Passivity-based Tracking Control of Port-Hamiltonian Mechanical Systems with only Position Measurements

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Abstract—Mechanical systems can be asymptotically stabilized without velocity measurements by applying a dynamic extension. By interconnecting the system with a (virtual) controller damping is indirectly injected into the system making velocity measurements unnecessary, provided that this damping propagates to the mechanical system. The approach has been shown for the case of potential energy shaping and for a class of systems requiring total energy shaping. In this paper we investigate the same idea for the trajectory tracking problem. Like with the stabilization problem we apply a dynamic extension to avoid having to measure the system velocities and still realize perfect tracking of a desired trajectory. This is done for fully-actuated port-Hamiltonian mechanical systems.

I. INTRODUCTION

In [9], [3] it was shown how a class of respectively Euler-Lagrange (EL) and port-Hamiltonian [7] mechanical systems could be asymptotically stabilized by a dynamic extension. A controller (the dynamic extension) is interconnected to the mechanical system which injects the necessary damping for asymptotic stability, provided that this damping can propagate to the original system. The damping input then depends on the dynamic extension and having to measure velocities, traditionally necessary for damping, can be avoided. Here we extend this idea to trajectory tracking.

Passivity-based trajectory tracking methods have been presented in [9], [5], for Euler-Lagrange (EL) mechanical systems and port-Hamiltonian systems respectively. In [8] a new method for the study of port-controlled Hamiltonian systems based on virtual non-homogenous fields of forces was introduced which provided more insights into the tracking control problem. With tracking control the energy shaping approach is complicated by having to modify the energy function into a time-varying one which in general spoils passivity. In [4] it was shown how port-Hamiltonian systems are stabilized by canonical transformation. A port-Hamiltonian system is transformed into another one while preserving the structure of the original system. The transformation also includes the time variable (even for time-invariant systems) which offers some possibilities for applying the theory on the tracking problem. In [5] the tracking problem is solved by using the theory of canonical transformation presented in [4] to stabilize an *error system*. The result is then perfect tracking

of the desired trajectories. In [1] the problem of tracking control without velocity measurements for EL systems was also solved. Additionally, the dynamic output feedback of [1] also realizes disturbance attenuation. A more recent paper [13] has presented the Interconnection and Damping Assignment Passivity-Based Control¹ (IDA-PBC) method for trajectory tracking. Tracking of desired trajectories is realized by a feedforward controller and a state feedback controller with an observer. The observer eliminates the need of measuring velocities. In the following we want to achieve the same but with the help of a dynamic extension. The difference with [13] is that they estimate the velocities, whereas we interconnect the error system with a virtual system and inject damping indirectly through this system. Here we describe this method for port-Hamiltonian systems, for the fully-actuated case. It can be seen as a continuation of the work presented in [3], where the dynamic extension was used for stabilization without velocity measurements. In [9] the idea for EL systems is also to stabilize an error system, however, the definition of an error system in [5] allows a larger class of systems. This fact could give extra advantages when investigating tracking control of underactuated systems which is a subject for future research.

This paper starts by explaining the theory of stabilization by canonical transformation in section II. In this section IDA-PBC is shown to be a special case of this theory and we also show the idea of stabilization by dynamic extension. In section III tracking by dynamic extension is described, which is actually the stabilization of an error system. Tracking control by dynamic extension is then applied on an example in section IV. The example concerns tracking control of a planar manipulator with two degrees of freedom (DOF). In the final section some concluding remarks are given.

II. STABILIZATION OF PORT-HAMILTONIAN SYSTEMS

Stabilization via canonical transformation

The conventional canonical transformation in classical mechanics is to transform a set of canonical equations into another set, which preserves the structure. In dealing

¹A technique for total energy shaping [10], [11]

with a dynamical system we are free to choose whatever coordinates we like, general dynamical theory is invariant under transformations

$$q \rightarrow \bar{q} \quad (1)$$

The canonical transformation is widely used for analysis of the structure of dynamical systems in classical mechanics. In [4] canonical transformation for port-Hamiltonian systems was introduced, which preserves the port-Hamiltonian structure of the original system. There it was shown how port-Hamiltonian systems are stabilized by using the canonical transformation. According to the authors the method works even when the system cannot be stabilized by conventional unity feedback without canonical transformation.

Describe a non-autonomous port-Hamiltonian system by

$$\begin{aligned} \dot{x} &= (J(x, t) - R(x, t)) \frac{\partial H}{\partial x}(x, t) + g(x, t)u \\ y &= g^\top(x, t) \frac{\partial H}{\partial x}(x, t) \end{aligned} \quad (2)$$

where x are the states of the system: $x = (q, p)^\top$ the vector of generalized configuration coordinates $q = (q_1, \dots, q_n)^\top$ and generalized momenta $p = (p_1, \dots, p_n)^\top$, (skew symmetric) interconnection matrix $J(x, t) \in \mathbb{R}^{n \times n}$, damping matrix $R(x, t) \in \mathbb{R}^{n \times n}$ and input matrix $g(x, t) \in \mathbb{R}^{n \times m}$, $m \leq n$. The Hamiltonian $H(x, t)$ is defined as the kinetic plus potential energy of the system. The following definition and theorems from [4], [5] are presented here.

Definition 1: A set of transformations

$$\bar{x} = \Phi(x, t) \quad (3)$$

$$\bar{H} = H(x, t) + U(x, t) \quad (4)$$

$$\bar{y} = y + \alpha(x, t) \quad (5)$$

$$\bar{u} = u + \beta(x, t) \quad (6)$$

that changes the coordinates x into $\bar{x}$, the Hamiltonian H into $\bar{H}$, the output y into $\bar{y}$ and the input u into $\bar{u}$ is said to be a generalized canonical transformation for the port-Hamiltonian system if it transforms the port-Hamiltonian system (2) into another one.

The class of generalized canonical transformations are characterized by the following theorem:

Theorem 1: Consider the port-Hamiltonian system described by (2). For any scalar function $U(x, t)$ and any vector function $\beta(x, t)$, there exists a pair of functions $\Phi(x, t)$ and $\alpha(x, t)$ that yields a generalized canonical transformation. The function $\Phi(x, t)$ yields a generalized canonical transformation with $U(x, t)$ and $\beta(x, t)$ if and only if the partial differential equation (PDE)

$$\frac{\partial \Phi}{\partial(x, t)} \left((J - R) \frac{\partial U}{\partial x} + (K - S) \frac{\partial(H+U)}{\partial x} + g\beta \right) = 0 \quad (7)$$

holds with a skew-symmetric matrix $K(x, t)$ and a symmetric matrix $S(x, t)$ satisfying $R + S \geq 0$. Further the change

of output $\alpha(x, t)$ and the matrices $\bar{J}(x, t)$ and $\bar{g}(x, t)$ are given by

$$\alpha(x, t) = g^\top(x, t) \frac{\partial U}{\partial x}(x, t) \quad (8)$$

$$\bar{J}(\bar{x}, t) = \frac{\partial \Phi}{\partial x}(J + K) \frac{\partial \Phi}{\partial x}^\top \quad (9)$$

$$\bar{g}(\bar{x}, t) = \frac{\partial \Phi}{\partial x} g(x, t) \quad (10)$$

$$\bar{R}(\bar{x}, t) = \frac{\partial \Phi}{\partial x}(R + S) \frac{\partial \Phi}{\partial x}^\top \quad (11)$$

Before describing the stabilization procedure the definition of decrescent is given

Definition 2 ([6], [12]): A scalar function $W(x, t)$ is said to be decrescent if $W(0, t) = 0$ and if there exists a time-invariant positive definite function $W_1(x)$ such that

$$\forall t \geq 0, \quad W(x, t) \leq W_1(x)$$

Theorem 2: Consider the port-Hamiltonian system described by (2) and transform it by the generalized canonical transformation with $U(x, t)$ and $\beta(x, t)$ such that $H + U \geq 0$. Then the new input-output mapping $\bar{u} \mapsto \bar{y}$ is passive with storage function $\bar{H}$ if and only if

$$\frac{\partial(H + U)^\top}{\partial(x, t)} \left((J - R) \frac{\partial U}{\partial x} - S \frac{\partial(H+U)}{\partial x} + g\beta \right) \geq 0 \quad (12)$$

Suppose that (12) holds, that $H + U$ is positive-definite and that the system is zero-state detectable. Then the feedback $u = -\beta - C(x, t)(y + \alpha)$ with $C(x, t) \geq \epsilon I > 0$ renders the system asymptotically stable. Suppose moreover that $H + U$ is decrescent and that the transformed system is periodic. Then the feedback renders the system uniformly asymptotically stable.

In [4] it has already been shown that the canonical transformation theory includes potential energy shaping as a special case. It can be shown that total energy shaping by IDA-PBC [11] can be derived from the just presented theorems. First a short description is given of the IDA-PBC technique. Consider system (2) and assume it to be time-invariant and without physical damping ($R = 0$). By applying IDA-PBC we want to change this system into a new port-Hamiltonian system which has the desired equilibrium points, that is a new system

$$\dot{x} = (J_d(x) - R_d(x)) \frac{\partial H_d}{\partial x}(x) \quad (13)$$

where $J_d = -J_d^\top \in \mathbb{R}^{n \times n}$ is the new interconnection matrix, $R_d \geq 0 \in \mathbb{R}^{n \times n}$ is the damping matrix and new energy function $H_d(x)$. The success of IDA-PBC lies in finding an energy function H_d which qualifies as a Lyapunov function. This new energy function is found by solving the PDE

$$g(x)^\perp [(J_d(x) - R_d(x)) \frac{\partial H_d}{\partial x}(x) - J(x) \frac{\partial H}{\partial x}(x)] = 0 \quad (14)$$

with $g^\perp$ a left annihilator of g , i.e., $g^\perp g = 0$. PDE (14) can be divided into a kinetic energy PDE and a potential energy

PDE. These PDEs are also called the matching equations (or matching conditions) [2]. If a solution can be found the system (13) is realized by the input

$$u = (g^\top g)^{-1} g^\top [(J_d - R_d) \frac{\partial H_d}{\partial x} - J \frac{\partial H}{\partial x}] \quad (15)$$

Proposition 1: Consider the port-Hamiltonian system (2) and assume it to be time-invariant with $R = 0$. Applying IDA-PBC is a special case of theorem 1 with $\Phi(x) = x$.

Proof. From definition 1 and theorem 1 with $\Phi = x$, $R = 0$ and $S = R_d$ we have that

$$\begin{aligned} \bar{H} &= H_d = H + U \\ \bar{J} &= J_d = J + K \end{aligned}$$

The PDE (7) reduces to

$$g\beta = J \frac{\partial H}{\partial x} - J_d \frac{\partial H_d}{\partial x} + R_d \frac{\partial H_d}{\partial x}$$

Multiplying from the left with $g^\perp$ yields

$$g^\perp [J \frac{\partial H}{\partial x} - J_d \frac{\partial H_d}{\partial x} + R_d \frac{\partial H_d}{\partial x}] = 0$$

which is the same as (14). Take $\bar{u} = 0$ in definition 1 which results in $u = -\beta$. The solution of the PDE results in the input signal

$$u = -\beta = -(g^\top g)^{-1} g^\top [J \frac{\partial H}{\partial x} - (J_d - R_d) \frac{\partial H_d}{\partial x}]$$

which is the same as (15). $\square$

Stabilization via dynamic extension

As mentioned in the introduction a class of port-Hamiltonian mechanical systems can be asymptotically stabilized without velocity measurements. By interconnecting the system with a virtual system (the extension) and applying IDA-PBC an extended closed-loop system can be realized, described by

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} J_d(x) & 0 \\ 0 & J_c(x_c) - R_c(x_c) \end{bmatrix} \begin{bmatrix} \frac{\partial \tilde{H}_d}{\partial x} \\ \frac{\partial \tilde{H}_d}{\partial x_c} \end{bmatrix} \quad (16)$$

with $J_d = -J_d^\top$ the new interconnection matrix of the mechanical system, $J_c = -J_c^\top$ the controller interconnection matrix, $R_c > 0$ the controller damping matrix and x_c the controller states. The function $\tilde{H}_d$ is the new energy function of the mechanical system which is for one part the solution of a partial differential equation (the matching equations) and the other part arbitrary realizing the desired equilibrium points and the interconnection between controller and system. For this closed-loop system we have

$$\begin{aligned} \frac{d\tilde{H}}{dt} &= -\frac{\partial \tilde{H}_d^\top}{\partial x_c} R_c \frac{\partial \tilde{H}_d}{\partial x_c} \\ &\leq 0 \end{aligned} \quad (17)$$

We now limit the class of systems to

- Systems that need only potential energy shaping (e.g. fully actuated systems), or

- Underactuated systems with constant mass matrix.

For this class of systems velocity measurements are not necessary for energy shaping.

Theorem 3: A dynamic extension for port-Hamiltonian systems resulting in the closed-loop system (16) asymptotically stabilizes the time-invariant system (2) belonging to the class described above, with no damping, to x^* if

- 1) The closed-loop matching condition, with $\tilde{H}_d$ depending on both system and controller coordinates, is satisfied.
- 2) The dissipation propagates: $\frac{\partial \tilde{H}_d}{\partial x_c} = 0 \Leftrightarrow x = x^*, x_c = x_c^*$

Proof. Stability is proven by LaSalle's principle. $\square$

III. TRACKING VIA DYNAMIC EXTENSION

In [5] a coordinate transformation was found which transformed the original system (2), without dynamic extension, into an error system. Asymptotically stabilizing the error system (in the equilibrium point 0) implies that the tracking error converges to zero and the state x then tracks the desired trajectory. The desired trajectory of $x(t)$ for $t \in [t_0, \infty]$ is given by $x_d(t) = (q_d(t), M(q_d(t)\dot{q}_d(t))^\top$ and assume it to be *realizable*. An error system is defined in [5] by

Definition 3: A system described by

$$\bar{x} = \tilde{f}(\bar{x}, \bar{u}, t), \quad \bar{x}(t_0) = \phi(x(t_0), x_d(t_0)) \quad (18)$$

with a smooth function $\phi : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$, is said to be an *error system* of the general nonlinear system

$$\dot{x} = f(x, u, t) \quad (19)$$

with respect to the desired trajectory $x_d(t)$ if the following holds for $t \geq t_0$:

$$\bar{x} = 0 \Leftrightarrow x(t) = x_d(t). \quad (20)$$

We consider only fully-actuated systems. Equation (2) in terms of q and p becomes

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u \quad (21)$$

with input matrix $G = I$. The Hamiltonian H is defined as

$$H(q, p) = \frac{1}{2} p^\top M^{-1}(q) p + V(q) \quad (22)$$

where M is the system mass matrix and V the potential energy. In [5] it has been shown that (21) can be transformed into a *passive* error system with a Hamiltonian $H + U$ positive definite with respect to the tracking error by,

$$U(q, p, t) = \frac{1}{2} \dot{q}_d^\top M(q) \dot{q}_d - p^\top \dot{q}_d - V(q) + \bar{U}(q - q_d(t)) \quad (23)$$

and the vector function β in (6)

$$\begin{aligned} \beta(q, p, t) &= -M\ddot{q}_d(t) - \frac{\partial V}{\partial q} + \frac{\partial \bar{U}}{\partial q} + \left(\frac{1}{2} \frac{\partial (M(q)\dot{q}_d)^\top}{\partial q} \right. \\ &\quad \left. - \frac{1}{2} \frac{\partial (M(q)\dot{q}_d)}{\partial q} + \frac{1}{2} M \frac{\partial (M^{-1}(q)p)}{\partial q} \right) \dot{q}_d(t) \end{aligned} \quad (24)$$

Here $\bar{U}$ is a free design parameter which realizes a unique equilibrium in $\bar{q} = 0$. It has to be noted that the error system is not yet asymptotically stable. The feedback law given by theorem 2 is applied to asymptotically stabilize the error system and thus to realize trajectory tracking. Notice that (24) depends on velocity measurements. We will now investigate the possibilities of avoiding these velocity measurements. Consider $\Phi = (\bar{q}, \bar{p})^\top$:

$$\bar{q} = q - q_d(t) \quad (25)$$

$$\bar{p} = p - M(q)\dot{q}_d(t) \quad (26)$$

and the function U as in (23), which results in a Hamiltonian in terms of $\bar{q}, \bar{p}$, positive definite with respect to the tracking error. Theorem 1 is now applied to find another solution for β , which does not depend on p :

$$\beta = \frac{\partial U}{\partial q} - \frac{\partial(M(q)\dot{q}_d)}{\partial q} \dot{q}_d - M(q)\ddot{q}_d \quad (27)$$

It is necessary to verify whether the transformed system satisfies the passivity condition (12), i.e.,

$$\left(-\frac{1}{2} \frac{\partial(p^\top M^{-1}(q)p)}{\partial q} - \frac{1}{2} \frac{\partial(\dot{q}_d^\top M(q)\dot{q}_d)}{\partial q} - (M^{-1}p - \dot{q}_d)^\top \frac{\partial(M(q)\dot{q}_d)}{\partial q} \right) \dot{q}_d \geq 0 \quad (28)$$

Now we interconnect the error system with a dynamic extension realizing an extended error system:

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} \bar{J} & 0 \\ 0 & J_c - R_c \end{bmatrix} \begin{bmatrix} \frac{\partial \tilde{H}}{\partial \bar{x}}(\bar{x}, x_c, t) \\ \frac{\partial \tilde{H}}{\partial x_c}(\bar{x}, x_c, t) \end{bmatrix} + \begin{bmatrix} \bar{g} \\ 0 \end{bmatrix} \bar{u} \quad (29)$$

with Hamiltonian

$$\tilde{H} = H + U + V_c(\bar{x}, x_c) \quad (30)$$

From this point on we continue to work with the extended error system (29). The function V_c is the controller potential energy and realizes the interconnection between the error system and the controller (the dynamic extension). Having the controller to be passive and interconnecting it with a passive error system results in a passive extended error system (29). The controller dynamics can be described by potential energy $V_c(q_c)$ and controller dissipation energy² $\mathcal{F}_c(\dot{q}_c)$, $q_c \in \mathbb{R}^n$. By taking $\bar{u} = 0$, which means that $u = -\beta$ from definition 1, the extended error system is realized with input

$$u = -\frac{\partial U}{\partial q} - \frac{\partial V_c}{\partial q} + \frac{\partial(M(q)\dot{q}_d)}{\partial q} \dot{q}_d + M(q)\ddot{q}_d \quad (31)$$

Describe the controller by the functions

$$V_c(\bar{q}, q_c) = \frac{1}{2}(\bar{q} - q_c)^\top K_c(\bar{q} - q_c) \quad (32)$$

$$\mathcal{F}_c(\dot{q}_c) = \frac{1}{2}\dot{q}_c^\top K_d \dot{q}_c \quad (33)$$

²The controller kinetic energy plays no role in the stabilization task, so it can be chosen equal to zero

where $K_c = \text{diag}\{k_{c1}, \dots, k_{cn}\} > 0$ is the matrix of controller constants and $K_d = \text{diag}\{k_{d1}, \dots, k_{dn}\} > 0$ the matrix of controller damping constants. The controller dynamics become

$$\begin{aligned} \dot{q}_c &= -K_d^{-1} \frac{\partial V_c}{\partial q_c} \\ &= K_d^{-1} K_c(\bar{q} - q_c) \end{aligned} \quad (34)$$

The function $\bar{U}$ is chosen as

$$\bar{U} = \frac{1}{2} \bar{q}^\top K \bar{q} \quad (35)$$

with constant matrix $K = \text{diag}\{k_1, \dots, k_n\} > 0$.

Theorem 4: Consider system (21) and assume it to have a constant mass matrix. The input signal (31) realizes tracking of a desired trajectory with the functions (23), (32) and (33). Moreover, tracking is realized (i.e., the error converges to zero asymptotically) without velocity measurements.

Proof. For a constant mass matrix condition (28) is satisfied and it is straightforward to obtain

$$\frac{d\tilde{H}}{dt} = -\dot{q}_c^\top K_d \dot{q}_c \quad (36)$$

Because the system is passive the states $\bar{q}, \bar{p}$ and q_c are bounded. Since the function $\frac{d^2 \tilde{H}}{dt^2}$ depends on these states it is also bounded, implying that $\frac{d\tilde{H}}{dt}$ is uniformly continuous in time. Since we have also chosen our $\tilde{H}$ to be positive definite, and thus bounded from below, we can apply Barbalat's lemma. By Barbalat's lemma [6], [12] $\dot{q}_c$ converges to zero, so q_c becomes constant. From the dynamics of the extended error system it can be shown that when q_c is constant $\bar{q} = q - q_d(t)$ also has to be constant. The only points that are constant are the equilibrium points (the origin) implying asymptotic stability of the extended error system. The tracking error converges to zero. The input signal (31) depends only on the coordinates q and q_c . $\square$

Remark 1: Barbalat's lemma is invoked here since, as is usually the case with mechanical systems, the time derivative of the energy function is negative semi-definite. For autonomous systems LaSalle's principle would be enough to prove stability, however as mentioned in [12] the invariance theorem cannot be used for nonautonomous systems. Nevertheless, there are some special classes of nonautonomous systems for which invariance theorems exist, as mentioned in [6].

Remark 2: From [6], [12] we know that if $\tilde{H}$ in proposition 4 is also decrescent that the extended error system will be uniformly asymptotically stable, i.e., we then have uniform tracking.

The tracking problem without velocity measurements has also been presented in [9] for EL systems. There it was shown that the error system could be semi-globally exponentially stabilized. They also mention that the tracking problem, without velocity measurements, for any initial conditions and for n -degrees-of-freedom systems remains open. In this

section we have shown that this problem can be solved if the mass matrix of the system is constant. We can show that under certain conditions the input signal (31) can also realize tracking for a more general system, a system with mass matrix depending on the coordinates q .

Proposition 2: Consider system (21). The input signal (31) realizes tracking of a desired trajectory, without velocity measurements, with the functions (23), (32) and (33) if

- $\frac{\partial M^{-1}(\bar{q}+q_d)}{\partial t} \leq 0 \quad \forall \bar{q}, q_d$ or
- $\frac{1}{2}\bar{p}^\top \frac{\partial M^{-1}(\bar{q}+q_d)}{\partial t} \bar{p} < \dot{q}_c^\top K_d \dot{q}_c \quad \forall \bar{q}, q_d, q_c, \bar{p}$

Proof. In the new coordinates, $\bar{q}$ and $\bar{p}$, the Hamiltonian $\tilde{H}$ becomes

$$\tilde{H} = \frac{1}{2}\bar{p}^\top M^{-1}(\bar{q} + q_d)\bar{p} + \bar{U}(\bar{q}) + V_c(\bar{q}, q_c) \quad (37)$$

which is positive definite with respect to the tracking error. Taking $\tilde{H}$ as Lyapunov candidate function results in

$$\frac{d}{dt}\tilde{H} = -\dot{q}_c^\top K_d \dot{q}_c + \frac{1}{2}\bar{p}^\top \frac{\partial M^{-1}(\bar{q} + q_d)}{\partial t} \bar{p} \quad (38)$$

which is negative semi-definite only when one of the above mentioned conditions hold. Asymptotic stability of the extended error system (29) can then be proven with Barbalat's lemma, in the same way as in proposition 4. $\square$

Remark 3: Uniform tracking can be realized under the same conditions as in remark 2.

IV. EXAMPLE

2R planar manipulator

To show how tracking works when we apply a dynamic extension we apply the methods of the previous section on a fully actuated 2 DOF planar manipulator (2R planar manipulator). The system is shown in figure 1. The manipulator has links with length l_i , angles θ_i , mass m_i , the center of the mass is denoted by r_i and the moment of inertia I_i with $i = 1, 2$. The system works in the horizontal plane so gravity

Fig. 1. 2R planar manipulator.

influence can be neglected. This means that the system has Hamiltonian defined only by kinetic energy:

$$H(q, p) = \frac{1}{2}p^\top M(q)^{-1}p \quad (39)$$

with $q = (\theta_1, \theta_2)^\top$ and $p = M(q)\dot{q}$. Define the constants

$$\begin{aligned} a_1 &= m_1 r_1^2 + m_2 l_1^2 + I_1 \\ a_2 &= m_2 r_2^2 + I_2 \\ b &= m_2 l_1 r_2 \end{aligned}$$

The mass matrix becomes

$$M(q) = \begin{bmatrix} a_1 + a_2 + 2b \cos \theta_2 & a_2 + b \cos \theta_2 \\ a_2 + b \cos \theta_2 & a_2 \end{bmatrix} \quad (40)$$

We also assume that there is no physical damping present, $R = 0$. This system can be described as a standard port-Hamiltonian mechanical system, (21), with G a 2×2 identity matrix since the system is fully actuated and input signal $u = (u_1, u_2)$ which are the control torques on the two joints.

Simulation Results

For simplicity we have chosen the system parameters to be all equal to one and the controller parameters $k_1 = k_2 = 10$, $k_{c1} = k_{c2} = 10$ and $d_{c1} = d_{c2} = 5$. In this example the desired joint angles are

$$q_{1d}(t) = \theta_{1d}(t) = c_1 \sin \omega_1 t \quad (41)$$

$$q_{2d}(t) = \theta_{2d}(t) = \frac{c_2(t - t^*)}{\sqrt{d + (t - t^*)^2}} - 1.5 \quad (42)$$

where $c_1 = c_2 = \omega_1 = 1$, $d = 50$ and $t^* = 20$ ($d, t^* > 0$). We want to remark that it is the desired trajectory $q_{2d}(t)$, shown in figure 2 (which looks like a smooth step), which is constrained by proposition 2 since the mass matrix only depends on this coordinate. Figure 3 shows some simulation

Fig. 2. Desired trajectory for θ_2

results for the 2R manipulator. Although we are not able to show that the conditions of proposition 2 hold globally we can still claim that the extended error system for the 2R planar manipulator is locally uniformly asymptotically stable. To show this we write the first condition of proposition 2 in the form

$$\frac{\partial M^{-1}(\bar{q} + q_d)}{\partial t} = -M^{-1} \frac{\partial M}{\partial t} M^{-1} \leq 0 \quad (43)$$

implying that the matrix $\frac{\partial M}{\partial t}$ should be positive semi-definite. It can be verified that the desired trajectories $q_{2d}(t)$ and

Fig. 3. Trajectories for the 2R planar manipulator. Initial conditions: $[q(0) \ p(0)] = [1 \ 1 \ 0 \ 0]$. The dotted lines represent the results when velocity measurements are used (no extension)

$\dot{q}_{2d}(t)$ in this example satisfy this condition in a neighborhood of $\bar{q}_2 = 0$ ³. By again applying Barbalat's lemma asymptotic stability of the extended error system can be shown, however only locally. Additionally we know that the function $\tilde{H}$ is decrescent (since from (40) we know that M is also positive definite and bounded) implying local uniform asymptotic stability of the extended error system. The results show that the tracking error converges to zero. The drawback, however, is that the tracking error at the start is larger than in the case where velocity measurements are used. The input signal when dynamic extension is used is also larger at the beginning. There is clearly a tradeoff with performance when controlling the system without velocity measurements.

V. CONCLUDING REMARKS

We have shown how tracking control is achieved without having to measure system velocities. From an applications point of view this can be interesting since less sensors are required for tracking control. The idea was firstly inspired by [4], [5] where by coordinate transformations one could also deal with time-varying port-hamiltonian systems. By an appropriate choice of new coordinates one can change a system into an error system. The tracking control problem is then reduced to asymptotic stabilization of the error system. In [3] it was shown how for a certain class of port-Hamiltonian mechanical systems asymptotic stability could be realized without velocity measurements. The approach was to interconnect the system with a controller and let the controller damping *propagate* to the original system. We also call this controller the dynamic extension of the system. For tracking control without velocity measurements a similar approach was taken with the dynamic extension. However, now the controller is (virtually) interconnected with the error system. Tracking of a desired trajectory is achieved by proving asymptotic stability of the error system. Barbalat's lemma was used to prove asymptotic stability because the time

derivative of the energy function is negative semi-definite and the system is nonautonomous. For a decrescent energy function one can additionally realize uniform asymptotic stability of the extended error system (i.e., uniform tracking). How the theory works was shown with a two degrees of freedom planar manipulator as example. This system was assumed to be fully actuated. The simulation results showed convergence of the tracking error to zero. The drawback however was larger deviations and input signals at the start of the simulations compared to tracking control with velocity measurements.

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³Although not yet proven, the control input (31) also works for $q_{2d}(t) = c_2 \sin \omega_2 t$