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A Cyclodissipativity Condition for Power Factor Improvement under Nonsinusoidal Source with Significant Impedance

Dunstano del Puerto-Flores, Romeo Ortega, and Jacquelien M. A. Scherpen

Abstract— **The main contribution of this paper is an extension of a recent result that reformulates and solves the power factor compensation for nonlinear loads under nonsinusoidal regime in terms of cyclodissipativity. In the aforementioned result the generator was assumed to be ideal, that is, with negligible impedance. In this work, we formulate the power factor compensation problem in a way that explicitly accounts for the effects of a significant source impedance.**

I. Introduction

Recently, in [1] it has been established that the classical problem in electrical engineering of optimizing energy transfer from an alternating current (ac) source to a load with non-sinusoidal (but periodic) source voltage is equivalent to imposing the property of cyclodissipativity to the source terminals. Since this framework is based on the cyclodissipativity property, see [2], the improvement of power factor (PF) is done independent of the reactive power definition, which is a matter of discussions in the power community, see [3].

Using this framework the classical capacitor or inductor compensators were interpreted in terms of energy equalization, see [1] for more details. We have presented an extension of the results in [4] where we considered arbitrary lossless linear time-invariant (LTI) filters, and proved that for general lossless LTI filters the PF is reduced if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. Although the aforementioned results were obtained by considering nonlinear loads, the generator was assumed to be ideal, that is, with negligible impedance. However, in practice the source impedance is often significant.

We consider the energy transfer from a known voltage source v_s to a given fixed load i_ℓ, see Figure 1. The standard approach to improve the PF is to place a lossless compensator, Y_c, between the source and the load. The PF compensation configuration considered in the paper is depicted in Figure 2.

In this work, our task is to formulate the power factor compensation problem in a way that explicitly accounts for the effects of a non-negligible source impedance, Z_s, on the load voltage and current. We prove that cyclodissipativity provides a rigorous mathematical framework useful to analyze and design power factor compensators for general nonlinear loads operating in nonsinusoidal regimes with significant source impedance.

First, we briefly review our result from [4]. Then, in Section IV we give a cyclodissipativity characterization of the lossless LTI PF Compensation problem with source impedance. Next we provide a geometric characterization in Section V and we end with an example in Section VI.

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II. Power Factor Improvement

A. Framework

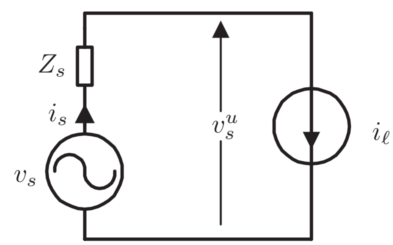

Fig. 1. Power delivery system with significant source impedance.

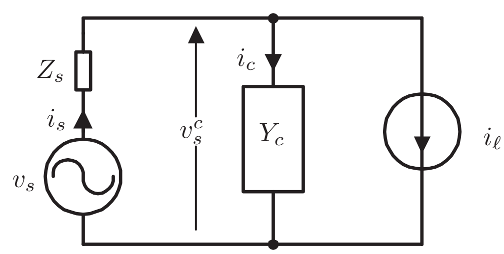

Fig. 2. Parallel load compensation in a power delivery system with significant source impedance.

We consider the energy transfer from an n-phase ac generator to a load where the source is not assumed to be ideal, but has a significant impedance, see Figure 1. All signals are assumed to be periodic and have finite power, that is, they belong to the space

$$\mathcal{L}_2^n = \left\{ x : [0,T) \to \mathbb{R}^n : \|x\|^2 := \frac{1}{T}\int_0^T |x(\tau)|^2 d\tau < \infty \right\}$$

where $\|\cdot\|$ is the rms value and $|\cdot|$ is the Euclidean norm. We also define the inner product in $\mathcal{L}_2^n$ as

$$\langle x, y \rangle := \frac{1}{T}\int_0^T x^\top(t) y(t) dt.$$

The process of power factor correction is an attempt to reduce the apparent voltamperes of a load to the value of the average power consumed. The universally accepted definition of PF is given as [5]:

Definition 1 (Power factor): Consider the power delivery systems of the Figure 1. The PF of an AC electric power system is defined by

$$PF := \frac{P}{S}, \tag{1}$$

where

$$P := \langle v_s^u, i_s \rangle, \quad S := \|v_s^u\| \|i_s\| \tag{2}$$

are the active (real) power,[1] and the apparent power, respectively.

From (2), it follows that $P \leq S$. Hence $PF \in [-1, 1]$ is a dimensionless measure of the energy-transmission efficiency. Cauchy–Schwartz also tells us that a necessary and sufficient condition for the apparent power to equal the active power is that v_s and i_s are collinear. If this is not the case, $P < S$ and compensation schemes are introduced to maximize the PF.

The condition for unity power factor is that the input current to a systems is proportional at all times to the instantaneous supplied voltage.

B. The Power Factor compensation problem

The PF compensation configuration considered in the paper is depicted in Figure 2, where $Y_c : \mathcal{L}_2^n \to \mathcal{L}_2^n$ is the admittance operator of the compensator. That is,

$$i_c = Y_c v_s^c$$

where $i_c \in \mathcal{L}_2^n$, is the compensator current. In the simplest LTI case the operator Y_c can be described by its admittance transfer matrix, which we denote by $\hat{Y}_c(s) \in \mathbb{R}^{n \times n}(s)$. We make the following fundamental assumption.

Following standard practice, we consider only lossless compensators, that is,

$$\langle i_c, v_s^c \rangle = 0, \quad \forall v_s^c \in \mathcal{L}_2^n. \tag{3}$$

We recall that, if Y_c is a passive LTI LC-network, losslessness implies

$$\mathrm{Re}\{\hat{Y}_c(j\omega)\} = 0. \tag{4}$$

for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of $\hat{Y}_c$ and all alternate zeros and poles are simple and lie on the imaginary axis and where $\mathrm{Re}\{\hat{Y}_c(j\omega)\}$ is the real part of the admittance transfer $\hat{Y}_c(j\omega)$, see [6].

Definition 2 (Power factor improvement): Given a n-phase source voltage $v_s(t)$, a linear time-invariant (n-phase) source impedance $Z_s : \mathcal{L}_2^n \to \mathcal{L}_2^n$, and a fixed load current i_ℓ, as in Figure 2, power-factor improvement is achieved with the lossless compensator Y_c if and only if

$$PF > PF_u := \frac{\langle v_s^u, i_\ell \rangle}{\|v_s^u\| \|i_\ell\|} \tag{5}$$

[1] Also called average power [5].

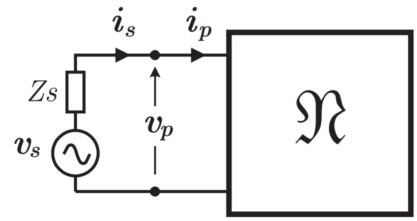

Fig. 3. Circuit schematic of an n-phase non-ideal generator connected to a n-port (possible nonlinear and time varying) load

where PF_u denotes the uncompensated power factor, that is, the value of PF with $Y_c = 0$ and $i_s = i_\ell$, and, by Kirchhoff's Voltage Law (KVL), the uncompensated voltage is

$$v_s^u = v_s - Z_s i_\ell. \tag{6}$$

III. PF compensation and cyclodissipativity: Ideal Case

The framework to be discussed carries abstract power connotations (as does the term itself). This is derived from the interpretation of the supply rate as an input power. To formulate our results we need the following.

Definition 3 (Cyclodissipative system, [2]): Given a mapping $w : \mathcal{L}_2^n \times \mathcal{L}_2^n \to \mathbb{R}$. The n-port system of Figure 3 is cyclodissipative with respect to the supply rate $w(v_p, i_p)$ if and only if

$$\int_0^T w(v_p(t), i_p(t))dt > 0. \tag{7}$$

for all $(v_p, i_p) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$.

A. Cyclodissipative Framework for Power Factor Compensation

To place or results in context, and make the paper self–contained, we recall the following results from [1]. Assume v_s is ideal, i.e., $Z_s = 0$.

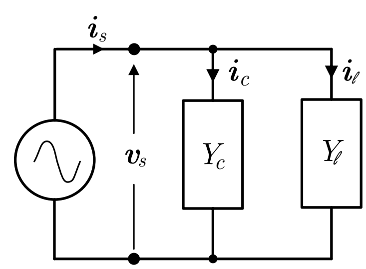

Fig. 4. Parallel load compensation in a power delivery system with negligible source impedance.

Proposition 4 ([1]): Consider the system of Figure 4 and fixed Y_ℓ. The compensator Y_c improves the PF if and only if the system is cyclodissipative with respect to the supply rate

$$w(v_s, i_s) := (Y_\ell v_s + i_s)^\top (Y_\ell v_s - i_s). \tag{8}$$

The next result follows from Proposition 4 and it characterizes the set of all compensators Y_c that improve the power-factor for a given Y_ℓ.

Corollary 5 ([1]): Consider the system of Figure 4 Then Y_c improves the PF for a given Y_ℓ if and only if Y_c satisfies

$$2\langle Y_\ell v_s, Y_c v_s\rangle + \|Y_c v_s\|^2 < 0, \quad \forall v_s \in \mathcal{L}_2^n. \tag{9}$$

Dually, given Y_c, the PF is improved for all Y_ℓ that satisfy (9).

B. Compensation with Lossless LTI Compensator equates Weighted Power Equalization.

In this section we review the results presented in [4], which extended the results in [1]. Similarly to [1], the class of RLC circuits that we consider as load models consists of possibly nonlinear lumped dynamic elements (n_L inductors, n_C capacitors) and nonlinear static elements (n_R resistors). Capacitors and inductors are defined by the physical laws and constitutive relations [7]:

$$i_C = \dot{q}_C, \quad v_C = \nabla H_C(q_C), \tag{10}$$

$$v_L = \dot{\phi}_L, \quad i_L = \nabla H_L(\phi_L), \tag{11}$$

respectively, where i_C, v_C, $q_C \in \mathbb{R}^{n_C}$ are the capacitors currents, voltages and charges, and $i_L, v_L, \phi_L \in \mathbb{R}^{n_L}$ are the inductors currents, voltages and flux–linkages, $H_L : \mathbb{R}^{n_L} \to \mathbb{R}$ is the magnetic energy stored in the inductors, $H_C : \mathbb{R}^{n_C} \to \mathbb{R}$ is the electric energy stored in the capacitors, and ∇ is the gradient operator. We assume that the energy functions are twice differentiable. For linear capacitors and inductors

$$H_C(q_C) = \frac{1}{2} q_C^\top C^{-1} q_C, \; H_L(\phi_L) = \frac{1}{2}\phi_L^\top L^{-1}\phi_L,$$

respectively, with $L \in \mathbb{R}^{n_L \times n_L}$, $C \in \mathbb{R}^{n_C \times n_C}$. To avoid cluttering the notation we assume L, C are diagonal matrices. Finally, we distinguish between two sets of nonlinear static resistors: n_{R_i} current–controlled resistors and n_{R_v} voltage–controlled resistors, for which the characteristic are given by

$$v_{R_i} = \hat{v}_{R_i}(i_{R_i}), \;\text{ and }\; i_{R_v} = \hat{i}_{R_v}(v_{R_v}),$$

respectively, where i_{R_i}, $v_{R_i} \in \mathbb{R}^{n_{R_i}}$ are the currents, voltages of the current-controlled resistors, and $i_{R_v}, v_{R_v} \in \mathbb{R}^{n_{R_v}}$ are the currents, voltages of the voltage-controlled resistors, with $n_R = n_{R_i} + n_{R_v}$.

Recalling the definition of real power (2) we introduce the following.

Definition 6 (Weighted (real) power): Given a compensator admittance Y_c the weighted (real) power of a single–phase circuit with port variables $(v, i) \in \mathcal{L}_2 \times \mathcal{L}_2$ is given by

$$P^w := \langle Y_c v, i\rangle. \tag{12}$$

For instance, if Y_c is LTI

$$P^w = \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}[k]\hat{I}^*[k] \tag{13}$$

where $\hat{V}[k], \hat{I}[k]$ are the k-th spectral lines of v and i, respectively, and $\hat{Y}_c[k] := \hat{Y}_c(k\omega_0)$, with $\omega_0 := \frac{2\pi}{T}$. That is, P^w is the sum of the power components of the circuit modulated by the frequency response of Y_c—hence the use of the "weighted" qualifier.[2] The aforementioned definition motivates the next result.

Proposition 7: Consider the system of Figure 2 with $n = 1$,[3] a full nonlinear RLC load and a fixed LTI lossless compensator Y_c with admittance transfer function $\hat{Y}_c(j\omega)$ which has a zero at the origin.

i) PF is improved if and only if

$$\frac{1}{2}V_s^w + \sum_{q=1}^{n_L} P_{L_q}^w + \sum_{q=1}^{n_C} P_{C_q}^w < 0 \tag{14}$$

where V_s^w is the rms value of the filtered voltage source, that is,

$$V_s^w := ||Y_c v_s||^2 = \sum_{k=1}^{\infty} |\hat{Y}_c(k)\hat{V}_s(k)|^2$$

and

$$\begin{aligned} P_{C_q}^w &:= \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}_{C_q}[k]\hat{I}_{C_q}^*[k] \\ P_{L_q}^w &:= \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}_{L_q}[k]\hat{I}_{L_q}^*[k], \end{aligned}$$

are the weighted powers of the q–th inductor and capacitor, respectively.

ii) Condition (14) may be equivalently expressed as

$$\left\langle (\tfrac{1}{p}Y_c)v_L, \nabla^2 H_L v_L\right\rangle - \left\langle i_C, (\tfrac{1}{p}Y_c)\nabla^2 H_C i_C\right\rangle > \frac{1}{2}V_s^w \tag{15}$$

where $p := \frac{d}{dt}$.

iii) If the inductors and capacitors are linear their weighted powers become

$$\begin{aligned} P_{C_q}^w &:= 2\omega_0 \sum_{k=1}^{\infty}\left\{k\mathcal{I}_m\{\hat{Y}_c[k]\}\sum_{q=1}^{n_C} C_q|\hat{V}_{C_q}[k]|^2\right\} \\ P_{L_q}^w &:= -2\omega_0 \sum_{k=1}^{\infty}\left\{k\mathcal{I}_m\{\hat{Y}_c[k]\}\sum_{q=1}^{n_L} L_q|\hat{I}_{L_q}[k]|^2\right\}. \end{aligned}$$

iv) Furthermore, the results i-iii can be extended for a general LTI lossless compensator, if the resistors of the load are linear time-invariants.

Condition 14 indicates that the power factor improvement if and only if a certain equalization condition between the weighted powers of compensator and load is ensured. We now continue with extending these result to the non-ideal source case.

IV. A Cyclodissipativity Condition for Power Factor Improvement: Non-ideal Case

Although the problem at hand is posed as a problem in networks, it can be equally well interpreted as a feedback problem; the circuit of Fig. 2 is represented by the system of Fig. 5, which consists of two systems in a feedback loop.

[2]Since the spectral lines of real signals satisfy $\hat{F}[-k] = \hat{F}^*[k]$, the weighted power is a real number.

[3]This condition is imposed, without loss of generality, to simplify the presentation of the result.

Specifically, the inputs are v_s and i_ℓ and the outputs are i_s and v_s^c, and products are related with the instantaneous delivered power by the source $i_s^\top v_s$ and the instantaneous input power into the load, $i_\ell^\top v_s^c$.

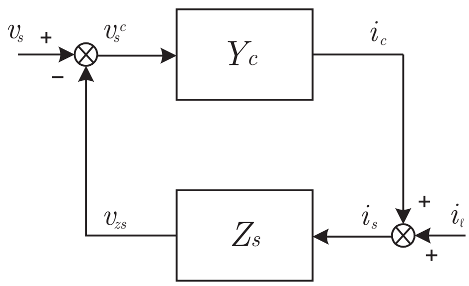

Fig. 5. Feedback configuration

Proposition 8: Consider the system of Figure 2. Given a n-phase source voltage $v_s(t)$, a linear time-invariant (n-phase) source impedance Z_s and a fixed load current i_ℓ. The compensator Y_c improves the PF if and only if the system has finite gain and is cyclodissipative with respect to the supply rate

$$w(i_s, i_\ell) := \delta^2 i_\ell^\top i_\ell - i_s^\top i_s. \tag{16}$$

for all $(i_s, i_\ell) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$, where δ is the upper gain bound[4] and is given by

$$\delta = \frac{\langle v_s^c, i_\ell\rangle}{\|v_s^c\|} \frac{\|(I + Z_sY_c)v_s^c\|}{\langle (I + Z_sY_c)v_s^c, i_\ell\rangle}, \tag{17}$$

with $1 < \delta < \infty$.

Proof: From Kirchhoff's Current Law (KCL), we have $i_s = i_c + i_\ell$, and using KVL,

$$v_s^c = v_s - Z_s(i_c + i_\ell), \tag{18}$$

Substituting (19) into (18), we obtain

$$v_s^u = v_s^c + Z_s i_c \tag{19}$$

From the definition of power factor and the lossless condition of the compensator, we have

$$PF := \frac{\langle v_s^c, i_\ell\rangle}{\|v_s^c\|\|i_s\|} \tag{20}$$

And, we define

$$\tilde{\alpha} := \frac{\langle v_s^c, i_\ell\rangle}{\|v_s^c\|}. \tag{21}$$

PF_u is given by

$$PF_u := \frac{\langle v_s^u, i_\ell\rangle}{\|v_s^u\|\|i_\ell\|}$$

and by using (19) and $i_c = Y_c v_s^c$,

$$v_s^u = v_s^c + Z_sY_cv_s^c = (I + Z_sY_c)v_s^c,$$

then,

$$PF_u = \frac{\langle (I + Z_sY_c)v_s^c, i_\ell\rangle}{\|(I + Z_sY_c)v_s^c\|\|i_\ell\|}, \tag{22}$$

[4]See Table 1 of [8], Definition 2.1 of [9], and Definition 2 of [10].

and we define:

$$\alpha := \frac{\langle (I + Z_sY_c)v_s^c, i_\ell\rangle}{\|(I + Z_sY_c)v_s^c\|}. \tag{23}$$

From Definition (5), we conclude that $PF > PF_u$ if and only if

$$\frac{\langle v_s^c, i_\ell\rangle}{\|v_s^c\|\|i_s\|} > \frac{\langle (I + Z_sY_c)v_s^c, i_\ell\rangle}{\|(I + Z_sY_c)v_s^c\|\|i_\ell\|},$$

or, by (21) and (23), the inequality becomes

$$\|i_s\| < \frac{\tilde{\alpha}}{\alpha}\|i_\ell\|, \tag{24}$$

with $\delta := \frac{\tilde{\alpha}}{\alpha}$. If $Y_c = 0$, i.e., the uncompensated case, from (17) we have that $\delta > 1$ and because the fact that δ depends only on bounded signals, i_ℓ and v_s^c, and the operators Z_s and Y_c are LTI, we can conclude that $\delta < \infty$. ■

Remark 9: The results of Proposition 5 in [1] are a particular case of Proposition 8 assuming an ideal source, i.e., the source impedance $Z_s = 0$ and, from (17), we have $\delta = 1$.

The next corollary of this result is the characterization of all compensators that improve the power factor.

Corollary 10: Consider the system of Figure 2. Then Y_c improves the PF for a given i_ℓ if and only if Y_c satisfies

$$\|i_c\|^2 + 2\langle i_c, i_\ell\rangle < \left(\left(\frac{\tilde{\alpha}}{\alpha}\right)^2 - 1\right)\|i_\ell\|^2, \quad \forall v_s, i_\ell \in \mathcal{L}_2^n. \tag{25}$$

Proof: Using the fact $i_s = i_c + i_\ell$, and the square of (24),

$$\begin{aligned}
\|i_s\|^2 &< \left(\frac{\tilde{\alpha}}{\alpha}\right)^2\|i_\ell\|^2 \\
\|i_c\|^2 + 2\langle i_c, i_\ell\rangle + \|i_\ell\|^2 &< \left(\frac{\tilde{\alpha}}{\alpha}\right)^2\|i_\ell\|^2 \\
\|i_c\|^2 + 2\langle i_c, i_\ell\rangle &< \left(\left(\frac{\tilde{\alpha}}{\alpha}\right)^2 - 1\right)\|i_\ell\|^2.
\end{aligned}$$

and, from $i_c = Y_c v_s^c$, then we have

$$\|Y_cv_s^c\|^2 + 2\langle Y_cv_s^c, i_\ell\rangle < \left(\left(\frac{\tilde{\alpha}}{\alpha}\right)^2 - 1\right)\|i_\ell\|^2 \tag{26}$$

■

Remark 11: From the feedback configuration under consideration, see Fig. 5. The interconnected system is cyclodissipative with respect to (16) if the compensator Y_c is cyclodissipative with respect to the supply rate function

$$w_c(i_\ell, i_c) := -i_c^\top i_c - 2i_c^\top i_\ell - (1 - \delta)(1 + \delta)i_\ell^\top i_\ell.$$

where the input is i_ℓ and i_c is the output.

V. Geometrical interpretation

Referring to Fig. 6 we have a geometric interpretation of power factor compensation. Fig. 6 depicts the vector v_s^c, v_s^u, i_s, i_c and i_ℓ. The angles β and β_u are defined as

$$\beta := \cos^{-1} PF, \quad \beta_u := \cos^{-1} PF_u,$$

or, $\beta = \angle(v_s^c, i_s)$ and $\beta_u = \angle(v_s^u, i_\ell)$. Then, it is clear from Fig. 6 that $PF > PF_u$ if only if $\beta < \beta_u$.
From (17), by assuming that $\langle v_s^c, i_\ell\rangle > 0$ and $\langle (I + Z_s Y_c) v_s^c, i_\ell\rangle > 0$, then we have that $1 < \delta < \infty$.

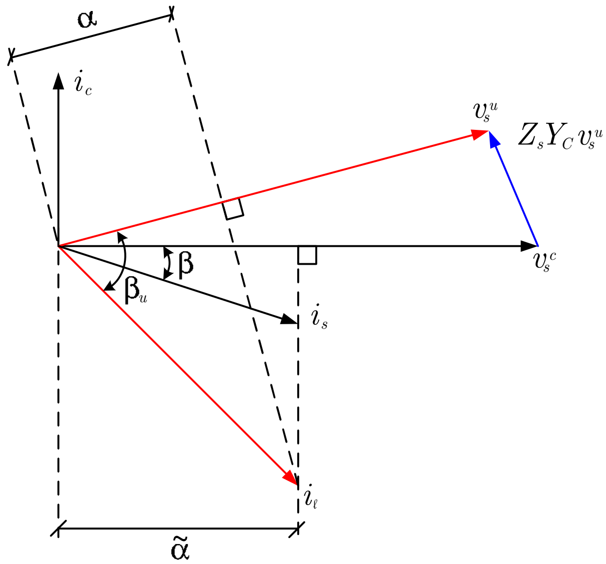

Fig. 6. Geometric interpretation of power factor compensation.

Remark 12: The projection of i_ℓ onto v_s^c is the vector denoted and defined by

$$\mathrm{proj}(i_\ell, v_s^c) := \frac{\langle v_s^c, i_\ell\rangle}{\|v_s^c\|^2} v_s^c,$$

with magnitude

$$\tilde{\alpha} := \frac{\langle v_s^c, i_\ell\rangle}{\|v_s^c\|}.$$

Remark 13: Consider the projection of i_ℓ onto v_s^u is the vector denoted and defined by

$$\mathrm{proj}(i_\ell, v_s^u) := \frac{\langle v_s^u, i_\ell\rangle}{\|v_s^u\|^2} v_s^u,$$

with magnitude

$$\alpha := \frac{\langle v_s^u, i_\ell\rangle}{\|v_s^u\|},$$

and with $v_s^u = (I + Z_s Y_c) v_s^c$, we obtain

$$\alpha := \frac{\langle (I + Z_s Y_c) v_s^c, i_\ell\rangle}{\|(I + Z_s Y_c) v_s^c\|}.$$

VI. Example

In this section we present an example that illustrate some of the points discussed in the paper.

Consider the three-phase distribution system of Fig 7. A linear, R-L load per phase, which admittance Y_{ℓ_i} with $i \in \{1, 2, 3\}$, is star-connected with a set of star-connected, compensated capacitors $\hat{Y}_{c_i}(s) = C_{c_i} s$, with $i \in \{1, 2, 3\}$, per phase. Assuming balanced, three-phase operation, with no d.c. component of current, the problem can be represented by the single-phase equivalent circuit in the Fig. 8, see [11]. $v_s(t)$, $v_o(t)$ are the supply and load instantaneous voltages per phase. The RMS voltage $\hat{V}_s$ is maintained constant at 33 kV. The distribution line has the following data: 3-phase, 20 miles of 336.4 MCM[5], 26/7 ACSR[6] with 14 ft. conductor spacing, $R = 0.278$ Ω/mile/conductor, $X_L = 0.516$ Ω/mile/conductor. The R-L load per phase is assumed to be lumped resistance in series with lumped, pure inductance with 10 MV-A, 0.65 PF lagging at 50 Hz.

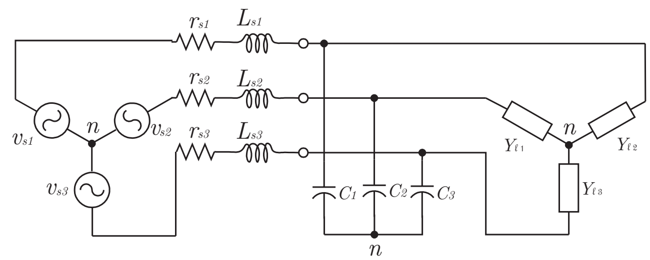

Fig. 7. Three-phase distribution system in parallel with a linear, balanced R-L load.

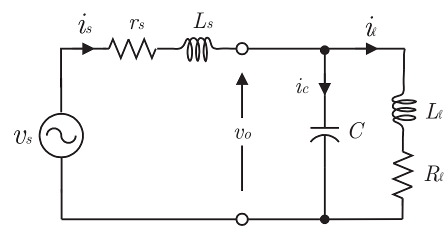

Fig. 8. Single-phase equivalent circuit of the three-phase system of Figure 7.

Now, consider the per-phase equivalent circuit, Fig. 8. The impedance line is $Z_s = 20(0.278 + j0.516)$ Ω, the linear load is $Y_\ell = 1/Z_\ell$ with $Z_\ell = 20(0.278 + j0.516)$ Ω, and the uncompensated total impedance is $Z_T^u = Z_s + Z_\ell$. The impedance of the compensator is $Z_c(C) = \frac{1}{j\omega C}$, then the compensated total impedance

$$Z_T^c(C) = Z_s + \frac{Z_c Z_\ell}{Z_c + Z_\ell}.$$

Condition (24) helps us to obtain the parameters for a given compensator Y_c, i.e., the capacitance for this example, such that the power factor is improved. Where, the bounded gain is

$$\delta(C) = \frac{\Re\{Z_\ell^c\}\left(\frac{\hat{V}_s}{\sqrt{3}} \frac{1}{\sqrt{|Z_T^c|}}\right)^2}{\Re\{Z_\ell\}\left(\frac{\hat{V}_s}{\sqrt{3}} \frac{1}{\sqrt{|Z_T^u|}}\right)^2} \frac{\frac{\hat{V}_s}{\sqrt{3}} \frac{|Z_\ell|}{|Z_T^u|}}{\frac{\hat{V}_s}{\sqrt{3}} \frac{|Z_\ell^c|}{|Z_T^c|}}, \tag{27}$$

the rms value of the input current is

$$\|i_s(C)\| = \frac{\hat{V}_s}{\sqrt{3}} \frac{1}{|Z_T^c|},$$

[5]The equivalent cross sectional area is 336,400 circular mils.
[6]An Aluminum Conductor Steel Reinforced (or ACSR) cable with 26 aluminum conductors and a core of 7 steel conductors.

and the load current is

$$\|i_\ell\| = \frac{\hat{V}_s}{\sqrt{3}} \frac{1}{|Z_T^u|}.$$

Variation of the power factor against capacitance is shown

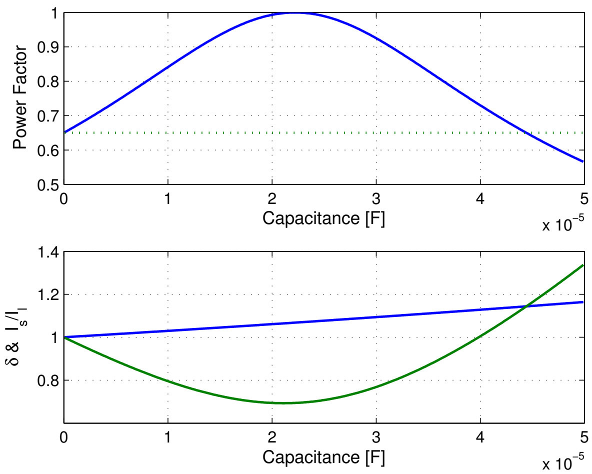

Fig. 9. The plots of the power factor and the inequality $\delta(C) > \frac{\|i_s\|}{\|i_\ell\|}$ against capacitance. Blue $\delta(C)$ and green $\frac{\|i_s\|}{\|i_\ell\|}$.

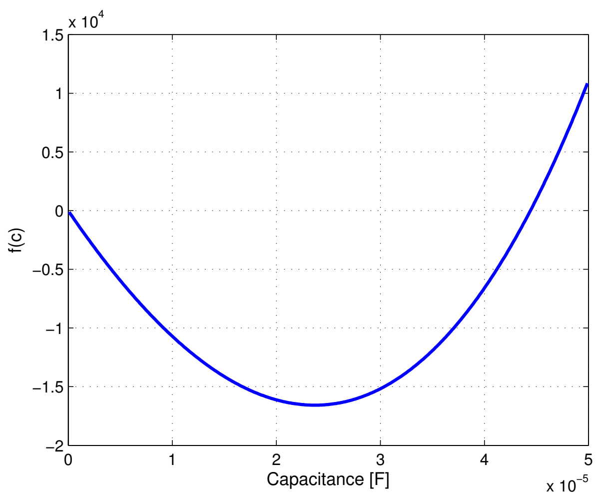

Fig. 10. The plot of the inequality $f(C) > 0$ against capacitance.

in Fig. 9, where it also shows that the optimal compensation is achieved at the capacitance $C_\star = 22.2\mu$ F. From Fig 10, for a fixed LTI capacitor compensator with admittance $\hat{Y}_c(s) = sC$, the power factor is improved if and only if $0 < C < 2C_\star$.

Let us now explain the characterization of the set of compensator based on the Corollary 10. Toward this end, define the function

$$f(C) = \|i_c\|^2 + 2\langle i_c, i_\ell\rangle + \left(1 - \delta^2\right) \|i_\ell\|^2,$$

that, for this example, is of the form

$$f(C) = C^2\omega^2 \hat{I}_s^2 |Z_\ell^c| + 2C\omega \left(\frac{\hat{V}_s}{\sqrt{3}}\right)^2 \frac{|Z_\ell^c|}{|Z_T^c||Z_T^u|} + (1-\delta^2)\hat{I}_\ell^2. \tag{28}$$

where $\hat{I}_s$, $\hat{I}_\ell$ are the rms values of i_s and i_ℓ, resp., and δ is given by (27). Figure shows the plot of $f(C)$, which shows that there exists a set of compensator parameter C such that $f(C) < 0$. The elements of this set, $0 < C < 44.4\mu$ F, clearly improve the power factor. Through this example we illustrate that the result reported here can be used for the formulation of a problem of optimization of the compensator.

VII. Conclusions

In this paper, a cyclo-dissipativity characterization of power factor improvement for non-sinusoidal networks with a significant source impedance was introduced. Our main goal is to point out that the cyclo-dissipative framework benefits the design by giving additional physical insights: namely, we show that the shunt compensation can be interpreted as feedback interconnection between the compensator and the uncompensated system. Based on this, the obtained results with the dissipativity framework can be used in order to increase system efficiency.

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