Pionic fusion in light-ion systems
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6. Results

6.1 Introduction

The main objective of this work is to measure the differential cross sections of the pionic fusion in a wide angular range for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction. In the previous chapter, the analysis procedure that led us to the final set of data was introduced. In this chapter the final results of this thesis are presented. In Sections 6.2-6.5, a few general remarks about the extraction of the $\pi^0$ signal from the data compared with the results obtained from the Monte-Carlo calculations will be given. The Monte-Carlo simulations are based on phase-space distributions and do not contain any specific reaction dynamics (Chapter 3). In Section 6.6, the physics inferred from the differential cross section of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction will be discussed.

6.2 Reconstructed $\pi^0$ energy

The $\pi^0$ energy can be reconstructed using the measured observables in the two different approaches. The first one is simply to add the energy of two photons which were measured by the ADCs during the experiment

$$E_{\pi^0} = E_{\gamma\text{low}} + E_{\gamma\text{high}},$$

where $E_{\gamma\text{low}}$ and $E_{\gamma\text{high}}$ are the energy of the photon with the lower and the higher energy, respectively. Since we have achieved a measurement in over-determined kinematics under the condition with the present experimental setup, we have another useful observable to calculate the $\pi^0$ energy. The module position of the Heavy Ion detector gives us the momentum (and therefore the energy) of the fusion product. According to Method 1 of the $\pi^0$ invariant mass reconstruction which was explained in Section 5.4.2, in the second approach the measured energy of $^{10}\text{B}$ ions was used to reconstruct the total $\pi^0$ energy

$$E_{\pi^0} = E_{\text{total}} - E_{^{10}\text{B}}.$$
energy higher than 140 MeV. This corresponds to more than 50% of the total energy available in the centre-of-mass frame and implies that more than half of the available centre-of-mass energy is transferred to produce a pion. The sharp cut off in the lowest $\pi^0$ energy range is due to the results of the kinematical constraints.

### 6.3 Two photon energies

For the energy of the photon with a high energy deposition, the values measured by the ADCs were used. According to Method 1 of the $\pi^0$ invariant mass reconstruction, the $\pi^0$ energy was used to reconstruct the energy of the photon with a low energy deposition

$$E_{\gamma_{\text{low}}} = E_{\pi^0} - E_{\gamma_{\text{high}}}.$$  \hspace{1cm} (6.3)

Figure 6.2-(a) shows the simulated results of the initial and the reconstructed photon energies. The initial photon energy is the simulated energy of the photon in the laboratory system right after the $\pi^0$ decay in the $^6\text{Li}$ target. The dashed-dotted and dotted curves are the initial energies of the photon with high and low energies, respectively. The solid and dashed curves are the reconstructed energies of the photon with high and low energies, re-
Figure 6.2: (a): The dashed-dotted and dotted curves are the energy spectra of initial photons with high and low energies, respectively. The solid and dashed curves are the reconstructed distributions of photons with high and low energies, respectively. (b): The measured photon energies for the photon with the high (solid curve) and the low (dashed curve) energy. Labels “initial” and “reconstructed” indicate the initial and reconstructed energy distributions.
Figure 6.3: The two-photon invariant mass distribution. The solid and dashed curves are the results of the measurement and the Monte-Carlo simulation, respectively.

respectively. Due to the incomplete shower collection in the Plastic Ball, which was explained in Section 5.3.2, the width of the energy peak for the reconstructed photon energies is not the same as the original width. In Fig. 6.2-(b), the solid and dashed curves represent the distributions of reconstructed measured photon energies for the photon with the high and the low energy, respectively. The obvious cutoff at energies about 75 MeV is caused by the effect of the $E_{\pi^0}$ cutoff (which can be seen in Fig. 6.1 as well) in Eq. 6.3 ($E_{\gamma_{\text{low}}} > 0$, therefore, $E_{\pi} > E_{\gamma_{\text{high}}}$). A good agreement between the simulated and measured results in the widths as well as the peak positions has been observed.

### 6.4 Reconstructed $\pi^0$ mass

The identification of neutral pions is usually based on the invariant mass analysis of two-photon events. The observables needed for the identification of $\pi^0$ through the invariant mass analysis are two photon energies $E_{\gamma_{\text{high}}}$ and $E_{\gamma_{\text{low}}}$ and the opening angle $\theta_{\gamma\gamma}$ between them:

$$ M_{\gamma\gamma} = \sqrt{2E_{\gamma_{\text{high}}}E_{\gamma_{\text{low}}}(1 - \cos(\theta_{\gamma\gamma}))}. \quad (6.4) $$

Figure 6.3 shows the two-photon invariant mass distribution. The solid and dashed curves are the results of the measurement and the Monte-Carlo simulation, respectively.
6.4. Reconstructed $\pi^0$ mass

A very good agreement is observed, giving a high confidence in the data reconstruction. The tail structure in the low region of the invariant mass distribution originates from the detector energy resolution. In fact, events in the tail are associated with the situation that $E_{\gamma_{\text{high}}} > 110$ MeV and $E_{\gamma_{\text{low}}} < 40$ MeV. The peak resolution of the invariant mass is directly due to the results of the experimental angular and energy resolutions. In addition, the sharp cut around 150 MeV is caused by the kinematical limits imposed by the energy

**Figure 6.4:** (a): The polar angle distribution of the produced $\pi^0$ in the laboratory system. (b): The polar angle distribution of the produced $\pi^0$ in the centre-of-mass system. (c): The momentum distribution of the produced $\pi^0$ in the laboratory system. (d): The momentum distribution of the produced $\pi^0$ in the centre-of-mass system. The solid and dashed curves represent the measured and the simulated results, respectively.
conservation.

6.5 Pion angular and momentum distributions

The polar angle distributions of the produced $\pi^0$ in the laboratory and the centre-of-mass system are shown in Fig. 6.4-(a) and -(b), respectively. Figure 6.4-(c) and -(d) represents the momentum distributions of the produced $\pi^0$ in the laboratory and the centre-of-mass system, respectively. The solid and dashed curves are the measured and the simulated results, respectively. As can be noticed, the measured angular and momentum distributions are well reproduced by the simulation. The original $\pi^0$ angular and momentum distributions are shown in Figs. 3.6-(b) and 3.5-(b), respectively. It should be noted that the original $\pi^0$ angular and momentum distributions display double peak structures when the BBS acceptance is introduced into the simulation (hatched area in Figs. 3.6-(b) and 3.5-(b)). However, since no Plastic Ball module is positioned in the region $0^\circ \leq \theta \leq 50^\circ$, by applying the full geometrical acceptances of the detector setup most of the highest energetic pions moving into the forward direction are not accepted. The momentum of those pions is associated with the momentum in the region of the second peak in Fig. 3.5-(b). As result only one peak out of every double-peak structure from Figs. 3.6-(b) and 3.5-(b) will remain.

6.6 Differential cross section

Section 5.7 and Table 5.3 explain how the cross section of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction is corrected for the detector efficiencies, target thickness and the beam intensity. Figure 6.5 shows the differential cross sections as a function of $\pi^0$ polar angle in the centre-of-mass system integrated over the covered azimuthal angle. Full circles are the results of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ measurement at KVI at the incident energy of $T_{\text{beam}/\text{nucleon}}=59.1$ MeV. Empty circles are the preliminary results of the KVI measurement for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction at the incident energy of $T_{\text{beam}/\text{nucleon}}=85.3$ MeV. Triangles are the observed data from the ORSAY experiment [14] for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction at the incident energy of $T_{\text{beam}/\text{nucleon}}=88.8$ MeV. The curves represent the results of the theoretical predictions [8]. The thin black curve is the result obtained using the cluster model wave function of $^7\text{Be}$ in the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction. The thick grey and black curves are the results calculated with the use of, respectively, the cluster model and the shell model wave functions of $^7\text{Li}$ from the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction. The results shown by the solid curves were obtained when the adopted strength of the imaginary part of the optical potential in the entrance channel, $W_D$, is -25 MeV (see Chapter 2), while the dashed curves are the results when $W_D=0$ MeV. The theoretical predictions have been given for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ and $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ cross sections at $T_{\text{beam}/\text{nucleon}}=88.8$ MeV. Up to now, there is no theoretical calculation available for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction. Calculations for the other reactions are shown to guide the eyes and to give an indication how and where the results should be expected. Only the statistical uncertainties of the measured results are displayed in Fig. 6.5. The systematic error includes uncertainties in the measurement of the beam current (8%), the live time of the data acquisition electronics (12%), and the determination of the detector acceptances (2%), summing up to $\pm 15\%$. 


### 6.6. Differential cross section

#### Figure 6.5: Angular distribution of the cross sections for the various pionic fusion reactions at subthreshold energies.

The full and empty circles are the results of the KVI experiments for the $^6$Li($^4$He,π$^0$)$^7$B$^*$ and $^4$He($^3$He,π$^0$)$^7$Be reactions at the incident energies of $T_{beam}/nucleon=59.1$ and 85.3 MeV, respectively. The triangles are the observed result from the ORSAY experiment for the $^4$He($^3$He,π)$^7$Li reaction at the incident energy of $T_{beam}/nucleon=88.8$ MeV. The thin black curve is the calculated result obtained using the cluster model wave functions for the $^4$He($^3$He,π$^0$)$^7$Be reaction. The thick grey and black curves are the calculated results obtained with the use of, respectively, the cluster model and the shell model wave functions for the $^4$He($^3$He,π$^+$)$^7$Li reaction. All the solid curves were obtained with the adopted strength of the imaginary part of the optical potential in the entrance channel $W_D=0$ MeV, while the dashed curves are the results with $W_D=25$ MeV. The theoretical predictions for the $^4$He($^3$He,π$^0$)$^7$Be and $^4$He($^3$He,π$^+$)$^7$Li cross sections have been performed at $T_{beam}/nucleon=88.8$ MeV.
In the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ experiment, with an angular binning of $10^\circ$ in the centre-of-mass system, we have not measured any pion at $\theta_{\text{c.m.}} = 10^\circ$, $30^\circ$, $50^\circ$ and $170^\circ$. Moreover, at $\theta_{\text{c.m.}} = 20^\circ$ and $40^\circ$ only one pion has been measured. Therefore, the statistical uncertainties are of the same size as the cross sections at these two angles. The Plastic Ball covers the laboratory polar angles between $50^\circ$ and $160^\circ$ and since the $^{10}\text{B}$ mass is about 67 times bigger than the $\pi^0$ mass, these angles are converted to almost the same angles in the centre-of-mass frame. This experimental constraint is the reason why we did not measure pions in the most forward and backward directions.

First of all it should be noted that the general behaviour of the measured differential cross sections of both reactions, $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^{7}\text{Be}$, is as predicted for the $^4\text{He}(^3\text{He},\pi^0)^{7}\text{Be}$ and $^4\text{He}(^3\text{He},\pi^+)^{7}\text{Li}$ reactions by taking strong clustering correlations into account [8]. The measured angular distribution of the $^4\text{He}(^3\text{He},\pi^0)^{7}\text{Be}$ reaction in particular in the backward direction is well reproduced by using the cluster model. It is impossible for us to draw the same conclusion for the $^4\text{He}(^3\text{He},\pi^+)^{7}\text{Li}$ reaction since there is no measurement in the backward direction. It should be noted that the incident wave in the entrance channel is attenuated due to absorption by the local imaginary potential $W_D$. In case of the $^4\text{He}(^3\text{He},\pi^+)^{7}\text{Li}$ reaction, it was found that in the cluster model the local imaginary potential plays a significant role at the most backward angles (see the grey solid and dashed curves). Therefore, employing the correct imaginary potential may improve the calculation for the $^4\text{He}(^3\text{He},\pi^0)^{7}\text{Be}$ reaction.

There is a remarkable disagreement between the cross sections for the two reactions $^4\text{He}(^3\text{He},\pi^0)^{7}\text{Be}$ and $^4\text{He}(^3\text{He},\pi^+)^{7}\text{Li}$, although the target, the projectile and the beam kinetic energies are the same. It implies that the total pionic fusion cross section is not only defined by the available energy and the number of available nucleons, but also the isospin affects the strength of the pionic fusion cross section. The ratio of a factor of 2 between the $^4\text{He}(^3\text{He},\pi^0)^{7}\text{Be}$ and the $^4\text{He}(^3\text{He},\pi^+)^{7}\text{Li}$ cross sections is explained by charge symmetry.

For a phenomenological analysis of the angular distributions, the following expression for the differential cross section has been assumed:

$$
\frac{d\sigma}{d\Omega}_{\text{c.m.}} = \sum_{n=0}^i a_n P_n(\cos\theta_{\text{c.m.}}),
$$

where $P_n(\cos\theta_{\text{c.m.}})$ are the Legendre Polynomials and $\theta_{\text{c.m.}}$ is the centre-of-mass angle of $\pi^0$. We call $a_n$ the Legendre coefficients. Employing Eq. 6.5, we fitted the calculated results of the clustering model for the $^4\text{He}(^3\text{He},\pi^0)^{7}\text{Be}$ reaction (the solid curve in Fig. 6.5). There are two reasons for employing Eq. 6.5 in the fitting. The first aim was to study which different contributions of the Legendre polynomials are responsible to reproduce the asymmetric behaviour of the angular distribution and produce the same shape of the differential cross section as the prediction of the cluster model. Furthermore, it would be interesting to explore which component of the Legendre Polynomial expansion plays the most important role. By comparing the fitted results of the $^4\text{He}(^3\text{He},\pi^0)^{7}\text{Be}$ calculation and the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ measurement, one may compare the structure of the angular distributions, which are influenced by the clustering correlations, and draw conclusions on the importance of clustering correlations in case of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction. Since the measured differential cross sections at $\theta_{\text{c.m.}} = 20^\circ$ and $40^\circ$ are associated with only
one measured pion and the statistical uncertainties are too large, those points may not determine the shape of the cross section at forward angles in a confident way. For comparison of the sensitivity, we decided to exclude those two points from the polynomial fit and extrapolate the fit in order to cover the full angular range. Extrapolation is needed to calculate the total cross section. Thus the second aim of fitting the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation was to understand how the cross section data at forward angles influence the obtained fit parameters and the total cross section.

### Table 6.1: Legendre coefficients obtained from the fits to the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculated results based on the clustering correlations. The fitted curves are shown in Fig. 6.6-(a). The fits were performed in the full $\pi^0$ angular range. $i$ indicates the order of the Legendre polynomials which have been fitted to the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation. The $\chi^2$ is normalised to the number of degrees of freedom.

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### Table 6.2: Legendre coefficients obtained from the fits to the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculated results based on the clustering correlations. The fitted curves are shown in Fig. 6.6-(b). The fits were performed in the $-1 \leq \cos \theta_{\pi^0}^{\text{c.m.}} \leq 0.5$ angular range. $i$ indicates the order of the Legendre polynomials which have been fitted to the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation. The $\chi^2$ is normalised to the number of degrees of freedom.

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Figure 6.6 shows the results obtained by fitting Eq. 6.5 to the model result for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction (solid curve), when assuming $i$ in Eq. 6.5 is equal to 3, 4,..., 9. In Fig. 6.6, the differential cross section as a function of $\cos \theta_{\pi^0}^{\text{c.m.}}$ is displayed. It should be noted that the range of the vertical logarithmic scale of the graph is different from that used in Fig. 6.5. The results shown in Fig. 6.6-(a) have been obtained when the full $\pi^0$ angular range of the calculated cross section has been fitted. The results shown in Fig. 6.6-(b) correspond to the fits in the restricted angular range of $-1 \leq \cos \theta_{\pi^0}^{\text{c.m.}} \leq 0.5$. An average value of the experimental (statistical) errors of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction
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Figure 6.6: The results obtained by fitting the Legendre polynomials from Eq. 6.5 to the angular distribution of the cross sections for the $^4$He($^3$He,$\pi^0$)$^7$Be reaction (the solid curve) calculated in the cluster model, when assuming $i$ in Eq. 6.5 is 3, 4,..., 9. (a) and (b) correspond to the fit results for the full and restricted $\pi^0$ angular range, $-1 \leq \cos\theta_{c.m.} \leq 1$ and $-1 \leq \cos\theta_{c.m.} \leq 0.5$, respectively. The Legendre coefficients ($a_n$) as well as the $\chi^2$ of the fits are listed in Tables 6.1 and 6.2, for the fits to the full and restricted $\pi^0$ angular range, respectively.
(the errors for the empty circles in Fig. 6.5) was employed to calculate the \( \chi^2 \) per degree of freedom of the fits. The Legendre coefficients \( (a_n) \) as well as the \( \chi^2 \) of the fits are listed in Tables 6.1 and 6.2, for the fits to the full \((-1 \leq \cos \theta_{c.m.} \leq 1)\) and restricted \((-1 \leq \cos \theta_{c.m.} \leq 0.5)\) \( \pi^0 \) angular range, respectively.

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<td>11.47</td>
<td>8.44</td>
<td>5.4</td>
<td>2.92</td>
<td>0.9</td>
<td></td>
<td></td>
<td>0.6</td>
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<tr>
<td>8</td>
<td>-8.7</td>
<td>-25.3</td>
<td>-35</td>
<td>-35.9</td>
<td>-31.2</td>
<td>-21.9</td>
<td>-12.3</td>
<td>-5.4</td>
<td>-1.8</td>
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<tr>
<td>9</td>
<td>-34.1</td>
<td>-94.9</td>
<td>-131.4</td>
<td>-137.6</td>
<td>-119.4</td>
<td>-86.1</td>
<td>-51.3</td>
<td>-24.7</td>
<td>-8.8</td>
<td>-1.7</td>
<td>1.8</td>
</tr>
</tbody>
</table>

By changing the \( \pi^0 \) angular range in the fit from the full to the restricted range, the results of the polynomial fit to the \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) calculation change more drastically for higher-order polynomials. The lowest order polynomial with a \( \chi^2 \) close to one is found for \( i=3 \) in both the full and the restricted angular range. For higher-order polynomials, the \( \chi^2 \) increases far above one in the restricted angular range, which gains much less confidence in the obtained parameters. It is concluded that the most stable fit corresponds to the fit up to the third polynomial order \( (i=3) \). In Fig. 6.6, the results of the polynomial fit up to the third order are shown by the dashed curves. This fit is not only the most stable fit but also the best lowest-order polynomial fit to the clustering calculation. When the \( \pi^0 \) angular range changes, the rest of the fits change very much in the forward direction.

Figure 6.7 shows the same procedure for the experimental \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) results. In addition, for reasons of comparison the results produced for the \( ^4\text{He}(^4\text{He},\pi^0)^7\text{Be} \) reaction by the clustering calculations (the solid curve), the two first polynomial fits \( (i=3,4) \) to the full angular range of that calculation (the upper dashed and dashed-dotted curves, respectively) and the measured \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) results (empty circles) are displayed in Fig. 6.7. The last two experimental points with large error bars were excluded in the fit. The Legendre coefficients \( (a_n) \) as well as the \( \chi^2 \) of the fits to the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) experimental results are listed in Table 6.3.

It is noted that the two polynomial fits with \( i=3 \) and 4 follow the same trend as the two fits to the theory. The fit with \( i=3 \) has a good overlap with all the data points. We conclude that the fit with \( i=3 \) is the best fit to explain the \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) experimental results in the \(-1 \leq \cos \theta_{c.m.} \leq 0.5 \) range. The result of the fit with \( i=3 \) is shown as the dashed-dotted curve in Fig. 6.8. In case of the \( ^4\text{He}(^4\text{He},\pi^0)^7\text{Be} \) reaction, the contribution of \( P_2(\cos \theta_{c.m.}) \) is the largest to reproduce the theoretical curve as well as the experimental result. In case of the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) reaction, the \( P_3(\cos \theta_{c.m.}) \) contribution is the largest. However, still the \( P_2(\cos \theta_{c.m.}) \) and \( P_3(\cos \theta_{c.m.}) \) contributions are comparable.

Figure 6.8 shows the final results of the Legendre polynomial fits with \( i=3 \). The dashed-dotted and dashed curves are the results of the polynomial fit with Eq. 6.5 to the
Figure 6.7: The results of fitting the Legendre polynomials to the measured angular distribution of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ cross sections (full circles), when assuming $i$ in Eq. 6.5 is 3, 4, ..., 9. The fitting procedure has been applied to part of the $\pi^0$ angular range $-1 \leq \cos(\theta_{\text{c.m.}}) \leq 0.5$. The solid curve, upper dashed and upper dashed-dotted curves are the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ results predicted by the clustering calculations, the polynomial fit with $i=3$ and the polynomial fit with $i=4$ to that calculation, respectively. The empty circles are the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ experimental results. The Legendre coefficients ($a_n$) as well as the $\chi^2$ of the fits to the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ experimental results are listed in Table 6.3.
6.6. Differential cross section

\[ \frac{d\sigma}{d\Omega} = 0.942*P_0 + 0.854*P_1 + 0.899*P_2 + 1.020*P_3, \chi^2 = 0.26 \]

\[ \frac{d\sigma}{d\Omega} = 4.876*P_0 + 3.280*P_1 + 5.013*P_2 + 0.193*P_3, \chi^2 = 6.95 \]

\[ \frac{d\sigma}{d\Omega} = 3.775*P_0 + 1.766*P_1 + 4.415*P_2 + 2.869*P_3, \chi^2 = 0.65 \]

Figure 6.8: Angular distribution of the pionic fusion reaction cross sections at subthreshold energies. The full and empty circles are the results of the KVI experiments for the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) and \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) reactions, respectively. The solid curve is the calculated result obtained using the cluster model wave function of \( ^7\text{Be} \) for the \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) reaction, when the local imaginary potential \( W_D \) in the entrance channel is set to -25 MeV. The dashed-dotted and dashed curves are the results of the polynomial fit to the \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) experimental results and to the \( ^4\text{He}(^3\text{He},\pi^0)^7\text{Be} \) calculated results of the cluster model, respectively. The dotted curve is the fitted result to the \( ^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^* \) experimental data.
$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ experimental result and calculated result of the cluster model, respectively. The dotted curve is the resulting fit to the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ experimental data. The fits to the experimental data were performed in the angular range of $-1 \leq \cos\theta^0_{\text{c.m.}} \leq 0.5$, while the fit on the theoretical prediction of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction was performed in the angular range of $-1 \leq \cos\theta^0_{\text{c.m.}} \leq 1$.

By integrating the fitted curves, the total cross sections for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reactions were obtained. The obtained Legendre coefficients ($a_n$) and the total cross sections are listed in Table 6.4. The variance in the fit parameters $a_n$ is the diagonal entry in the covariance matrix so that the errors that are shown in Table 6.4 are the square root of that number. It should be noted that the calculated total cross section of 47.4 nb for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction from the polynomial fit using the full angular range is in good agreement with 47.3 nb from the cluster model.

**Table 6.4:** Legendre coefficients and the total cross section obtained from the fitting of Eq. 6.5 to the present measured results and to the calculations of the cluster model for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction. The uncertainties are purely statistical. “pf” and “cc” indicate “polynomial fit” and “cluster calculation”, respectively.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$T_{\text{beam}}$ [MeV]</th>
<th>$a_0$ [nb/sr]</th>
<th>$a_1$ [nb/sr]</th>
<th>$a_2$ [nb/sr]</th>
<th>$a_3$ [nb/sr]</th>
<th>$\sigma_{\text{tot}}$ [nb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$</td>
<td>$T_{\text{beam}}=236.4$ KVI measurement</td>
<td>0.94±0.38</td>
<td>0.85±1.11</td>
<td>0.90±0.27</td>
<td>1.02±0.32</td>
<td>11.8±1.2</td>
</tr>
<tr>
<td>$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$</td>
<td>$T_{\text{beam}}=256$ KVI measurement</td>
<td>4.88±2.22</td>
<td>3.28±4.32</td>
<td>5.01±3.15</td>
<td>0.19±1.93</td>
<td>61.3±2.3</td>
</tr>
<tr>
<td>$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$</td>
<td>$T_{\text{beam}}=266.4$ Cluster calculation Restricted angular range</td>
<td>4.28±0.32</td>
<td>2.98±0.76</td>
<td>5.58±2.16</td>
<td>3.32±2.36</td>
<td>using pf 53.7</td>
</tr>
<tr>
<td>$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$</td>
<td>$T_{\text{beam}}=266.4$ Cluster calculation Full angular range</td>
<td>3.78±0.38</td>
<td>1.77±1.11</td>
<td>4.42±0.27</td>
<td>2.87±0.32</td>
<td>using pf 47.4</td>
</tr>
<tr>
<td>$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$</td>
<td>$T_{\text{beam}}=266.4$ Cluster calculation</td>
<td>using cc 47.3</td>
<td></td>
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</tbody>
</table>

The total cross section of the neutral pion production for the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ and $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reactions follows the same trend as was predicted by the Sudden Overlap model. The results have already been shown in Fig. 2.4 by cross and square, respectively. Using the polynomial fits, the extrapolated cross section of the $^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}^*$ reaction at $\theta_{\text{c.m.}} = 0^\circ$ is 3.5 nb/sr and is lower than the predicted value by the cluster model [8] for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ reaction. This confirms the decreasing trend of the cross section with increasing mass, which has been predicted by the cluster model. This model predicts
values of 11.4 and 2.3 nb/sr for the $^4\text{He}(^3\text{He},\pi^+)^7\text{Li}$ and the $^{16}\text{O}(^3\text{He},\pi^+)^{19}\text{F}$ reactions, respectively. For the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction, the extrapolated cross section at $\theta_{\text{c.m.}}=0^\circ$ is about 12 nb/sr (again using the polynomial fit). Using the polynomial fit, the total measured cross section of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ reaction was found to be 61.3±2.3 nb. This value is comparable with the predicted result of the cluster model. In this model assuming $W_D=-25$ MeV, the total cross section of 47.3 nb was obtained.

The ratios of the Legendre coefficients are presented in Table 6.5. An asymmetry ($a_1/a_0$) is obtained over the angular range in all four fits. The ratio $a_2/a_0$ is close to unity in all cases except for the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ calculation when the polynomial fit is applied in the restricted angular range $-1 \leq \cos \theta_{\text{c.m.}} \leq 0.5$ (1.30). Furthermore, it should be noted that in case of the $^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$ measurement, $a_3/a_0$ is much smaller (0.04) compared to the other measurements or calculations.

In the next chapter, the consequences of the presented analysis will be discussed and conclusions will be drawn.

Table 6.5: Comparison of ratios of the Legendre coefficients from the fits to the calculation and to the experimental data.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$T_{\text{beam}}$ [MeV]</th>
<th>$a_1/a_0$</th>
<th>$a_2/a_0$</th>
<th>$a_3/a_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{Li}(^4\text{He},\pi^0)^{10}\text{B}$</td>
<td>$T_{\text{beam}}=236.4$ KVI measurement</td>
<td>0.91</td>
<td>0.95</td>
<td>1.08</td>
</tr>
<tr>
<td>$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$</td>
<td>$T_{\text{beam}}=256$ KVI measurement</td>
<td>0.67</td>
<td>1.03</td>
<td>0.04</td>
</tr>
<tr>
<td>$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$</td>
<td>$T_{\text{beam}}=266.4$ Cluster calculation Restricted angular range</td>
<td>0.70</td>
<td>1.30</td>
<td>0.78</td>
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<tr>
<td>$^4\text{He}(^3\text{He},\pi^0)^7\text{Be}$</td>
<td>$T_{\text{beam}}=266.4$ Cluster calculation Full angular range</td>
<td>0.47</td>
<td>1.17</td>
<td>0.76</td>
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