The Interacting boson approximation model and applications
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CHAPTER VII  SUMMARY AND CONCLUSIONS

In the Interacting Boson Approximation (IBA) Model as proposed by Arima and Iachello 1) a nucleus is described as a system of mutually interacting bosons. A boson can be either in an \( L = 0 \) (s) or in an \( L = 2 \) (d) state. Furthermore, since the bosons are thought of as collective states of nucleon pairs, the total number of bosons (s and d) is a conserved quantity.

The IBA Hamiltonian can be regarded as a general transformation acting on a six dimensional space, spanned by the s-boson and the five components of the d, leaving the total number of bosons invariant. Consequently, the group structure underlying the IBA-model is U(6).

In chapter 2 where the macroscopic aspects of the model are discussed, this symmetry assumes a central role.

In the U(6) group three different chains of subgroups can be distinguished, if one requires that O(3) (angular momentum) be a subgroup. We label the three dynamical 2) symmetries by the first subgroup of each chain, SU(5) 3), SU(3) 4) and O(6) 5), respectively. The solutions for these three limits show many similarities with three different cases in the geometrical picture 6), namely the anharmonic vibrator, the axially symmetric deformed rotor and the gamma-unstable model, respectively. In contrast to the geometrical picture the three limits in the IBA-model are merely special cases of a more general Hamiltonian which can be diagonalized numerically. The IBA-model can therefore provide a detailed description not only at the limits, but also for intermediate cases. For this purpose a computer program 7), PHINT, has been written. Calculations for the Sm isotopes 8) are shown, illustrating the transition from an SU(5)-type of spectrum to an SU(3)-like spectrum.

In the third chapter the microscopic aspects of the model are discussed. Here the same approach is taken as in ref. 9. The IBA-model can be seen as an approximation to the shell model, where, instead of describing the nucleus in terms of single nucleon degrees of freedom, it is described in terms of degrees of freedom of collective pairs of nucleons. To facilitate the calculations these pairs of nucleons are treated as bosons. From the properties of the underlying nucleon-nucleon interaction for like nucleons it turns out that only \( J = 0 \) (S) and \( J = 2 \) (D) pairs need to be considered to describe the low-lying collective states.
The difference between the neutron-proton interaction compared with the interaction between like nucleons suggests that it may be necessary to introduce explicitly neutron and proton bosons. The major part of the interaction between like nucleons is accounted for by the boson energies $\varepsilon_s$ and $\varepsilon_d$. We also show how the quadrupole term which plays a dominant role in the neutron-proton interaction gives rise to a strong neutron-proton boson-quadrupole interaction.

This model has been applied to many different chains of isotopes\textsuperscript{10}, and good agreement has been obtained. As an example calculations for the Xe, Ba and Ce region\textsuperscript{11} are shown. In the numerical calculations the computer program NPBOS\textsuperscript{12} has been used.

The observation that in the version of the IBA-model in which neutron and proton bosons are distinguished explicitly (IBA-2) the calculated low-lying states are symmetric in the neutron and proton degrees of freedom, suggests that a relation can be derived between IBA-2 and the older version of the model, IBA-1, in which no distinction is made between neutron and proton bosons. In chapter four a projection technique is proposed to relate the parameters of the two models. This relation is important since from numerical point of view the IBA-1 model is easier to implement while in the microscopic treatment the IBA-2 model is more transparent. Application to the Nd, Sm and Gd region shows that this technique works reasonably well.

The theory of odd-A nuclei has been discussed in chapter five. They are described as an odd-particle coupled to the system of $s$- and $d$-bosons\textsuperscript{13} (IBFA-model). The particle-boson interaction is derived from the neutron-proton quadrupole force. Two terms dominate: a particle-boson quadrupole interaction and a so-called exchange force. The former is closely related to the conventional particle-core interaction. The latter is new and can be related to the fermion seniority changing part of the quadrupole operator for the odd particle.

In chapter six the typical spectra that can be obtained when coupling and odd particle to an even-even core are shown, with core Hamiltonians chosen at each of the three limiting cases, SU(5), SU(3) and O(6). In these calculations\textsuperscript{15} the particle-boson interaction was varied to simulate gradual filling of the shell occupied by the odd particle. The calculations show that the IBFA-model reproduces characteristic properties of the spectra of odd-A nuclei. For example, in
the vibrational limit, the first excited unique parity level has spin \( J = (j-2) \) if the single particle orbit \( j \) is nearly empty, versus \( J = (j-1) \) if the level is half occupied. For a particle coupled to an SU(3) core the properties of the Nilsson scheme are reproduced while for a particle coupled to an O(6) core there are many similarities with similar calculations in the geometrical model for gamma-unstable nuclei\(^{25}\). It is important to note that all these examples are merely special cases of a single Hamiltonian. In the IBFA-model it is therefore as simple to include intermediate cases as it was in the IBA-model for even-even nuclei. As an example calculations for Europium isotopes are shown in which the Samarium cores vary from SU(5)-like to SU(3)-like. Another aspect of odd-A nuclei is the occurrence of supersymmetries. This field is still open for further investigation. Only for few cases the occurrence of a dynamical symmetry has been demonstrated\(^{16}\). In section 6.2.4 this is shown for the case of a \( j = 3/2 \) particle coupled to an O(6) core by the explicit construction of the generators of the underlying groups.

Several applications of the IBA-model have hardly been mentioned at all in this work. These include i) the calculation of negative parity states\(^{8,17}\) in even-even nuclei through the introduction of a \( j^\pi = 3^- \) degree of freedom, ii) the calculation of form-factors for elastic scattering such as \((e,e')\)^{18}, which appears to be rather sensitive to the microscopic structure of the bosons, and iii) the calculation of the spreading width of deeply-bound hole states\(^{19}\).

Another interesting aspect of the IBA-model is the relation with the geometrical model. Ref. 20 shows how the classical shape of the nucleus is related to a general IBA-Hamiltonian. Furthermore phase transitions, arising for example in the Sm and Gd isotopes, can be calculated algebraically\(^{21}\).

We conclude with a list of some questions which require further attention.

In the phenomenological IBA-2 calculations the ad hoc introduction of a Majorana force appeared to be necessary. This term might have its origin in the omission of terms in the Hamiltonian such as the monopole and hexadecapole component in the neutron-proton interaction. It might also arise in the effective interaction from the coupling of states in the (S-D) space to non-collective states. Furthermore,
applications of the IBA-Hamiltonian to the Pt isotopes\(^{22}\) suggest that an introduction of a term cubic in the U(6) generators might improve the fit considerably. A detailed comparison with shell-model calculations might clarify some of these points.

Shell-model or other microscopic calculations\(^{23}\) will also be useful to determine the coefficients \(\alpha_j\) and \(\beta_{jj}\), defining the microscopic structure of the s- and d-boson. These are of special importance in the calculation of quantities more directly related to the single nucleon degrees of freedom such as form factors. These play an important role in the calculation of odd-A nuclei as well.

For the application of the IBA-model to odd-A nuclei it is important to test in detail the validity of the various approximations made in deriving the expression for the particle-boson interaction as used in the IBFA-model. Further applications will reveal the extent to which all important terms have been included in the IBFA Hamiltonian. They will also show whether the coupling of an odd proton is determined solely by the neutron quadrupole operator, as suggested by the microscopic theory, or rather the matter quadrupole operator, as suggested from preliminary calculations in the Platinum region.

The relation of the IBA-model with the geometrical description has now been studied by several authors\(^ {20,21}\). The relation of the IBA and IBFA-models with other boson-type models\(^ {23,24}\) is only now beginning to be investigated\(^ {26}\).

REFERENCES VII