Chapter 1
Introduction

Cooperative control refers to a control system which aims at achieving a desired collective behavior for a group of agents with sensing or communication abilities. In contrast to the traditional centralized control design, cooperative control systems benefit decentralized or distributed control schemes. In particular, cooperative motion coordination of mobile agents aims at motion coordination for a group of agents using local feedback laws. This problem has attracted increasing attention in recent years owing to its wide range of applications from biology and social networks to sensor/robotic networks.

In problems of decentralized motion coordination, an important component, besides the dynamics of the agents and the graph topology, is the flow of information among the agents. In fact, the usual assumption in the literature on cooperative control is that a continuous flow of perfect information is exchanged among the agents. However, due to the coarseness of sensors and/or to communication constraints, the latter might be a restrictive requirement. To cope with this restriction, quantized information and control have been proposed and studied in the literature. In particular, there has been a growing interest in binary quantizers and controllers owing to the recent developments in cyber-physical systems [18, 19, 62]. A binary controller (quantizer) is a controller (quantizer) whose output takes a value in a set of two values, in particular \{-1, +1\}.

This thesis focuses on coordination with binary controllers. In this chapter, first we introduce motivation and background on different aspects of the problem which has been studied in the thesis. Next, Section 1.2 presents the contribution of the thesis. Finally, Section 1.3 gives the outline of the thesis.

1.1 Motivation and Background

There is a variety of group objectives in motion coordination of a group of agents such as flocking, formation keeping and consensus (agreement). In this thesis, we focus on formation keeping and consensus problems, mostly with binary controllers.

- Formation keeping problem: Formation keeping control is a motion coordination problem which aims at achieving a desired geometrical shape for the position of the agents using local feedback laws. In addition to a desired shape (and orientation), some other collective behaviors can be aimed, such as tracking a desired velocity. This problem has been addressed with different approaches e.g. [1], [4], [7], [56],
There are two main classifications in formation control problems: position-based and distance-based formations [4]. The former refers to a problem setting where the goal is to achieve a desired shape and orientation for the agents, however, the latter refers to the case where achieving a desired shape for the agents is the only target.

- **Consensus**: The consensus (agreement) problem is a motion coordination problem where the goal is to steer the variables of interest, e.g. position, orientation, to a common value. This problem has been widely addressed in the literature [7], [9], [56], [68] and [79] to name a few. Consensus is a specific case of position-based formation keeping where the desired distance among the agents is zero.

Among key components in problems of motion coordination of mobile agents are the communication topology, exchange of information and dynamics of the agents. The communication topology determines the way in which agents of a network interact and exchange information. In this thesis we assume a connected and bidirectional communication topology (graph). The assumption of a bidirectional (undirected) graph allows us to use passivity as a tool in designing the local feedback laws. Proposed by Arcak [1], passivity-based design in coordination problems has been further investigated in the recent literature e.g. [4], [8], [21], [45], [46]. Passivity is a property of a large class of mechanical systems including Euler-Lagrangian and port-Hamiltonian systems. In a nutshell, the maximum amount of energy that a passive system can store is equal to the energy that is supplied to it from outside. The property guarantees internal stability (boundedness) of the system. Under special interconnection structures, the interconnection of passive systems inherits the passivity property. Hence, the stability of the network (interconnection of passive agents) can be guaranteed which is a big achievement considering the complexity of a network system. In addition, within the passivity framework and with some additional assumptions a desired common control objective, which is the asymptotic stability, can be achieved for the network. This approach will be further investigated in the next chapters.

Apart from the communication topology, this thesis mainly investigates the other two key components in motion coordination problems: exchange of information and dynamics of the agents. Considering the position-based formation keeping control, we study control of different classes of dynamical agents using binary quantizers and controllers. Moreover, the robustness of the formation against external disturbances, which has a great practical relevance, is considered. Considering communication constraints, we study a self-triggered coordination algorithm, which has been designed in [18], using analytical tools from hybrid dynamical systems. In what follows, motivation and background behind these different components are provided.
1.1. Motivation and Background

1.1.1 Quantized information and control

Considering real world constraints in coordination problems, the use of distributed quantized feedback control has been proposed in the literature to cope with the information constraints for both continuous-time (e.g. [13], [11], [12], [30]) and discrete-time agents ([10], [31], [51], [59] to name a few). In the continuous-time formation control by quantized feedback control, information is transmitted among the agents whenever measurements cross the thresholds of the quantizer. At these times, the corresponding quantized value taken from a discrete set is transmitted. This allows to deal naturally both with the continuous-time nature of the agents’ dynamics and with the discrete nature of the transmission information process without the need to rely on sampled-data models [11], [17], [20].

In this thesis we focus on continuous-time agents controlled by binary controllers which are a specific type of quantizers. Considering continuous-time agents and motivated by the problem of reaching a consensus in finite-time, Cortés [13] has adopted binary control laws and cast the problem in the context of discontinuous control systems. Also, a consensus control algorithm in which the information collected from the neighbors is in binary form has been proposed in [12]. A similar problem but in the presence of quantized measurements has been investigated in [26], while Ceragioli et al. [11] have rigorously cast the problem in the framework of non-smooth control systems. They have also introduced a new class of hybrid quantizers to deal with possible chattering phenomena. A rendezvous problem for Dubins cars based on a ternary feedback, which depends on whether or not the neighbor’s position is within the range of observation of the agent’s sensor, has been studied in [87]. Deployment on the line of a group of kinematic agents has been shown to be achievable by distributed feedback which uses binary information control [17]. Formation control problems for groups of agents with second-order non-linear dynamics and in the presence of quantized measurements have been studied in [17, 20]. Leader-follower coordination for single-integrator agents and constant disturbance rejection by proportional-integral controllers with quantized information and time-varying topology have been studied in [86].

Motivated by the above background, we summarize our interest in considering binary controllers in the two following reasons: (i) The use of binary information in coordination problems ([12,13]) has been proven useful to the design and real-time implementation of distributed controls for systems of first- or second-order agents in a cyber-physical environment (see e.g. [18,19,62]). We envision that a similar role will be played by the results in this thesis for a larger class of coordination problems. (ii) The resulting control laws are implemented by very simple directional commands (such as “move north”, “move north-east”, “stay still”, etc). We also show that the binary nature of these controllers does not affect their ability to achieve the formation in a leader-follower setting in which the desired reference velocity is only known to the leader.
1.1.2 Dynamics of agents

Another of the important components in formation control problems is the dynamics of the agents. In this regard, different classes of dynamic agents have been considered in the literature, for example [4, 64, 67, 77]. In this thesis, we exploit strictly output passive agents, unicycles, nonholonomic wheeled robots, and agents subject to a discontinuous friction law.

For dynamic agents, the dissipation due to friction forces plays an important role in stability analysis of the whole network e.g. [4, 45, 82]. In the current literature, only continuous friction forces are considered for the formation control problem of networks which motivate us to consider the formation control of a group of agents in the presence of Coulomb friction which is a discontinuous friction law [84]. Coulomb friction is a quantification of the friction force that exists between two (dry) surfaces in contact with each other. It renders the networked system nonsmooth, thereby requiring tools from nonsmooth systems for the analysis.

Some of the results of this thesis are presented in the port-Hamiltonian framework. In line with the passivity-based approach, the port-Hamiltonian framework, which is an energy-based modeling framework, describes a large class of (nonlinear) systems including passive mechanical and electrical systems (see [29, 76]). The use of energy based-models in providing a clear physical interpretation of engineering problems has been shown in [65]. The port-Hamiltonian framework interconnects the various sub-systems in a power preserving manner using so-called power ports [29]. Moreover, the Hamiltonian equals the total energy stored in the system and can be used as a Lyapunov function. In addition, one can design controllers using energy-related elements, such as springs, in order to shape the total energy of the system into a desired one. The term virtual spring used throughout this thesis refers to the application of this concept in the control design. Recent results of [77] have introduced the concept of port-Hamiltonian systems on graphs, which provides tools for the analysis of (complex) networks.

Respecting various classes of dynamic agents, there has been a strong interest in formation control of nonholonomic dynamic agents, for instance unicycles and nonholonomic wheeled robots. The term nonholonomic refers to motion constraints which depend on both configurations and velocities. Stabilizing the position and heading of this type of robot is challenging, since it does not satisfy the Brockett’s necessary condition for stabilization using smooth feedback [6]. The latter motivate us to consider nonholonomic agents to be controlled by discontinuous binary controllers. In view of the challenges in control of nonholonomic agents, discontinuous, hybrid and time varying control laws have been studied to stabilize a single robot ( [2], [38], [54], [72]). The multiple robot case, which is naturally more challenging, has also been studied (see e.g. [68], [28], and [71]). To cope with the restriction of smooth controllers to stabilize both of the position and orientation of the robots, formation keeping control of the front end (‘hand position’) of the robots has been considered in [68]. The front end of a wheeled robot lies at a distance
$L \neq 0$ along the line that is normal to the wheel axle. This assumption simplifies the problem, however, it is interesting in practice due to numerous applications i.e. controlling the gripper position located on the front end of a wheeled robot. Considering a network of unicycles, the feasibility of stabilization of this network over a directed graph using smooth time-varying control laws has been studied in [53]. The consensus of unicycles using discontinuous controllers over static and switching topologies has been investigated in [27].

1.1.3 Disturbance Rejection

In a general sense, the goal of a control system is to steer the variables of interest of the system to the desired values. Disturbances are unknown signals coming from the environment and are often the cause of deviations in the desired behavior of a control system. Correspondingly, the disturbance rejection (attenuation), in which a controller is designed to suppress the effect of disturbances, is one of the central problems in the design of control systems. Similar to other control systems, the robustness of formations against external disturbances is of great importance. In this regard, we consider matched input disturbances which are assumed to occur in accordance with the control inputs. From practical point of view, we consider disturbances which occur at the actuator side. The disturbances can be constant, sinusoidal or a combination of the two. Examples of constant and sinusoidal disturbances include an offset (DC term) in the output of the actuator and disturbances caused by a rotating equipment, e.g. gearbox housing vibrations.

Considering the challenging problem of disturbance rejection for multi-agent systems, different approaches have been pursued in the literature for some classes of dynamical systems. The roles of the internal model principle and of the passivity property to deal with input disturbances in coordination of relative-degree-one and -two incrementally passive systems have been discussed in [21]. Moreover, the problem of output agreement in networks of nonlinear dynamical systems under time-varying disturbance has been studied in [8]. Disturbance rejection (both internal-model-based approach and observer-based designs) in formation control of strictly passive systems with coarse exchanged data has been investigated in [46]. A leader-follower coordination for single-integrator agents and constant disturbance rejection by proportional-integral controllers with quantized information and time-varying topologies have been considered in [86].

In this thesis, we take an internal-model-based approach [39] for disturbance rejection and also tracking desired reference velocities. The internal-model-based approach is capable of handling simultaneously uncertainties in plant parameters as well as in the trajectory which is to be tracked provided that the latter belongs to the set of all trajectories generated by some fixed dynamical system (exosystem).
1.1.4 Self-triggered coordination algorithms

Over the past decade, there has been a considerable interest to give more ‘autonomy’ to the agents of a network. The interest changed the focus from a central controller scheme to the distributed one. Nevertheless, the real world constraints, i.e. communication constraints, have demanded greater autonomy for the agents of a network. Among examples of the communication constraints are: limited bandwidth of the communication channels, difficulties in clock synchronization for all agents of a network, and the huge communication and computation cost of large networks. Motivated by the aforementioned constraints, there has been an interest to develop coordination algorithms which can rely on sporadic exchange of information and relax the requirement on clock synchronization. In this line, the use of event-triggered and self-triggered methods have been studied in the literature e.g., [11], [13], [18], [46], [78].

Considering communication constraints, event-triggered control attracted the attention of the control community as a method to oppose the traditional time-triggered control. In the event-based control, each agent is supposed to update its control law if a prescribed event occurs. To name a few papers in this framework we refer to [23], [55], [78], and [83]. The drawback of this method is that each agent requires continuous access to the network information in order to recognize the occurrence of the event. To overcome this limitation, self-triggered methods propose controllers that allows each agent to be more autonomous [18], [22], [25], [62], and [63]. These algorithms allow each agent to collect information from its neighboring agents only at its own suitably designed sampling times. These methods typically result in a drastic reduction of the traffic on the communication infrastructure which is a desirable feature given the large networks.

Motivated by the use of binary controllers in the self-triggered algorithm in [18], this thesis studies this algorithm in order to provide a more systematic analytical approach which will be useful for future extensions of such an algorithm.

1.2 Contribution

Section 1.1 presented motivation and background behind the main research questions which have been studied in this thesis. In this section, we summarize the contributions of this thesis.

1. We study the problem of distributed position-based formation keeping of a group of agents with strictly passive dynamics which exchange binary information. The binary information models a sensing scenario in which each agent detects whether or not the components of its current position vector from a neighbor are above or below the prescribed distance and apply a force (in which each component takes a binary value) to reduce or respectively increase the actual distance. Moreover, we investigate the formation control problem together with tracking known and
unknown reference velocities for the network of strictly passive agents. In the unknown reference velocity scenario, it is assumed that the reference velocity is only available to one of the agents (the so-called leader). In addition, the matched input disturbance rejection is studied considering both harmonic and constant disturbances. Although this thesis considers the aforementioned problems with binary controllers, the solutions to these problems even without quantization make a contribution to the field of formation keeping control.

**Compared with the literature:** We adopt a setting similar to the one in [17, 20] but controllers and analysis are different. Moreover, we investigate the formation control problem with unknown reference velocity tracking and matched input disturbance rejection which were not considered in [17, 20]. Compared with [87], where also coarse information has been used for rendezvous, the results in our contribution apply to a different class of systems and to a different cooperative control problem.

2. In this thesis, binary controllers (sign-based) are used in formation keeping control. As a result, the controller shows the known undesired phenomenon of fast switchings at the convergence. The latter is undesired in practice and can be mitigated by the adoption of the hysteretic quantizers (as in [11]), dynamic quantizer, or the self-triggered control algorithm (as in [18]). Accordingly, we propose alternative solutions to cope with the fast switchings of the control action.

3. This thesis considers the consensus problem for a group of unicycles communicating over a connected and undirected graph. We use only ternary (binary-based) controllers for the consensus of the positions and a hybrid-quantizer-based controller to reach an agreement on the orientations of the agents. Moreover, the consensus problem is studied in the presence of matched input disturbances. The design and rigorous analysis are presented in a hybrid framework while tools from the nonsmooth analysis and the internal model techniques are also applied. Distributed ternary controllers are designed to reach a finite-time consensus on the positions. Despite the binary nature of control laws, the control action shows a chattering-free behavior at the convergence.

**Compared with the literature:** We use only binary-based controllers to achieve the consensus on the positions of the continuous-time nonholonomic unicycles. The current literature mainly introduced sign-based binary controllers. However, here we use \( \text{sign}_\varepsilon \)-based controllers. Compared with [27] which considered discontinuous controller for consensus of unicycles, we use only \( \text{sign}_\varepsilon \)-based controllers. Furthermore, a hybrid-quantizer-based controller is proposed to control the orientations of the unicycles. Hybrid (hysteretic) quantizers were introduced in [11] to achieve a chattering-free quantized consensus on the positions of single integrators. This thesis proposes a planar hybrid quantizer in order to control the orientation of the unicycles. In addition, a convergence result for the consensus is achieved despite the presence of the matched input disturbances.
4. This thesis studies the problem of self-triggered coordination control of a network of agents with first-order dynamics. We present the model of the network with the self-triggered algorithm within the hybrid framework [36]. Our contribution is to reinterpret some of the results of [18] to shed a new light in the design of the self-triggered controllers of [18]. The importance of this alternative approach lies on the possibility to tackle complex coordination problems in a more systematic way than the way it was done in [18].

**Compared with the literature:** Comparing to [22], this thesis considers single-integrators and uses the hybrid invariance principle in [75]. Moreover, it is focused on interpreting the results of [18]. Comparing with [63], we present the analysis in the hybrid framework.

5. We present modeling and analysis of a network of planar heterogeneous dynamic point masses subject to Coulomb friction in the port-Hamiltonian framework. Moreover, we design a distributed discontinuous controller (discontinuous virtual springs) to achieve the desired goals of a formation control problem and compare the results with a continuous-time counterpart. Both the network and the controller are modeled within the port-Hamiltonian framework which provides a clear physical interpretation of the results.

**Compared with the literature:** In the current literature, only continuous friction forces are considered for the formation control problem of networks. However, we consider the formation control of a group of agents in the presence of Coulomb friction which is a discontinuous friction law. It is worth noting that the use of nonsmooth analytical tools for formation keeping control in the port-Hamiltonian framework has not been studied before.

6. This thesis studies the formation control of a group of nonholonomic wheeled robots in the presence of harmonic and constant input disturbances in the port-Hamiltonian framework using design tools of passivity-based [1] and internal-model-based [39] approaches. It also considers some relaxation on the assumption of strict output passivity condition of the robots’ dynamics, hence, a network of lossless robots.

**Compared with the literature:** The combination of problems that we considered, that is disturbance rejection of a network of nonholonomic agents within the port-Hamiltonian framework, has not been considered before. Comparing with the literature on disturbance rejection using internal-model-based approach within the port-Hamiltonian framework [34], [33], our contribution is to consider a network of agents rather than a single robot as well as considering robots with nonholonomic constraints.

### 1.3 Outline of the thesis

This thesis is structured as follows. Chapter 2 introduces preliminaries in the graph theory, passivity, discontinuous (nonsmooth) and hybrid frameworks. In addition, some
Chapter 3 analyzes a position-based formation control of a group of double integrators which exchange binary information. In addition to this assumption, two different velocity feedback laws are considered: continuous and discontinuous ones. We also study the case where the control law is a saturated function. Moreover, the practical pitfall of binary controllers, which is the fast switching of the control action at the convergence, is discussed. To mitigate this undesired phenomenon, we propose three alternative solutions. Some of the results of this chapter have been published in [41, 45].

Chapter 4 presents the results of formation control, velocity tracking and disturbance rejection for a group of strictly passive agents which exchange binary information. We use tools from passivity and the internal model approach to achieve a desired prescribed formation, track a desired velocity and cope with the disturbances. We consider two scenarios for velocity tracking. First, it is assumed that the reference velocity is known to all of the agents. Second, the latter assumption is relaxed to the case where the reference velocity is only known to one of the agents. Considering disturbance rejection, both constant and harmonic disturbances are studied. The results of this chapter have been published in [46].

Chapter 5 presents the results on the agreement of a group of unicycles using ternary (binary-based) controllers. We propose hybrid quantizers to achieve an agreement on the orientations and binary controllers for the agreement on the positions. The analysis is presented in the hybrid framework. In addition, we study disturbance rejection using the internal model approach. The results of this chapter are based on [42, 43].

Chapter 6 presents the analysis of the self-triggered coordination algorithm in [18] for a network of single integrators using a hybrid invariance principle [75]. Moreover, it reinterprets the design of the algorithm in [18] using a Lyapunov-based argument. The results of this chapter are based on [47].

Chapter 7 studies the formation control of a group of planar point masses in the presence of the Coulomb friction. We consider a Tree graph, which is an acyclic connected undirected graph, as the communication topology. The model of the network subject to the planar Coulomb friction is given in the port-Hamiltonian framework. The aim is to achieve a desired formation. The results of two different controllers are compared: distributed continuous state feedback controller and distributed binary controllers. The results of this chapter are based on [50].

In Chapter 8, we study the disturbance rejection in a network of nonholonomic wheeled robots in the port-Hamiltonian framework. In contrast to the other chapters, both agents’ dynamics and the controller are in the continuous-time. Two variations of a network of passive agents are considered. First, we assume that at least one of the agents is strictly output passive. Second, we consider a network of lossless agents. Using tools from the internal-model-based approach and passivity, we study disturbance rejection considering constant and harmonic disturbances. The results of this chapter are based on [49, 81].

Concluding remarks and recommendations for future research are given in Chapter 9.