Chapter 7
Conclusions and Future Research

This final chapter summarizes the main results presented in this thesis and discusses possible topics for future research.

7.1 Conclusions

In this thesis we have systematically studied distributed control of networked Lur’e systems, specifically, robust synchronization problems and cooperative robust output regulation problems. In such nonlinear multi-agent networks, the model of each agent dynamics is taken as a Lur’e system that consists of a nominal linear dynamics with an unknown static nonlinearity around it. More general than the slope-restrictedness condition, so called incremental passivity and incremental sector boundedness assumptions are proposed on the unknown Lur’e-type nonlinearities. For the interconnection topologies among these Lur’e agents, we consider two cases. One is that the interconnection topology is undirected and connected; the other is that the interconnection topology is directed and contains a directed spanning tree. In both cases, the interconnection topologies are time-invariant. The synchronizing protocols we designed can be divided into static relative state feedback protocols, dynamic relative output feedback protocols, dynamic state feedback protocols, and dynamic output feedback protocols. Except in Chapter 6, we consider identical Lur’e systems in other chapters. Note that the results obtained in Chapter 6 generalize those in other chapters in some sense. In addition, all the material in this thesis deal with continuous-time systems.

In Chapter 3, we study the robust synchronization problem for undirected dynamical networks of identical Lur’e systems interconnected through static relative state feedback protocols. The graphs representing these undirected networks are assumed to be connected. The feedback gain matrices are determined by the matrices defining the individual agent dynamics and the second smallest and the largest Laplacian eigenvalues associated with the graphs. Since the entire interconnection topology is required to be known and then its Laplacian matrix can be derived, the knowledge of the Laplacian eigenvalues is a kind of global information. In order to remove such requirement, an adaptive control law is utilized. The idea to solve this problem is to introduce adaptively updated coupling
weights in the original protocol, whose dynamics are driven by local synchronization errors over each edge in the network. Eventually, a fully distributed synchronizing protocol can be designed for undirected Lur’e dynamical networks.

According to the analysis in Chapter 3, the assumption that the graphs are undirected together with Lemma 2.1 plays an important role therein. However, in practice, the interconnection/communication between adjacent agents could be unidirectional and thus the topology is directed. In this case, Lemma 2.1 fails. The algebraic connectivity, i.e. the second smallest Laplacian eigenvalue turns out to be a critical design parameter in the computation of a suitable feedback gain matrix in Chapter 3. This motivates us to employ the general algebraic connectivities of directed graphs in our present problem. Using the concept of general algebraic connectivities, the robust synchronization problem for directed Lur’e networks is successfully solved via elaborate Lyapunov analysis in Chapter 4, where the directed graphs are assumed to contain a directed spanning tree. We note that the synchronization conditions obtained in Chapter 4 are quite similar to those in Chapter 3.

In both Chapters 3 and 4, we only consider static relative state feedback protocols. It is of course more reasonable to consider relative output feedback protocols since system state variables are not easily accessed. Whereas in output feedback stabilization of one single Lur’e system the feedback nonlinearity is always assumed to be known exactly, in Chapter 5 we adopt a linear dynamic output feedback protocol without using the feedback nonlinearities inspired by a $H_{\infty}$ control technique. Thus, our approach solves the robust synchronization problem for undirected networks of identical Lur’e systems interconnected through dynamic relative output feedback protocols, but also works in output feedback stabilization of one single Lur’e system without the exact knowledge of its feedback nonlinearity. Although only the undirected topology case has been discussed, the directed case could be addressed in a similar way.

Before, the agent dynamics are assumed to be identical. The Lur’e dynamical networks are robustly synchronized in the sense that the Lur’e-type nonlinearities are unknown. But, practically speaking, these unknown nonlinearities can be different for distinct agents as well. In this case the Lur’e networks are heterogeneous. In Chapter 6, we assume that the nominal linear parts of the Lur’e agents in a network are identical and their unknown nonlinearities are allowed to be non-identical. By using output regulation theory, the outputs of these agents are regulated to track a reference signal generated by an exosystem. Thanks to our designed fully distributed estimator for the state of the exosystem, the cooperative robust output regulation of heterogeneous directed Lur’e networks is achieved in a fully distributed fashion through dynamic state feedback protocols and dynamic output feedback protocols, respectively.
7.2 Future research

We identify three possible topics for future research.

- In Chapters 4 and 5, the synchronizing protocols cannot be implemented in a fully distributed way as those in Chapter 3. Chapter 4 addresses the robust synchronization problem for directed Lur’e networks using general algebraic connectivities of directed graphs. Similar to the algebraic connectivity used in Chapter 3, we need to know the entire interconnection topology to compute its general algebraic connectivity, which is a kind of global information as well. However, the adaptive control law in Chapter 3 does not work here due to the asymmetry of directed interconnection topologies. Thus a new idea to solve this problem is required. We also consider undirected Lur’e networks in Chapter 5 as in Chapter 3. But, the largest Laplacian eigenvalue together with the second smallest Laplacian eigenvalue, i.e. the algebraic connectivity, are involved in the protocol computation nonlinearly in Chapter 5. The idea of the fully distributed synchronizing protocol in Chapter 3 fails here as well. Certainly, in distributed coordination of multi-agent networks, any requirement on global information is not convincing. How to remove such requirement is a critical step to implement these synchronizing protocols in practice. Because it is impossible to get global information if the network is huge.

- Indeed, Lur’e systems are uncertain nonlinear systems in the presence of unknown feedback nonlinearities. In this sense we are dealing with robust distributed control of networked Lur’e systems. Furthermore, besides external uncertainties, a Lur’e system could be confronted with model uncertainties in the sense that it consists of an uncertain linear dynamics with an unknown static nonlinearity around it while the static nonlinearity could be possibly dominated by a passive or sector bounded nonlinearity along with an additional perturbation. By modeling each agent using such Lur’e system in a network, the robust synchronization problem for this Lur’e dynamical network would be nontrivial. In particular, the cooperative robust output regulation problem becomes more complex. Although a few general distributed control frameworks have been proposed, for given complex networks, we still need to look for particular control strategies.

- This thesis only considers time-invariant interconnection topologies. But, in dynamically changing environments, the interconnection topologies could be time-varying. This is a wide topic. In fact, time-varying interconnection topologies introduce nonlinear and particularly nonsmooth couplings among the agents in a network. While only linear couplings are considered in this
thesis, other nonlinear couplings that arise from engineering applications are of course interesting, for example, delayed, quantized or time/event-triggered interconnections. In addition, how to improve the network performances under these constraints demands urgent solutions.