4 FU(s); 1 sign. conn(s).</td>
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<td>3 FU(s); 0 sign. conn(s).</td>
</tr>
<tr>
<td>2.</td>
<td>2 FU(s); 1 sign. conn(s).</td>
<td>5 FU(s); 1 sign. conn(s).</td>
<td>5 FU(s); 1 sign. conn(s).</td>
<td>5 FU(s); 0 sign. conn(s).</td>
<td>5 FU(s); 0 sign. conn(s).</td>
</tr>
<tr>
<td>3.</td>
<td>5 FU(s); 8 sign. conn(s).</td>
<td>5 FU(s); 6 sign. conn(s).</td>
<td>6 FU(s); 4 sign. conn(s).</td>
<td>5 FU(s); 4 sign. conn(s).</td>
<td>4 FU(s); 1 sign. conn(s).</td>
</tr>
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<td>4.</td>
<td>2 FU(s); 1 sign. conn(s).</td>
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<td>7 FU(s); 7 sign. conn(s).</td>
<td>8 FU(s); 7 sign. conn(s).</td>
<td>8 FU(s); 6 sign. conn(s).</td>
</tr>
<tr>
<td>5.</td>
<td>6 FU(s); 8 sign. conn(s).</td>
<td>9 FU(s); 3 sign. conn(s).</td>
<td>6 FU(s); 1 sign. conn(s).</td>
<td>5 FU(s); 3 sign. conn(s).</td>
<td>5 FU(s); 3 sign. conn(s).</td>
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</tbody>
</table>

**Figure 3.8.** Novel IWB FU maps, older adults. Same parameters as in Fig. 3.5

(Fig. 3.5 1-12 Hz). This is possibly associated with the two most important sources of brain activity for this type of experiment, located anteriorly (known as P3a) and posteriorly (known as P3b) (Comerchero and Polich 1999).

FU maps show more lateral FUs (located on both sides of the head) for older adults than for younger adults in the same frequency band (Fig. 3.5, compare left to right column). This may indicate more bilateral activation for older than for younger adults, as was also observed in (Maurits et al. 2006).
<table>
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<th>Freq. (Hz)</th>
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<th>OLD</th>
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<tbody>
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<tr>
<td>4-7</td>
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<td>5 FU(s); 5 sign. conn(s).</td>
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<tr>
<td>8-12</td>
<td>4 FU(s); 1 sign. conn(s).</td>
<td>6 FU(s); 3 sign. conn(s).</td>
</tr>
<tr>
<td>13-20</td>
<td>3 FU(s); 0 sign. conn(s).</td>
<td>6 FU(s); 1 sign. conn(s).</td>
</tr>
<tr>
<td>21-30</td>
<td>3 FU(s); 0 sign. conn(s).</td>
<td>8 FU(s); 2 sign. conn(s).</td>
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</table>

Figure 3.9. **Group mean coherence maps** for younger (left) and older (right) adults, per frequency band (from top to bottom). The line color depends on the inter-FU coherence (see color bar, bottom right, with minimum corresponding to the coherence threshold $\phi \approx 0.22$ for $p = 0.01$; the color bar is the same for all FU maps). Above each group mean coherence map, the number of FUs and the number of connecting lines between FUs are displayed.
### Figure 3.10. Group FU size maps for younger (left) and older (right) adults, per frequency band (from top to bottom). The average FU size for each electrode, computed for the five individual FU maps of every group (see Fig. 3.9), is mapped to a gray value (see right bar, with maximum equal to the maximum average FU size). A lighter cell indicates that the corresponding electrode is on average part of larger and more interesting FUs. The gray scale range is adapted per group FU size map, to be able to better distinguish between light and dark regions in one FU size map.

#### Group FU Size Map: Comparison Between Groups

For younger adults (Fig. 3.10 left), average FU sizes are highest in a posterior region and an anterior region, for all frequencies. The lateral regions on both left and right sides have the
lowest average FU size.

Similarly, for older adults (Fig. 3.10 right), the highest average FU sizes occur in a posterior and an anterior region, although for older adults these regions are more widespread than for younger adults. Whereas the average FU sizes are lower on the sides than in the middle for both younger and older adults (Fig. 3.10), the difference between lower and higher average FU sizes is smaller for older than younger adults. This indicates more bilateral activation for older than younger adults, in correspondence with (Maurits et al. 2006).

Cells for younger adults are generally part of FUs with a lower average size than corresponding cells for older adults (Fig. 3.10 compare color bars of the left and right column), once more confirming the observation of higher coherence for older than younger adults (Maurits et al. 2006). Moreover, the average FU size decreases with increasing frequency, in agreement with the presence of simultaneous activity at a more global scale for lower EEG frequencies and at a more local scale for higher EEG frequencies (Nunez et al. 1997).

**Comparison of Hypothesis-Driven and Data-Driven Approaches**

For the same type of data, a hypothesis-driven subselection of 12 out of 119 scalp electrodes (Fp1, Fp2, F3, F4, C3, C4, P3, P4, O1, O2, O3, O4; see Fig. 3.1, right) and 15 coherences was made (Maurits et al. 2006). In contrast to this hypothesis-driven approach, FU maps together with group mean coherence maps and group FU size maps all contribute to a data-driven selection of electrodes of interest. In addition to the coherences studied in (Maurits et al. 2006), our data-driven results suggest to include left and right temporal electrodes (e.g., T7 and T8), and to include both intrahemispheric and interhemispheric connections between anterior and posterior regions.

### 3.7.3 Threshold Effect

Two thresholds applied in the FU visualization are coherence thresholds. One concerns the initial coherence graph, the other the inter-FU coherence. Both are chosen to be equal to each other and are related to a value $p$ (Eqn. 3.1). A third threshold is the minimum FU size.

Threshold variation is illustrated by FU maps for one dataset and one frequency band, varying $p$ and the minimum FU size (Fig. 3.11). Obviously, larger FUs are found for a higher $p$ corresponding with a lower coherence threshold. For the value $p = 0.1$, the highest inter-FU coherence occurs between largest FUs which are located anteriorly and posteriorly. The same is visible for the value $p = 0.001$. Other significant inter-FU coherence appear between smaller FUs located laterally on the left and the right side, for all values of $p$.

For other datasets and frequency bands, we have generally observed a similar pattern. Across different threshold values, the largest FUs remain in the same regions and the highest inter-FU coherences remain between FUs in the same regions.
3.8 Discussion and Conclusions

EEG coherence analysis is the study of coherence between functional units. Most current analyses use hypothesis-driven ROIs. Existing data-driven graph visualizations for EEG coherence commonly visualize vertices representing electrodes as dots and coherences as edges, resulting in clutter for multichannel EEG with up to 512 electrodes. However, without a hypothesis, all coherences should be considered. Therefore, we presented a data-driven visualization

<table>
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<th>min. FU size</th>
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<tr>
<td>10</td>
<td><img src="image6.png" alt="Image" /></td>
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<tr>
<td>12</td>
<td><img src="image7.png" alt="Image" /></td>
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Figure 3.11. Variation of the thresholds $p$ (horizontally) and the minimum FU size (vertically), for one dataset (dataset young 5, 1-3 Hz).
method for multichannel EEG coherence, which strongly reduces clutter and is referred to as functional unit (FU) map. An FU is a spatially connected set of electrodes recording pairwise significantly coherent signals, represented in the graph by a spatially connected clique. The visualization of an FU is a simplified representation of a spatially connected clique which does not explicitly visualize all edges within a clique.

We developed three methods to detect FUs. One is the maximal clique based (MCB) method (time complexity $O(3^{n/3})$, with $n$ the number of vertices) (ten Caat et al. 2007d), another is the watershed based (WB) method designed with an interactive visualization in mind (time complexity $O(n^2 \log n)$) (ten Caat et al. 2007e). Third, the improved watershed based (IWB) method (time complexity $O(n^2 \log n)$) merges two spatially neighboring FUs if their union is a clique in the coherence graph. We did not choose one of the common solutions for over-segmentation which uses the concept of dynamics (Grimaud 1992), because dynamics are defined for vertex values, whereas the EEG coherence graph has edge values. Moreover, the IWB method merges FUs during segmentation (and not afterwards, such as with hierarchical watersheds (Schultz et al. 2007)). FU detection with the WB and IWB method (taking about 0.04 s and 0.05 s, respectively) is up to a factor of 100,000 times faster than the MCB method, and makes interactive visualization of multichannel EEG coherence possible.

Comparing the methods, the greedy WB and IWB methods directly result in uniquely labeled electrodes contrary to the standard MCB method. Our three methods depend on the same thresholds: one for the initial coherence graph, one for the inter-FU coherence, and one for the minimum FU size. Threshold changes do not have a large effect on the most eminent FUs and inter-FU coherences. The MCB, WB, and IWB methods find FUs in approximately the same locations, and the inter-FU connections present in the MCB FU maps are generally also present in the WB and IWB FU maps. However, the average difference between the WB and the MCB method regarding the number of FUs is 1.8 (for our parameters). The same difference between the IWB and the MCB method has decreased to 0.9, for the case study that was presented.

Additionally, as an alternative to hypothesis-driven group analysis methods for multichannel EEG coherence, we proposed two novel data-driven group maps for visual group analysis. They are both extensions of the efficient IWB FU detection method. One is a group mean coherence map, which is a data-driven FU map based on the group mean coherence. The other is a group FU size map, showing for each electrode the average FU size across a collection of individual FU maps. Both novel group maps represent common elements from a set of individual FU maps.

Because conventional data-driven multichannel EEG coherence analysis is cumbersome, comparable conventional findings are rare. Nevertheless several conventional findings are confirmed by observations in the new data-driven visualizations. (a) Coherence is lower for younger than older adults (Maurits et al. 2006). Accordingly, the number of FUs in group mean coherence maps is lower for younger than for older adults for 1-30 Hz, group mean coherence maps show a larger number of white cells not part of any sufficiently large FU for younger adults than older adults, and group FU size maps show larger average FU sizes for older than for younger adults. (b) Older adults have more bilateral activation than younger adults (Maurits et al. 2006). In accordance, FU maps and group mean coherence maps display more FUs in lateral regions for older than younger adults, whereas group FU size maps show that the average FU size is generally higher for older than for younger adults and that the difference between lower and higher
average FU sizes is smaller for older than younger adults. (c) There is simultaneous activity at a more global scale for lower EEG frequencies and at a more local scale for higher EEG frequencies (Nunez et al. 1997). In agreement, group FU size maps indicate that the average FU size decreases with increasing frequency. (d) The two most important sources of brain activity for this type of data are located anteriorly (known as P3a) and posteriorly (known as P3b) (Comerchero and Polich 1999). Accordingly, FU maps and group mean coherence maps show connections between anterior and posterior FUs for lower frequencies.

Thus, the detection of data-driven ROIs for multichannel EEG coherence on the basis of the IWB method results in similar information as the MCB method, and this information is found to agree with conventional findings. Also, the two new data-driven group maps, referred to as group mean coherence map and group FU size map, yield results in accordance with conventional findings. Yet, our results suggest to expand an earlier selection of hypothesis-driven ROIs (Maurits et al. 2006) with additional data-driven ROIs. This demonstrates the usefulness of the IWB FU map, and both new data-driven group maps.

FU maps, group mean coherence maps, and group FU size maps all contribute to a data-driven subselection of electrodes of interest (EOIs): the number of EOIs, their location, and their region of influence can be derived directly from the combination of FU maps, group mean coherence maps, and group FU size maps. In other words, the novel IWB method together with the two new group maps make a data-driven subselection of the available electrophysiological signals possible. This can be used as a data-driven starting point for conventional quantitative group analysis. Our methods are currently applied to a multichannel EEG coherence study of mental fatigue (ten Caat et al. 2007a) by researchers from the Department of Experimental Psychology of the University of Groningen. In this study, the ROIs are obtained in a data-driven way since no strong hypotheses can be formulated based on existing evidence. Our approach overcomes the severe limitations of conventional hypothesis-driven methods and takes full advantage of all the available recordings. The presented visualization of (group) FU maps provides a very economical data summary of extensive experimental results, which otherwise would be very difficult and time-consuming to assess. Initial responses from the psychologists using our visualization methods are very favorable.

The IWB method will be available in FUmaplab on

http://www.rug.nl/informatica/onderzoek/programmas/svcg/demos.