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# On Linear Power Factor Compensation, Power Equalization and Cyclo-dissipativity of Nonlinear Loads

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**Abstract**—The main contribution of this paper is an extension of the recently introduced result that recasts the problem of power factor compensation for nonlinear loads with non-sinusoidal source voltage in terms of the property of cyclo-dissipativity. Using the cyclo-dissipativity framework the classical capacitor and inductor compensators can be interpreted in terms of energy equalization. Unfortunately, the supply rate is a function of the load, which is usually unknown, styming the applicability of this result for compensator synthesis. We extend the result in three directions. First, power factor compensation is shown to be equivalent to a new cyclo-dissipativity condition, whose supply rate is now function of the compensator. Second, we consider general lossless linear filters as compensators and show that the power factor is improved if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. Finally, we exhibit the gap between the ideal compensator, *e.g.*, the one that achieves unitary power factor, and the aforementioned equalization condition. This result naturally leads to the formulation of a problem of optimization of the compensator topology and its parameters.

## I. INTRODUCTION

Optimizing energy transfer from an alternating current (ac) source to a load is a classical problem in electrical engineering. The power factor (PF), defined as the ratio between the real or the active power (average of the instantaneous power) and the apparent power (the product of rms values of the voltage and current), captures the energy-transmission efficiency for a given load. The standard approach to improve the power factor is to place a lossless compensator between the source and the load.

If the load is scalar linear time-invariant (LTI) and the generator is ideal—that is, with negligible impedance and fixed sinusoidal voltage—it is well known that the optimal compensator minimizes the phase shift between the source voltage and current waveforms [1]. The task of designing compensators that aim at improving PF for nonlinear time-varying loads operating in non-sinusoidal regimes is, on the other hand, far from clear.

The effectiveness of capacitive compensation in systems with non-sinusoidal voltages and currents has been widely

studied, see *e.g.* [2] and [3]. In [4] it has been shown that capacitive compensation may not be effective for non-sinusoidal voltages. Therefore, a more complex compensator is required in such situations. Furthermore, most of the approaches used to improve PF are based on *ad-hoc* definitions of reactive power, [3], and a lack of consensus on these definitions produces misunderstanding of power phenomena in circuits with non-sinusoidal voltages and currents.

Recently, in [5] a new framework for analysis and design of (possibly nonlinear) PF compensators for electrical systems operating in non-sinusoidal (but periodic) regimes with nonlinear time-varying loads was presented. This framework proceeds from the aforementioned, universally accepted, definition of PF and does not rely on any axiomatic definition of reactive power. It is shown that PF is improved if and only if the compensated system satisfies a certain cyclo-dissipativity property, [6]. This result has been applied in [8] to analyze passive compensation of a classical half-bridge controlled rectifier with non-sinusoidal source voltage. Unfortunately, the supply rate in [5] depends explicitly on the load, which is typically unknown. Hence, the result cannot be used for compensator synthesis. One contribution of our work is the proof that PF improvement can also be characterized in terms of a new cyclo-dissipativity property where the supply rate is independent of the load and is solely determined by the compensator.

In [5] the case of LTI capacitive or inductive compensation are studied, showing that PF improvement is equivalent to energy equalization. Here, we extend this result to consider arbitrary LTI lossless filters, and prove that for general lossless LTI filters the PF is improved if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. A final contribution of our work is to exhibit the gap between the ideal compensator, *e.g.*, the one that achieves unitary PF, and the aforementioned equalization condition. This result naturally leads to the formulation of a problem of optimization of the compensator topology and its parameters.

## II. A NEW CYCLO-DISSIPATIVITY CHARACTERIZATION OF PF COMPENSATION

### A. Framework

We consider the energy transfer from an  $n$ -phase ac generator to a load, see Figure 1. All signals are assumed to be periodic and have finite power, that is, they belong to

$$\mathcal{L}_2^n = \left\{ x : [0, T) \rightarrow \mathbb{R}^n : \|x\|^2 := \frac{1}{T} \int_0^T |x(\tau)|^2 d\tau < \infty \right\}$$

where  $\|\cdot\|$  is the Euclidean norm. We also define the inner product in  $\mathcal{L}_2^n$  as

$$\langle x, y \rangle := \frac{1}{T} \int_0^T x^\top(t)y(t)dt.$$

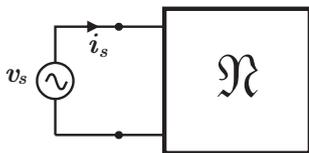


Fig. 1. Circuit schematic of an  $n$ -phase ac ideal generator connected to a (possibly nonlinear and time varying) load.

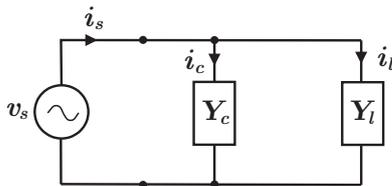


Fig. 2. Circuit schematic of shunt PF compensation configuration.

The universally accepted definition of PF is given as [1]:

*Definition 1:* The PF of the source is defined by

$$PF := \frac{P}{S}, \quad (1)$$

where

$$P := \langle v_s, i_s \rangle, \quad (2)$$

is the active (real) power,<sup>1</sup> and  $S := \|v_s\| \|i_s\|$  is the apparent power.

From (1) and the Cauchy–Schwartz inequality, it follows that  $P \leq S$ . Hence  $PF \in [-1, 1]$  is a dimensionless measure of the energy-transmission efficiency. Cauchy–Schwartz also tells us that a necessary and sufficient condition for the apparent power to equal the active power is that  $v_s$  and  $i_s$  are collinear. If this is not the case,  $P < S$  and compensation schemes are introduced to maximize the PF.

<sup>1</sup>Also called average power [7].

### B. The PF compensation problem

The PF compensation configuration considered in the paper is depicted in Figure 2, where  $Y_c, Y_\ell : \mathcal{L}_2^n \rightarrow \mathcal{L}_2^n$  are the admittance operators of the compensator and the load, respectively. That is,

$$i_c = Y_c v_s \quad i_\ell = Y_\ell v_s$$

where  $i_c, i_\ell \in \mathcal{L}_2^n$ , are the compensator and load currents, respectively. In the simplest LTI case the operators  $Y_c, Y_\ell$  can be described by their admittance transfer matrices, which we denote by  $\hat{Y}_c(s), \hat{Y}_\ell(s) \in \mathbb{R}^{n \times n}(s)$ , respectively.

The uncompensated PF, that is, the value of  $PF$  when  $Y_c = 0$ , is clearly given by

$$PF_u := \frac{\langle v_s, i_\ell \rangle}{\|v_s\| \|i_\ell\|}. \quad (3)$$

We make the following fundamental assumption.<sup>2</sup>

*Assumption 2:* The source is ideal, in the sense that  $v_s$  remains unchanged for all loads  $Y_\ell$ .

Following standard practice, we consider only lossless compensators, that is,

$$\langle Y_c v_s, v_s \rangle = 0, \quad \forall v_s \in \mathcal{L}_2^n. \quad (4)$$

We recall that, if  $Y_c$  is LTI, this is equivalent to

$$\text{Re}\{\hat{Y}_c(j\omega)\} = 0, \quad (5)$$

where  $\text{Re}\{\hat{Y}_c(j\omega)\}$  is the real part of the admittance transfer matrix  $\hat{Y}_c(j\omega)$ .

### C. PF compensation and cyclo-dissipativity

*Definition 3:* Given a mapping  $w : \mathcal{L}_2^n \times \mathcal{L}_2^n \rightarrow \mathbb{R}$ . The  $n$ -port system of Figure 1 is cyclo-dissipative with respect to the supply rate  $w(v_s, i_s)$  if and only if

$$\int_0^T w(v_s(t), i_s(t)) dt > 0. \quad (6)$$

for all  $(v_s, i_s) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$ .

To place our results in context, and make the paper self-contained, we recall the following results from [5].

*Proposition 4:* Consider the system of Figure 2 with fixed  $Y_\ell$ . The compensator  $Y_c$  improves the PF if and only if the system is cyclo-dissipative with respect to the supply rate

$$w(v_s, i_s) := (Y_\ell v_s + i_s)^\top (Y_\ell v_s - i_s). \quad (7)$$

*Proof:* From Kirchoff's current law  $i_s = i_c + i_\ell$ , the relation  $i_c = Y_c v_s$ , and the lossless condition (4), it follows that  $\langle v_s, i_s \rangle = \langle v_s, i_\ell \rangle$ . Consequently, (1) becomes

$$PF = \frac{\langle v_s, i_\ell \rangle}{\|v_s\| \|i_s\|}. \quad (8)$$

Comparing the equation above with (3) we conclude that  $PF > PF_u$  if and only if

$$\|i_s\|^2 < \|i_\ell\|^2 = \|Y_\ell v_s\|^2, \quad (9)$$

<sup>2</sup>Under Assumption 2, the apparent power  $S$  is the highest average power delivered to the load among all loads that have the same rms current  $\|i_s\|$ .

where we used  $i_\ell = Y_\ell v_s$  for the right hand side identity. Finally, note that (6) with (7) is equivalent to (9), which yields the desired result. ■

*Corollary 5:* Consider the system of Figure 2 Then  $Y_c$  improves the PF for a given  $Y_\ell$  if and only if  $Y_c$  satisfies

$$2\langle Y_\ell v_s, Y_c v_s \rangle + \|Y_c v_s\|^2 < 0, \quad \forall v_s \in \mathcal{L}_2^n. \quad (10)$$

Dually, given  $Y_c$ , the PF is improved for all  $Y_\ell$  that satisfy (10).

*Proof:* Substituting  $i_s = (Y_\ell + Y_c)v_s$  in (9) yields (10). ■

#### D. A new cyclo-dissipativity condition for PF compensation

We underscore the fact that the supply rate (7) explicitly depends on the, usually unknown, load admittance  $Y_\ell$ . Hence, the result of Proposition 4 can only be used for analysis of a given known load—as done in [8] for a TRIAC controlled rectifier. The proposition below gives an alternative cyclo-dissipative characterization of PF improvement where the supply rate depends instead on the compensator admittance.

*Proposition 6:* Consider the system of Figure 2 with fixed  $Y_c$ . The PF is improved for all  $Y_\ell$  such that the system is cyclo-dissipative with respect to the supply rate

$$w(v_s, i_s) := |Y_c v_s|^2 - 2i_s^\top Y_c v_s. \quad (11)$$

*Proof:* We have shown above that  $PF > PF_u$  if and only if  $\|i_s\|^2 < \|i_\ell\|^2$ . Using the fact that  $i_s = i_c + i_\ell$ , the latter inequality can be written as

$$\|i_c + i_\ell\|^2 < \|i_\ell\|^2, \quad (12)$$

which is equivalent to

$$\|i_c\|^2 + 2\langle i_c, i_\ell \rangle < 0. \quad (13)$$

Substituting  $i_\ell = i_s - i_c$  in (13) yields

$$\|i_c\|^2 - 2\langle i_c, i_s \rangle > 0. \quad (14)$$

The proof is completed replacing  $i_c = Y_c v_s$ . ■

As indicated above, the interest of the new cyclo-dissipativity property is that now the supply rate (11) depends on  $Y_c$ , that is to be designed. Current research is under way to exploit this new cyclo-dissipativity property to synthesize PF compensators.

### III. WEIGHTED POWER EQUALIZATION AND PF COMPENSATION FOR RLC LOADS

In this section we extend Proposition 5 in [5], where the PF compensators are assumed to be capacitors or inductors, to general lossless LTI filters. Similarly to [5], we assume that the load is a nonlinear RLC circuit consisting of lumped dynamic elements ( $n_L$  inductors,  $n_C$  capacitors) and static elements ( $n_R$  resistors). Capacitors and inductors are defined by the physical laws and constitutive relations [7]:

$$i_C = \dot{q}_C, \quad v_C = \nabla H_C(q_C), \quad (15)$$

$$v_L = \dot{\phi}_L, \quad i_L = \nabla H_L(\phi_L), \quad (16)$$

respectively, where  $i_C, v_C, q_C \in \mathbb{R}^{n_C}$  are the capacitors currents, voltages and charges, and  $i_L, v_L, \phi_L \in \mathbb{R}^{n_L}$  are the inductors currents, voltages and flux-linkages,  $H_L : \mathbb{R}^{n_L} \rightarrow \mathbb{R}$  is the magnetic energy stored in the inductors,  $H_C : \mathbb{R}^{n_C} \rightarrow \mathbb{R}$  is the electric energy stored in the capacitors, and  $\nabla$  is the gradient operator. We assume that the energy functions are twice differentiable. Resistors are assumed to be linear, that is,  $v_R = Ri_R$ , with  $R \in \mathbb{R}^{n_R \times n_R}$  a diagonal, positive definite matrix. For linear capacitors and inductors

$$H_C(q_C) = \frac{1}{2} q_C^\top C^{-1} q_C, \quad H_L(\phi_L) = \frac{1}{2} \phi_L^\top L^{-1} \phi_L,$$

respectively, with  $L \in \mathbb{R}^{n_L \times n_L}$ ,  $C \in \mathbb{R}^{n_C \times n_C}$ . To avoid cluttering the notation we assume  $L, C$  are diagonal matrices.

Recalling the definition of real power (2) we introduce the following.

*Definition 7:* Given a compensator admittance  $Y_c$  the weighted (real) power of a single-phase circuit with port variables  $(v, i) \in \mathcal{L}_2 \times \mathcal{L}_2$  is given by

$$P^w := \langle Y_c v, i \rangle. \quad (17)$$

If  $Y_c$  is LTI

$$P^w = \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}[k] \hat{I}^*[k] \quad (18)$$

where  $\hat{V}[k], \hat{I}[k]$  are the  $k$ -th spectral lines of  $v$  and  $i$ , respectively, and  $\hat{Y}_c[k] := \hat{Y}_c(k\omega_0)$ , with  $\omega_0 := \frac{2\pi}{T}$ . That is,  $P^w$  is the sum of the power components of the circuit modulated by the frequency response of  $Y_c$ —hence the use of the “weighted” qualifier.<sup>3</sup>

*Proposition 8:* Consider the system of Figure 2 with  $n = 1$ ,<sup>4</sup> a nonlinear RLC load, with linear resistors, and a fixed LTI lossless compensator with admittance transfer function  $\hat{Y}_c(s)$ .

i) PF is improved if and only if

$$\frac{1}{2} V_s^w + \sum_{q=1}^{n_L} P_{L_q}^w + \sum_{q=1}^{n_C} P_{C_q}^w < 0 \quad (19)$$

where  $V_s^w$  is the rms value of the filtered voltage source, that is,

$$V_s^w := \|Y_c v_s\|^2 = \sum_{k=1}^{\infty} |\hat{Y}_c(k) \hat{V}_s(k)|^2$$

and

$$P_{C_q}^w := \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}_{C_q}[k] \hat{I}_{C_q}^*[k]$$

$$P_{L_q}^w := \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}_{L_q}[k] \hat{I}_{L_q}^*[k],$$

are the weighted powers of the  $q$ -th inductor and capacitor, respectively

<sup>3</sup>Since the spectral lines of real signals satisfy  $\hat{F}[-k] = \hat{F}^*[k]$ , the weighted power is a real number.

<sup>4</sup>This condition is imposed, without loss of generality, to simplify the presentation of the result.

ii) Condition (19) may be equivalently expressed as

$$\left\langle \left( \frac{1}{p} Y_c \right) v_L, \nabla^2 H_L v_L \right\rangle - \left\langle i_C, \left( \frac{1}{p} Y_c \right) \nabla^2 H_C i_C \right\rangle > \frac{1}{2} V_s^w \quad (20)$$

where  $p := \frac{d}{dt}$ .

iii) If the inductors and capacitors are linear their weighted powers become

$$\begin{aligned} P_{C_q}^w &:= 2\omega_0 \sum_{k=1}^{\infty} \left\{ k \operatorname{Im}\{\hat{Y}_c[k]\} \sum_{q=1}^{n_C} C_q |\hat{V}_{C_q}[k]|^2 \right\} \\ P_{L_q}^w &:= -2\omega_0 \sum_{k=1}^{\infty} \left\{ k \operatorname{Im}\{\hat{Y}_c[k]\} \sum_{q=1}^{n_L} L_q |\hat{I}_{L_q}[k]|^2 \right\}, \end{aligned} \quad (21)$$

where  $\operatorname{Im}\{\hat{Y}_c[k]\}$  is the imaginary part of the admittance transfer function  $\hat{Y}_c[k]$ .

*Proof:* Corollary 5 shows that the PF is improved if and only if (10) holds, which may be equivalently expressed as

$$\|Y_c v_s\|^2 + 2\langle Y_c v_s, i_\ell \rangle < 0.$$

Applying Tellegen's theorem to the RLC load one gets

$$i_\ell^\top Y_c v_s = i_R^\top Y_c v_R + i_L^\top Y_c v_L + i_C^\top Y_c v_C, \quad (22)$$

Then, Equation (22) upon integration becomes

$$\langle i_\ell, Y_c v_s \rangle = \langle i_L, Y_c v_L \rangle + \langle i_C, Y_c v_C \rangle \quad (23)$$

where we have used the fact that, because of (4),  $\langle Y_c v_R, i_R \rangle = 0$  for LTI resistors. Condition (19) is obtained directly from Definition 7. Now,

$$\langle i_L, Y_c v_L \rangle = \left\langle \nabla H_L, Y_c \dot{\phi}_L \right\rangle = - \left\langle \nabla^2 H_L v_L, \left( \frac{1}{p} Y_c \right) v_L \right\rangle,$$

where the first identity follows from the relations (16) and the second uses the well-known property of periodic functions  $\langle f, \dot{g} \rangle = -\langle \dot{f}, g \rangle$ . Similar derivations with the term  $\langle i_C, Y_c v_C \rangle$  yield (20).

To prove iii) we use (18), the basic relations for LTI inductors and capacitors

$$\hat{I}_{C_q}[k] = jk\omega_0 C_q \hat{V}_{C_q}[k], \quad \hat{V}_{L_q}[k] = jk\omega_0 L_q \hat{I}_{L_q}[k],$$

and the fact that  $Y_c$  satisfies (5). ■

The following remarks are in order.

- R1 Condition (19) indicates that the PF will be improved if and only if the overall weighted power (supplied plus stored) is negative.
- R2 From (20) (or replacing (21) in (19)) we see that PF improvement is equivalent to average power equalization between inductors and capacitor—notice the minus signs—with the gap being determined by the weighted supplied power.
- R3 The results of Proposition 5 in [5] are a particular case of Proposition 8 taking  $Y_c = C_c p$ . In particular, the aforementioned equalization interpretation of PF improvement extends the one given in [5].

R4 In Proposition 8 it has been assumed that the resistors are linear, which is not required in [5]. This stems from the fact that—although we conjecture it is true—we have not been able to show that  $\langle i_R, Y_c v_R \rangle = 0$  for nonlinear resistors and general LTI lossless filters.

#### IV. IDEAL POWER-FACTOR COMPENSATION

We now use the framework presented in the previous section to explore the power transmission efficiency. In particular, we give two conditions to achieve unitary PF, the first one is only necessary, while the second one is necessary and sufficient. Although both conditions can be derived from standard considerations, giving them in the framework used in the paper allows, on one hand, to identify the gap between PF improvement, characterized in Proposition 4, and achieving unitary PF. On the other hand, with these conditions we can formulate a compensator synthesis problem—as illustrated in the next section.

*Proposition 9:* Consider the system of Figure (2) with fixed  $Y_\ell$  and a lossless compensator  $Y_c$ . A necessary condition to achieve unitary power is

$$\|i_c\|^2 + \langle i_c, i_\ell \rangle = 0. \quad (24)$$

*Proof:* From the definition of PF, (1) and Cauchy-Schwarz inequality, see Lemma 3.1 in [10], it follows that unity PF is achieved if and only if  $i_s(t)$  and  $v_s(t)$  are co-linear, *i.e.*,  $i_s(t) = \alpha v_s(t)$ , for some nonzero constant  $\alpha$ . Since the compensator is lossless we have

$$\begin{aligned} 0 &= \langle i_c, v_s \rangle \\ &= \alpha \langle i_c, v_s \rangle = \langle i_c, \alpha v_s \rangle. \end{aligned}$$

Hence,  $\langle i_c, i_s \rangle = 0$ , which means that  $i_c$  is orthogonal to  $i_s$ . Now, replacing  $i_s = i_\ell + i_c$  in the condition above we obtain the desired result. ■

We have shown in Proposition 4 that the PF is improved if and only if (13), which we repeat here for ease of reference,

$$\|i_c\|^2 + 2\langle i_c, i_\ell \rangle < 0, \quad (25)$$

holds. Comparing (24) with (25) we notice that there is a gap between PF improvement and optimality. Referring to Fig. 3 we have a (rather obvious) geometric interpretation of this gap. While the PF improvement condition (25) ensures that  $\|i_s\| < \|i_\ell\|$ , the optimality condition (24) places  $i_s$  orthogonal to  $i_c$ .

The proposition below, which follows directly from the proof of Proposition 4, gives a necessary and sufficient condition for optimal power transfer.

*Proposition 10:* Consider the system of the Figure 2 with fixed  $Y_\ell$ . The lossless compensator  $Y_c$  renders to unity PF if and only if

$$\langle v_s, i_\ell \rangle = \|v_s\| \|i_s\|. \quad (26)$$

*Proof:* Condition (26) follows directly setting  $PF = 1$  in (8), which was established in the proof of Proposition 4. ■

The result has, again, a very simple geometric interpretation, which also clarifies why condition (24), although necessary, is

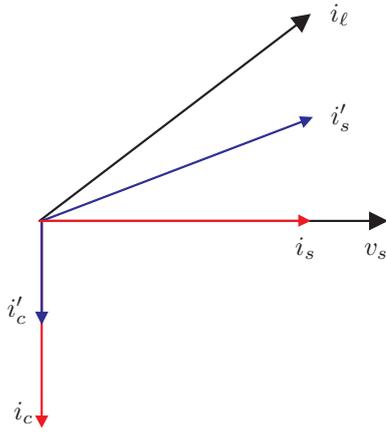


Fig. 3. Geometric interpretation of power-factor compensation. The currents  $i'_c, i'_s$  satisfy the condition of improved power factor, (25), and the currents  $i_c, i_s$  satisfy the necessary condition for unity power factor, (24).

not sufficient for optimal power transfer. Indeed, from Fig. 3 it is clear that although there are many currents orthogonal to  $v_s$ , there is only one ensures (26). Actually, this current is well-known in the power community and it is known as Fryze's current, defined by

$$i_F(t) = \frac{\langle i_\ell, v_s \rangle}{\|v_s\|^2} v_s(t).$$

Furthermore, the waveforms do not affect the transfer of active power, since only the norms of the voltage and current and the relation between the waveforms are relevant. The role of compensation in power system optimization for an ideal power source is that as much power from the source as possible is delivered to the load, see [9].

## V. APPLICATION OF THE PROPOSED FRAMEWORK

In this section we present an example that illustrates the points discussed in the paper.

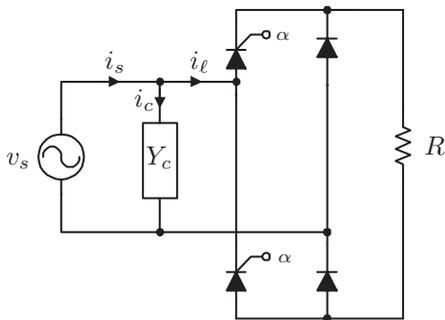


Fig. 4. Single-phase half semi-controlled bridge rectifier as linear time-varying resistor.

*Example: Single phase half semi-controlled bridge rectifier.*

Consider the classical single-phase semiconverter controlled rectifier load, terminated by a resistor in Figure 4. Under reasonable assumptions on  $v_s$ , the load can be modeled as a linear time-varying resistor with admittance operator

$$i_\ell(t) = \begin{cases} 0, & \text{if } t \in [\frac{kT}{2}, \frac{kT}{2} + \alpha), \quad k = 0, 1, \dots \\ \frac{v_s(t)}{R}, & \text{otherwise,} \end{cases}$$

where  $T = 2\pi/\omega_0$  is the fundamental period and  $0 < \alpha < T/2$  is the SCR's firing angle. The converter parameters are chosen as  $R = 10 \Omega$ ,  $\alpha = 3\pi/4 = 0.0075$  s. and the voltage source is taken as  $v_s(t) = 280 \cos(100\pi t)$  V. For these values, the uncompensated power factor is  $PF_u = 0.3014$ .

In [8] it has been proved that a capacitive compensator,  $Y_c = Cp$ , where  $C > 0$  is the value of the capacitance and  $p := \frac{d}{dt}$ , improves the power factor,  $PF > PF_u$ , if and only if

$$C^2 \omega_0^2 \sum_{n=-\infty}^{\infty} n^2 |\hat{V}(n)|^2 - \frac{C}{RT} \left[ v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right) \right] < 0,$$

holds for all  $v_s$  and  $C < C_{\max}$ , where

$$C_{\max} = \frac{T}{4\pi^2 R} \frac{v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right)}{\sum_{n=-\infty}^{\infty} n^2 |\hat{V}(n)|^2}.$$

In order to get a complete result, we now use the framework of our paper. We define the function  $f(C) = \|i_c\|^2 + 2\langle i_c, i_\ell \rangle$  which for  $v_s = V_s \sin \omega_0 t$  becomes

$$f(C) = C^2 \omega_0^2 V_s^2 - \frac{C}{RT} \left[ v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right) \right],$$

which is quadratic in the unknown C, and is minimal for

$$C_* = \frac{T}{8\pi^2 R V_s^2} \left[ v_s^2(\alpha) + v_s^2\left(\frac{T}{2} + \alpha\right) \right],$$

i.e.,  $C_* = 50.8 \mu\text{F}$ .

The power factor for our choice of  $Y_c$  can be written as a function of the capacitor and is given by

$$PF(C) = \frac{\frac{\pi}{2} - 1}{\pi \sqrt{\frac{4R^2}{X_c^2} - \frac{4R}{\pi X_c} + \frac{2\pi-2}{\pi}}},$$

where  $X_c = 1/\omega_0 C$ . Variation of the power factor against capacitance is shown in Fig. 5, where it also shows that the maximal compensated power factor is achieved at the minimum capacitance,  $PF(C_*) = 0.3549$ .

Condition (25) helps us to obtain the parameters for a given compensator  $Y_c$ , i.e., the capacitance for this example, that yield the highest possible power factor. In [8], the same example was treated, but no expression for the optimal C was obtained, and thus the optimal PF was not obtained either.

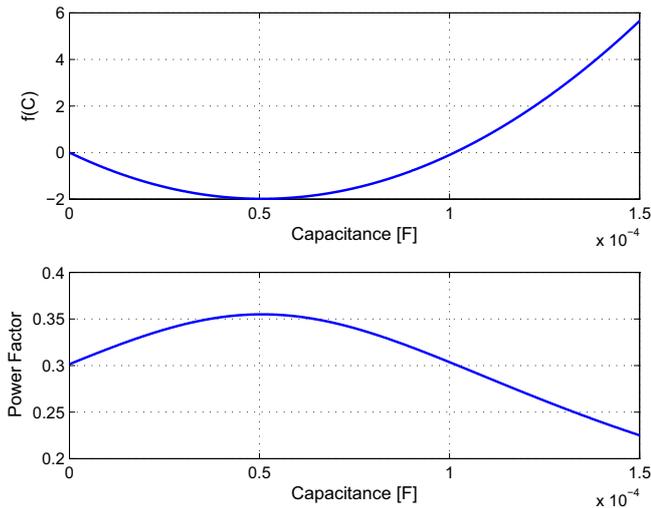


Fig. 5. The plots of the function  $f(C)$  and the power factor  $PF(C)$ .

## VI. CONCLUSION

In this paper, the framework for analysis of PF compensation for non-sinusoidal nonlinear networks based on cyclo-dissipativity introduced in [5] has been extended in several directions. First, a new cyclo-dissipativity characterization of PF improvement was introduced. Second, we have proved that the PF is improved with a general lossless LTI filter if and only if a certain equalization condition between the weighted powers of compensator and load is ensured. Third, the gap between the ideal compensator, *i.e.*, the one that achieves unitary PF, and one which satisfies the aforementioned equalization condition was described. Finally, through an example we have illustrated that the results reported here can be used for the formulation of a problem of optimization of the compensator.

Some issues that remain open, and are currently being explored, include:

- The extension of Proposition 8 for general nonlinear resistors in the load. In [5], where the cases of capacitive  $Y_c = C_p$  (or inductive  $Y_c = \frac{1}{L_p}$ ) compensation are considered, this assumption is not needed. In these cases, the key property  $\langle i_R, Y_c v_R \rangle = 0$  follows from the chain rule and the periodicity assumption. Given the phase shift characteristics of lossless compensators, we conjecture that this is also true in the general case.
- We have used the conditions of Propositions 9 and 10 to formulate an optimization problem for the compensator parameters. Also, we have revealed the existence of a gap between the cyclo-dissipativity condition for PF improvement and the necessary condition to achieve  $PF = 1$  of Proposition 9. However, it is not clear whether we can use the cyclo-dissipativity condition to design the PF compensator. More precisely, it is not clear whether minimizing the function  $\|i_c\|^2 + 2\langle i_c, i_\ell \rangle$  will yield an

optimal PF. Some preliminary calculations with some examples seem to confirm this fact.

- A key assumption to develop the framework is that the source is ideal, that is, that its impedance can be neglected. In many applications this is not the case. Some preliminary derivations prove that this assumption can be relaxed, preserving the essential principles, and will be reported elsewhere in the near future.

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