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Puerto-Flores, Dunstano del; Scherpen, Jacquelien M.A.; Ortega, Romeo

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On Power Factor Improvement by Lossless Linear Filters in the Nonlinear Nonsinusoidal Case

Dunstano del Puerto-Flores, and
Jacquelien M.A. Scherpen
Dept. of Discrete Technology and Automatization
Universiteit Groningen
Groningen, The Netherlands
Emails: d.del.puerto.flores@rug.nl,
j.m.a.scherpen@rug.nl

Romeo Ortega
Laboratoire des Signaux et Systèmes,
SUPELEC
Gif-sur-Yvette, France
Email: ortega@lss.supelec.fr

***Abstract*— Recently, it has been established that the problem of power factor compensation (PFC) for nonlinear loads with non-sinusoidal source voltage can be recast in terms of the property of cyclodissipativity. Using this framework we study the PFC for nonlinear loads containing a memoryless nonlinearity. We show that the power factor is improved by lossless linear time invariant filter if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured.**

I. INTRODUCTION

Recently, in [1] it has been established that the classical problem in electrical engineering of optimizing energy transfer from an alternating current (ac) source to a load with non-sinusoidal source (but periodic) voltage is equivalent to imposing the property of cyclodissipativity to the source terminals. Since this framework is based on the cyclodissipativity property, see [2], the improvement of the power factor (PF) is done independent of the reactive power definition, which is a matter of discussions in the power community, see [3].

Using the cyclodissipativity framework the classical capacitor and inductor compensators were interpreted in terms of energy equalization, see [1] for more details. And we have presented an extension of this result in [4] where we considered arbitrary lossless linear time invariant (LTI) filters, and proved that for general lossless LTI filters the PF is reduced if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. However, in [4] we have assumed that the resistors in the load are linear.

In this work, we extend this result to consider the case where the resistor are nonlinear time invariant statics.

The remaining of the paper is organized as follows. In Section II we briefly review of the framework proposed in [1]. We present the main result in Section III. In Section IV, we explore the power transmission efficiency. In Section V, an example is presented that illustrates the points discussed in the paper. Finally, we give the conclusions in Section VI.

II. A CYCLODISSIPATIVITY CHARACTERIZATION OF POWER FACTOR COMPENSATION

This section introduces the identification of the key role played by cyclodissipativity [2] in PF compensation.

A. Framework

We consider the energy transfer from an n-phase ac generator to a load, see Figure 1. The voltage and current of the source are denoted by the column vectors $v_s(t), i_s(t) \in \mathbb{R}^n$ and the load is described by a (possibly nonlinear and time varying) n-port network $\mathfrak{N}$. We make the following assumptions.

Assumption 1: All signals are assumed to be periodic and have finite power, that is, they belong to

$$\mathcal{L}_2^n = \left\{ x : [0,T) \to \mathbb{R}^n : \|x\|^2 := \frac{1}{T}\int_0^T |x(\tau)|^2 d\tau < \infty \right\}$$

where $|\cdot|$ is the Euclidean norm. We also define the inner product in $\mathcal{L}_2^n$ as

$$\langle x, y \rangle := \frac{1}{T}\int_0^T x^\top(t) y(t) dt.$$

Assumption 2: The source is ideal[1], in the sense that v_s remains unchanged for all loads Y_ℓ.

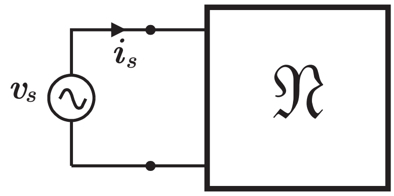

Fig. 1. Illustrating power delivered to a (possibly nonlinear and time varying) load from an n-phase ac ideal generator.

The universally accepted definition of PF is given as [5]:

[1] Under Assumption 2, the apparent power S is the highest average power delivered to the load among all loads that have the same rms current $\|i_s\|$.

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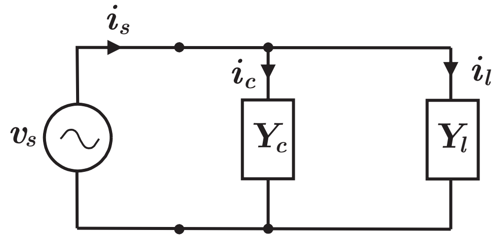

Fig. 2. Schematic diagram of shunt PF compensation configuration.

Definition 1: The PF of the source is defined by

$$PF := \frac{P}{S}, \tag{1}$$

where

$$P := \langle v_s, i_s \rangle, \tag{2}$$

is the active (real) power,[2] and $S := \|v_s\|\|i_s\|$ is the apparent power.
From (1) and the Cauchy–Schwartz inequality, it follows that $P \leq S$. Hence $PF \in [-1, 1]$ is a dimensionless measure of the energy-transmission efficiency. Cauchy–Schwartz also tells us that a necessary and sufficient condition for the apparent power to equal the active power is that v_s and i_s are collinear. If this is not the case, $P < S$ and compensation schemes are introduced to maximize the PF.

B. The PF compensation problem

The PF compensation configuration considered in the paper is depicted in Figure 2, where $Y_c, Y_\ell : \mathcal{L}_2^n \to \mathcal{L}_2^n$ are the admittance operators of the compensator and the load, respectively. That is,

$$i_c = Y_c(v_s) \quad i_\ell = Y_\ell(v_s)$$

where i_c, $i_\ell \in \mathcal{L}_2^n$, are the compensator and load currents, respectively. In the simplest LTI case the operators Y_c, Y_ℓ can be described by their admittance transfer matrices, which we denote by $\hat{Y}_c(s)$, $\hat{Y}_\ell(s) \in \mathbb{R}^{n\times n}(s)$, respectively, where s represents the complex frequency variable $s = j\omega$.

The uncompensated PF, that is, the value of PF when $Y_c = 0$, is clearly given by

$$PF_u := \frac{\langle v_s, i_s \rangle}{\|v_s\|\|i_s\|}. \tag{3}$$

Following standard practice, we consider only lossless compensators, that is,

$$\langle Y_c(v_s), v_s \rangle = 0, \quad \forall v_s \in \mathcal{L}_2^n. \tag{4}$$

We recall that, if Y_c is LTI, this is equivalent to

$$\mathrm{Re}\{\hat{Y}_c(j\omega)\} = 0. \tag{5}$$

where $\mathrm{Re}\{\hat{Y}_c(j\omega)\}$ is the real part of the admittance transfer matrix $\hat{Y}_c(j\omega)$.

[2]Also called average power [6].

C. Power-Factor compensation and cyclodissipativity

Dissipativity provides us with a useful tool for the analysis of nonlinear systems, which relates nicely to Lyapunov and $\mathcal{L}_2$ stability, [7], [8], [9]. In accordance with physical concepts, a system is called dissipative if it does not produce energy, in some abstract sense. Typical examples of dissipative systems are: passive electrical networks, mechanical systems, viscoelastic materials, etc.

The concept of cyclodissipativity is inspired by the fact that cyclodissipative systems exhibit a dissipative behavior in cyclic motions. As explained in [10], cyclodissipativity is understood here in terms of the available generalized energy. The idea is borrowed from thermodynamics, where the notion is formulated in a conceptually clearer manner than in circuits and systems theory. Thermodynamical systems define cyclodissipative systems as do, for example, less "physical" systems as electrical networks with positive resistors and capacitors and inductors with either sign.

Definition 2: Given a mapping $w : \mathcal{L}_2^n \times \mathcal{L}_2^n \to \mathbb{R}$. The n-port system of Figure 1 is cyclo–dissipative with respect to the supply rate $w(v_s, i_s)$ if and only if

$$\int_0^T w(v_s(t), i_s(t))dt > 0. \tag{6}$$

for all $(v_s, i_s) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$.

Remark 1: In words, a system is cyclodissipative when it can not create (abstract) energy over closed paths in the state-space. It might, however, produce energy along some initial portion of such a trajectory; if so, it would not be dissipative.

To place or results in context, and make the paper self–contained, we recall the following results from [1].

Proposition 3: Consider the system of Figure 2 with fixed Y_ℓ. The compensator Y_c improves the PF if and only if the system is cyclo–dissipative with respect to the supply rate

$$w(v_s, i_s) := (Y_\ell(v_s) + i_s)^\top (Y_\ell(v_s) - i_s). \tag{7}$$

Proof: From Kirchhoff's current law $i_s = i_c + i_\ell$, the relation $i_c = Y_c(v_s)$, and the lossless condition (4), it follows that $\langle v_s, i_s \rangle = \langle v_s, i_\ell \rangle$. Consequently, (1) becomes

$$PF = \frac{\langle v_s, i_\ell \rangle}{\|v_s\|\|i_s\|}. \tag{8}$$

Comparing the equation above with (3) we conclude that $PF > PF_u$ if and only if

$$\|i_s\|^2 < ||i_\ell||^2 = \|Y_\ell(v_s)\|^2, \tag{9}$$

where we used $i_\ell = Y_\ell(v_s)$ for the right hand side identity. Finally, note that (6) with (7) is equivalent to (9), which yields the desired result. ■

Corollary 4: Consider the system of Figure 2 Then Y_c improves the PF for a given Y_ℓ if and only if Y_c satisfies

$$2\langle Y_\ell(v_s), Y_c(v_s) \rangle + \|Y_c(v_s)\|^2 < 0, \quad \forall v_s \in \mathcal{L}_2^n. \tag{10}$$

Dually, given Y_c, the PF is improved for all Y_ℓ that satisfy (10).

Proof: Substituting $i_s = (Y_\ell + Y_c)(v_s)$ in (9) yields (10). ■

Remark 2: The key advantage of cyclodissipativity is that it restricts the set of inputs of interest to those generate periodic solutions (a feature that is intrinsic in PF compensation problems) it furthermore deals with "abstract" energies.

III. WEIGHTED POWER EQUALIZATION AND POWER FACTOR COMPENSATION FOR RLC LOADS

In this section we extend Proposition 5 in [1], where the PF compensators are assumed to be capacitors or inductors, to general lossless LTI filters. Similarly to [1], we assume that the load is a nonlinear RLC circuit consisting of lumped dynamic elements (n_L inductors, n_C capacitors) and static elements (n_R resistors). Capacitors and inductors are defined by the physical laws and constitutive relations [6]:

$$i_C = \dot{q}_C, \quad v_C = \nabla H_C(q_C), \tag{11}$$

$$v_L = \dot{\phi}_L, \quad i_L = \nabla H_L(\phi_L), \tag{12}$$

respectively, where i_C, v_C, $q_C \in \mathbb{R}^{n_C}$ are the capacitors currents, voltages and charges, and i_L, v_L, $\phi_L \in \mathbb{R}^{n_L}$ are the inductors currents, voltages and flux–linkages, $H_L : \mathbb{R}^{n_L} \to \mathbb{R}$ is the magnetic energy stored in the inductors, $H_C : \mathbb{R}^{n_C} \to \mathbb{R}$ is the electric energy stored in the capacitors, and ∇ is the gradient operator. We assume that the energy functions are twice differentiable and for linear capacitors and inductors,

$$H_C(q_C) = \frac{1}{2} q_C^\top C^{-1} q_C, \; H_L(\phi_L) = \frac{1}{2} \phi_L^\top L^{-1} \phi_L,$$

respectively, with $L \in \mathbb{R}^{n_L \times n_L}$, $C \in \mathbb{R}^{n_C \times n_C}$. To avoid cluttering the notation we assume L, C are diagonal matrices.

Finally, we distinguish between two sets of nonlinear static resistors: n_{R_i} current–controlled resistors and n_{R_v} voltage–controlled resistors, for which the characteristics are given by the following one-to-one real-valued functions:

$$v_{R_i} = \hat{v}_{R_i}(i_{R_i}), \tag{13}$$

and

$$i_{R_v} = \hat{i}_{R_v}(v_{R_v}), \tag{14}$$

respectively, where i_{R_i}, $v_{R_i} \in \mathbb{R}^{n_{R_i}}$ are the currents, voltages of the current-controlled resistors, and i_{R_v}, $v_{R_v} \in \mathbb{R}^{n_{R_v}}$ are the currents, voltages of the voltage-controlled resistors, with $n_R = n_{R_i} + n_{R_v}$.

Recalling the definition of real power (2) we introduce the following.

Definition 5: Given a compensator admittance Y_c the weighted (real) power of a single–phase circuit with port variables $(v, i) \in \mathcal{L}_2 \times \mathcal{L}_2$ is given by

$$P^w := \langle Y_c(v), i \rangle. \tag{15}$$

If Y_c is LTI

$$P^w = \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}[k] \hat{I}^*[k] \tag{16}$$

where $\hat{V}[k], \hat{I}[k]$ are the k-th spectral lines of v and i, respectively, and $\hat{Y}_c[k] := \hat{Y}_c(k\omega_0)$, with $\omega_0 := \frac{2\pi}{T}$. That is, P^w is the sum of the power components of the circuit modulated by the frequency response of Y_c—hence the use of the "weighted" qualifier.[3]

The aforementioned definition motivates the next lemma.

Lemma 6: Consider a nonlinear time invariant (TI) current-controlled {voltage-controlled} one-port resistor characterized by (13) {(14)} and a fixed LTI lossless compensator Y_c with $n = 1$. Let $\hat{Y}_c(j\omega)$ denote the associated admittance transfer function. If $\hat{Y}_c(j\omega)$ has a zero at the origin, then the weighted averaged power, along periodic trajectories,

$$P^w_{R_i} := \langle Y_c v_{R_i}, i_{R_i} \rangle = 0, \tag{17}$$

$\{P^w_{R_v} := \langle Y_c v_{R_v}, i_{R_v} \rangle = 0\}$ for all admissible pair $(v_{R_i}, i_{R_i}) \in \mathcal{L}_2 \times \mathcal{L}_2$ $\{(v_{R_v}, i_{R_v}) \in \mathcal{L}_2 \times \mathcal{L}_2\}$, and for all $\omega \in \mathbb{R}$ for which $j\omega$ is not a pole of $\hat{Y}_c(j\omega)$.

Proof: From the Foster's reactance theorem, see [13] and [14], the impedance function of LTI lossless can be written in the form

$$\hat{Z}(s) = \frac{g(s^2 + \omega_{z_1}^2)(s^2 + \omega_{z_2}^2) \cdots}{s(s^2 + \omega_{p_1}^2)(s^2 + \omega_{p_2}^2) \cdots},$$

where $g > 0$ and $0 \le \omega_{z_1} < \omega_{p_1} < \omega_{z_2} < \omega_{p_2} \cdots$. Furthermore, ω_{z_1} can be zero or not depending upon whether $\hat{Z}(s)$ has a zero or a pole at the origin. We have that $\hat{Y}_c(s) = \frac{1}{\hat{Z}_c(s)}$. Since Y_c admits a factorization $Y_c = Y_{c_1}(Y_{c_2})$, then

$$\langle i_{R_i}, Y_c v_{R_i} \rangle = \langle i_{R_i}, Y_{c_1}(Y_{c_2} v_{R_i}) \rangle,$$
$$\langle i_{R_v}, Y_c v_{R_v} \rangle = \langle i_{R_v}, Y_{c_1}(Y_{c_2} v_{R_v}) \rangle = \langle i_{R_v}, Y_{c_2}(Y_{c_1} v_{R_v}) \rangle,$$

where we used the fact that Y_{c_1} and Y_{c_2} commute[4]. For a lossless n-ports we have that Y_c is skew Hermitian, i.e., $\hat{Y}_c(s) + \hat{Y}_c^*(s) = 0$ for all $s = j\omega$, where Y_c^* is the adjoint (or the conjugate transpose) of Y_c, see [14] and [15].

Consider the case of the nonlinear TI current-controlled resistor. By the assumption that $\hat{Y}(s)$ has a zero at the origin, we define $\hat{Y}_{c_1}(s) = s$ and thus we have

$$\langle i_{R_i}, Y_c v_{R_i} \rangle = \langle Y_{c_1}^* i_{R_i}, Y_{c_2} v_{R_i} \rangle,$$

Since Y_{c_1} is skew-Hermitian, and $Y_{c_1} = \frac{d}{dt}$, then the last expression become

$$\langle i_{R_i}, Y_c v_{R_i} \rangle = -\left\langle \frac{di_{R_i}}{dt}, Y_{c_2} \hat{v}_{R_i}(i_{R_i}) \right\rangle, \tag{18}$$

and the right-hand side of (18) can be written as

$$\left\langle \frac{di_{R_i}}{dt}, Y_{c_2} \hat{v}_{R_i}(i_{R_i}) \right\rangle = \frac{1}{T} \int_0^T (Y_{c_2} \hat{v}_{R_i}(i_{R_i})) \frac{di_{R_i}}{dt} dt,$$

[3]Since the spectral lines of real signals satisfy $\hat{F}[-k] = \hat{F}^*[k]$, the weighted power is a real number.

[4]Since Y_{c_1} and Y_{c_2} are two continuous, linear, time-invariant operators, then there is an invertible operator S such that $SY_{c_1}S^{-1} = H_1$ and $SY_{c_2}S^{-1} = H_2$, where H_1 and H_2 denote multiplication operators. Since H_1 and H_2 commute, then Y_{c_1} and Y_{c_2} commute

By substitution, we obtain

$$\left\langle \frac{di_{R_i}}{dt}, Y_{c_2}\hat{v}_{R_i}(i_{R_i}) \right\rangle = \frac{1}{T}\int_{i_{R_i}(0)}^{i_{R_i}(T)} Y_{c_2}\hat{v}_{R_i}(i_{R_i})di_{R_i}.$$

Since the input is periodic with period T, i.e., $i_{R_i}(0) = i_{R_i}(T)$, then the inner product (18) is zero. The convolution $Y_{c_2}\hat{v}_{R_i}(i_{R_i})$ also is periodic with period T in steady state, see Theorem 4.1.2 in [16], and the existence and uniqueness of the composition can be proved by Volterra serie, see Theorem 3.2.1 in [16]. An analogous result holds for the case of the nonlinear TI voltage-controlled resistor, i.e., $\left\langle Y_{c_2}\hat{i}_{R_v}(v_{R_v}), \frac{dv_{R_v}}{dt} \right\rangle = 0.$ ■

Proposition 7: Consider the system of Figure 2 with $n = 1$,[5] a nonlinear RLC load, with linear resistors, and a fixed LTI lossless compensator Y_c with admittance transfer function $\hat{Y}_c(s)$.

i) PF is improved if and only if

$$\frac{1}{2}V_s^w + \sum_{q=1}^{n_L} P_{L_q}^w + \sum_{q=1}^{n_C} P_{C_q}^w < 0 \qquad (19)$$

where V_s^w is the rms value of the filtered voltage source, that is,

$$V_s^w := ||Y_c v_s||^2 = \sum_{k=1}^{\infty} |\hat{Y}_c(k)\hat{V}_s(k)|^2$$

and

$$\begin{aligned} P_{C_q}^w &:= \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}_{C_q}[k]\hat{I}_{C_q}^*[k] \\ P_{L_q}^w &:= \sum_{k=-\infty}^{\infty} \hat{Y}_c[k]\hat{V}_{L_q}[k]\hat{I}_{L_q}^*[k], \end{aligned}$$

are the weighted powers of the q–th capacitor and inductor, respectively.

ii) Condition (19) may be equivalently expressed as

$$\left\langle (\frac{1}{p}Y_c)v_L, \nabla^2 H_L v_L \right\rangle - \left\langle i_C, (\frac{1}{p}Y_c)\nabla^2 H_C i_C \right\rangle > \frac{1}{2}V_s^w \qquad (20)$$

where $p := \frac{d}{dt}$.

iii) If the capacitors and inductors are linear their weighted powers become

$$\begin{aligned} P_{C_q}^w &:= 2\omega_0 \sum_{k=1}^{\infty} \left\{ k \ \mathrm{Im}\{\hat{Y}_c[k]\} \sum_{q=1}^{n_C} C_q |\hat{V}_{C_q}[k]|^2 \right\} \\ P_{L_q}^w &:= -2\omega_0 \sum_{k=1}^{\infty} \left\{ k \ \mathrm{Im}\{\hat{Y}_c[k]\} \sum_{q=1}^{n_L} L_q |\hat{I}_{L_q}[k]|^2 \right\}. \end{aligned} \qquad (21)$$

where $\mathrm{Im}\{\hat{Y}_c[k]\}$ is the imaginary part of the admittance transfer function $\hat{Y}_c[k]$.

iv) Furthermore, the results i-iii can be extended for load with nonlinear TI resistors, if the admittance transfer function $\hat{Y}(j\omega)$ of the LTI lossless compensator has a zero at the origin.

[5]This condition is imposed, without loss of generality, to simplify the presentation of the result.

Proof: Corollary 4 shows that the PF is improved if and only if (10) holds, which may be equivalently expressed as

$$||Y_c v_s||^2 + 2\langle Y_c v_s, i_\ell \rangle < 0.$$

Applying the generalized form of Tellegen's theorem to the RLC load one gets

$$i_\ell^\top Y_c v_s = i_{R_v}^\top Y_c v_{R_v} + i_{R_i}^\top Y_c v_{R_i} + i_L^\top Y_c v_L + i_C^\top Y_c v_C,$$

see [12], which upon integration yields

$$\langle i_\ell, Y_c v_s \rangle = \langle i_L, Y_c v_L \rangle + \langle i_C, Y_c v_C \rangle \qquad (22)$$

where we have used the fact that, because of Lemma (6), $\langle i_{R_v}, Y_c v_{R_v} \rangle = 0$ and $\langle i_{R_i}, Y_c v_{R_i} \rangle = 0$ for LTI resistors.

Then, Condition (19) is obtained directly from Definition 5.

Now,

$$\begin{aligned} \langle i_L, Y_c v_L \rangle &= \left\langle \nabla H_L, Y_c \dot{\phi}_L \right\rangle \\ &= -\left\langle \nabla^2 H_L v_L, (\frac{1}{p}Y_c)v_L \right\rangle, \end{aligned}$$

where the first identity follows from the relations (12)and the second uses the well–known property of periodic functions $< f, \dot{g} >= - < \dot{f}, g >$. Similar derivations with the term $\langle i_C, Y_c v_C \rangle$ yield (20).

To prove iii) we use (16), the basic relations for LTI inductors and capacitors

$$\hat{I}_{C_q}[k] = jk\omega_0 C_q \hat{V}_{C_q}[k], \quad \hat{V}_{L_q}[k] = jk\omega_0 L_q \hat{I}_{L_q}[k],$$

and the fact that Y_c satisfies (5).

Finally, the proof of iv) is a consequence of Lemma 6. ■

Remark 3: Condition (19) indicates that the PF will be improved if and only if the overall weighted power (supplied plus stored) is negative.

Remark 4: From (20) (or replacing (21) in (19)) we see that PF improvement is equivalent to average power equalization between inductors and capacitor—notice the minus signs—with the gap being determined by the weighted supplied power.

IV. Towards the Optimal Power-Factor Compensation

We now use the framework presented in the previous section to explore the power transmission efficiency. In particular, we give two conditions to achieve unitary PF, the first one is only necessary, while the second one is necessary and sufficient. Although both conditions can be derived from standard considerations, giving them in the framework used in the paper allows, on one hand, to identify the gap between PF improvement, characterized in Proposition 3, and achieving unitary PF. On the other hand, with these conditions we can formulate a compensator synthesis problem—as illustrated in the next section.

Proposition 8: Consider the system of Figure (2) with fixed Y_ℓ and a lossless compensator Y_c. A necessary condition to achieve unitary power is

$$||i_c||^2 + \langle i_c, i_\ell \rangle = 0. \quad (23)$$

Proof: From the definition of PF, (1) and Cauchy-Schwarz inequality, see Lemma 3.1 in [17], it follows that unity PF is achieved if and only if $i_s(t)$ and $v_s(t)$ are co-linear, *i.e.*, $i_s(t) = \alpha v_s(t)$, for some nonzero constant α. Since the compensator is lossless we have

$$\begin{aligned} 0 &= \langle i_c, v_s \rangle \\ &= \alpha \langle i_c, v_s \rangle = \langle i_c, \alpha v_s \rangle. \end{aligned}$$

Hence, $\langle i_c, i_s \rangle = 0$, which means that i_c is orthogonal to i_s. Now, replacing $i_s = i_\ell + i_c$ in the condition above we obtain the desired result. ∎

We have shown in Proposition 3 that the PF is improved if and only if (10), which we repeat here for ease of reference,

$$||i_c||^2 + 2\langle i_c, i_\ell \rangle < 0, \quad (24)$$

holds. Comparing (23) with (24) we notice that there is a gap between PF improvement and optimality. Referring to Fig. 3 we have a (rather obvious) geometric interpretation of this gap. While the PF improvement condition (24) ensures that $\|i_s\| < \|i_\ell\|$, the optimality condition (23) places i_s orthogonal to i_c.

The proposition below, which follows directly from the proof of Proposition 3, gives a necessary and sufficient condition for optimal power transfer.

Proposition 9: Consider the system of the Figure 2 with fixed Y_ℓ. The lossless compensator Y_c renders to unity PF if and only if

$$\langle v_s, i_\ell \rangle = \|v_s\| \|i_s\|. \quad (25)$$

Proof: Condition (25) follows directly setting $PF = 1$ in (8), which was established in the proof of Proposition 3. ∎

The result has, again, a very simple geometric interpretation, which also clarifies why condition (23), although necessary, is not sufficient for optimal power transfer. Indeed, from Fig. 3 it is clear that although there are many currents orthogonal to v_s, there is only one ensures (25). Actually, this current is well-known in the power community and it is known as Fryze's current, defined by

$$i_F(t) = \frac{\langle i_\ell, v_s \rangle}{\|v_s\|^2} v_s(t).$$

Furthermore, the waveforms do not affect the transfer of active power, since only the norms of the voltage and current and the relation between the waveforms are relevant. The role of compensation in power system optimization for an ideal power source is that as much power from the source as possible is delivered to the load, see [18].

V. APPLICATION OF THE PROPOSED FRAMEWORK

In this section we present an example that illustrates the points discussed in the paper.

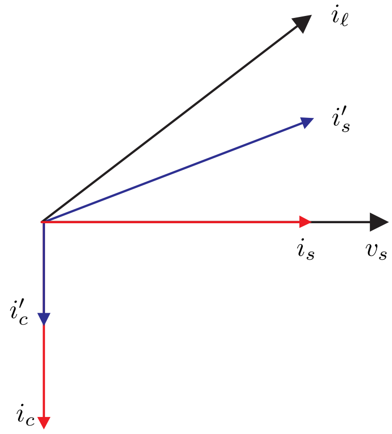

Fig. 3. Geometric interpretation of power-factor compensation. The currents i'_c, i'_s satisfy the condition of improved power factor, (24), and the currents i_c, i_s satisfy the necessary condition for unity power factor, (23).

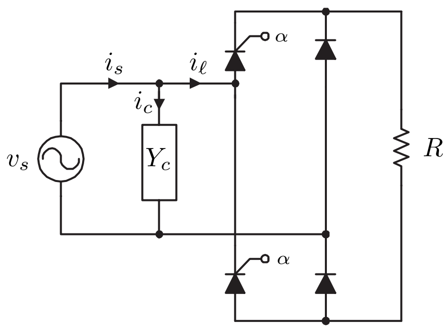

Fig. 4. Single–phase half semi-controlled bridge rectifier as linear time-varying resistor.

Example: Single phase half semi-controlled bridge rectifier.

Consider the classical single-phase semiconverter controlled rectifier load, terminated by a resistor in Figure 4. Under reasonable assumptions on v_s, the load can be modeled as a linear time-varying resistor with admittance operator

$$i_\ell(t) = \begin{cases} 0, & \text{if } \; t \in \left[\frac{kT}{2}, \frac{kT}{2} + \alpha\right), \quad k = 0, 1, \ldots \\ \frac{v_s(t)}{R}, & \text{otherwise,} \end{cases}$$

where $T = 2\pi/\omega_0$ is the fundamental period and $0 < \alpha < T/2$ is the SCR's firing angle. The converter parameters are chosen as $R = 10\ \Omega$, $\alpha = 3\pi/4 = 0.0075$ s. and the voltage source is taken as $v_s(t) = 280\cos(100\pi t)$ V. For these values, the uncompensated power factor is $PF_u = 0.3014$.

In [11] it has been proved that a capacitive compensator, $Y_c = Cp$, where $C > 0$ is the value of the capacitance and $p := \frac{d}{dt}$, improves the power factor, $PF > PF_u$, if and only if

$$C^2\omega_0^2 \sum_{n=-\infty}^{\infty} n^2 |\hat{V}(n)|^2 - \frac{C}{RT}\left[v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right)\right] < 0,$$

holds for all v_s and $C < C_{\max}$, where

$$C_{\max} = \frac{T}{4\pi^2 R} \frac{v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right)}{\sum_{n=-\infty}^{\infty} n^2 |\hat{V}(n)|^2}.$$

In order to get a complete result, we now use the framework of our paper. We define the function $f(C) = \|i_c\|^2 + 2\langle i_c, i_\ell \rangle$ which for $v_s = V_s \sin \omega_0 t$ becomes

$$f(C) = C^2 \omega_0^2 V_s^2 - \frac{C}{RT}\left[v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right)\right],$$

which is quadratic in the unknown C, and is minimal for

$$C_\star = \frac{T}{8\pi^2 R V_s^2}\left[v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right)\right],$$

i.e., $C_\star = 50.8\ \mu$F.

The power factor for our choice of Y_c can be written as a function of the capacitor and is given by

$$PF(C) = \frac{\frac{\pi}{2} - 1}{\pi\sqrt{\frac{4R^2}{X_c^2} - \frac{4R}{\pi X_c} + \frac{2\pi - 2}{\pi}}},$$

where $X_c = 1/\omega_0 C$. Variation of the power factor against capacitance is shown in Fig. 5, where it also shows that the maximal compensated power factor is achieved at the minimum capacitance, $PF(C_\star) = 0.3549$.

Condition (10) helps us to obtain the parameters for a given

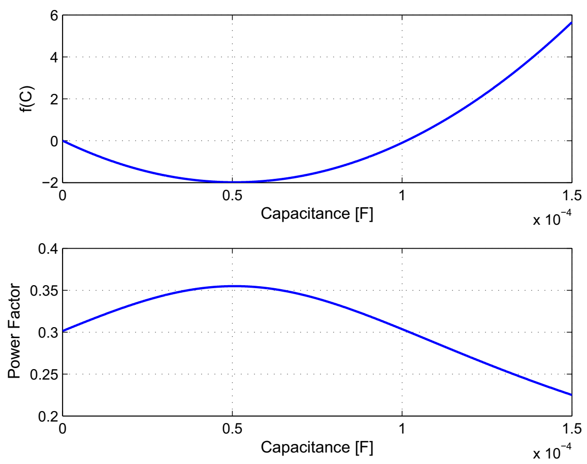

Fig. 5. The plots of the function $f(C)$ and the power factor $PF(C)$.

compensator Y_c, i.e., the capacitance for this example, that yield the highest possible power factor. In [11], the same example was treated, but no expression for the optimal C was obtained, and thus the optimal PF was not obtained either.

VI. CONCLUSIONS

In this paper, extensions to the analysis of power factor compensation of nonsinusoidal networks based on cyclo–dissipativity were presented. We have studied the concept of weighted (real) power and showed that the power factor by general LTI compensators is improved if and only if a certain equalization condition between the weighted powers of compensator and load is ensured. Furthermore, we extend the result to the case where the resistors are nonlinear.

REFERENCES

[1] E. Garcia-Canseco, R. Grino, R. Ortega, M. Salichs, and A. Stankovic, "Power-factor compensation of electrical circuits: a framework for analysis and design in the nonlinear nonsinusoidal case," *Control Systems Magazine, IEEE*, vol. 27, pp. 46–59, April 2007.

[2] D. J. Hill and P. J. Moylan, "Dissipative dynamical systems: Basic input-output and state properties," *The Journal of the Franklin Institute*, vol. 309, pp. 327–357, May 1990.

[3] L. S. Czarnecki, "Budeanu and fryze: Two frameworks for interpreting power properties of circuits with nonsinusoidal voltages and currents," *Electrical Enfineering*, vol. 80, pp. 359–367, 1997.

[4] D. del Puerto-Flores, R. Ortega, and J. M. A. Scherpen, "Power factor compensation with lossless linear filters is equivalent to (weighted) power equalization and a new cyclodissipativity characterization," *Conference on Decision and Control, 2009. CDC09*, Dec 2009.

[5] *Definitions for the Measurement of Electric Quantities Under Sinusoidal, Nonsinusiodal, Balance, or Unbalance Conditions*. IEEE Std 1450-2000, March 2010.

[6] L. O. Chua, C. A. Desoer and E. S. Kuh, *Linear and Non-Linear Circuits*. McGraw-Hill College, 1987.

[7] A. van der Schaft, *L_2-gain and passivity techniques in nonlinear control.* Springer Verlag, London: U.K., 2000.

[8] A. Astolfi, R. Ortega, and R. Sepulchre. "Stabilization and disturbance attenuation of nonlinear systems using dissipativity theory," *European Journal of Control*, vol. 8, no. 5, 2002. pp. 61–80, 1973.

[9] B. Brogliato, R. Lozano, B. Maschke, and O. Egeland, *Dissipative Systems Analysis and Control: Theory and Applications.* Springer Verlag, London: U.K., 2007.

[10] J. C. Willems, "Qualitative behaviour of interconnected systems," *Ann. Systems Res.*, vol. 3, pp. 61–80, 1973.

[11] R. Ortega, M. Hernandez-Gomez, F. Lamnabhi-Lagarrigue, and G. Escobar, "Passive power factor compensation of a controlled rectifier with non-sinusoidal generator voltage," *Conference on Decision and Control, 2008. CDC'08*, pp. 3755–3760, Dec 2008.

[12] P. Penfield, R. Spence, and S. Duinker, "A generalized form of Tellegen's theorem," *Circuit Theory, IEEE Transactions on*, vol. 17, pp. 302–305, Aug. 1970.

[13] M. E. Van Valkenburg, *Introduction to Modern Network Synthesis*. John Wiley & Sons, Inc., 1960.

[14] V. Belevitch, *Classical Network Theory*. Holden–Day, Inc., 1968.

[15] A. W. Naylor and G. R. Sell, *Linear Operator Theory in Engineering and Science*. Springer-Verlag, 1982.

[16] S. Boyd, L. O. Chua, and C. A. Desoer, "Analytical Foundations of Volterra Series," *IMA Journal of Mathematical Control and Information*, Oxford University Press, vol. 1(3), pp. 243-282, 1984.

[17] D. G. Luenberger, *Optimization by Vector Space Methods*. John Wiley & Sons, Inc, 1969

[18] J. L. Willems, "Reflections on Apparent Power and Power Factor in Nonsinusoidal and Polyphase Situations," *Power Delivery, IEEE Transactions on*, vol. 19, pp. 835–840, April 2004.