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Introduction

"Give me a place to stand and a lever long enough and I will move the world." - Archimedes of Syracuse

This thesis deals with the analysis and control of nonlinear port-Hamiltonian systems. We start with a literature survey on port-Hamiltonian systems and present a couple of physical examples. Towards the end of this chapter, we give a brief outline of the contents in the subsequent chapters of the thesis.

We begin by giving a formal introduction to port-Hamiltonian systems.

1.1 Port-Hamiltonian systems

Port-Hamiltonian system models [87], [89] directly arise from network modeling of lumped-parameter physical systems. Under some assumptions, the main one being that the storage elements are independent, a general explicit port-Hamiltonian system with dissipation (PHSD) is represented as:

\[
\begin{align*}
\dot{x} &= [J(x) - R(x)]\nabla H(x) + g(x)u, \\
y &= g^T(x)\nabla H(x),
\end{align*}
\]  

(1.1)

where \(x \in \mathcal{X}\) are the energy variables with \(\mathcal{X}\) being an \(n\) dimensional manifold, the smooth function \(H(x) : \mathcal{X} \to \mathbb{R}\) represents the total energy stored and \(u, y \in \mathbb{R}^m, m \leq n\) are the port variables. The port variables \(u\) and \(y\) are conjugated variables, in the sense that their product defines the power flows exchanged between the system and the environment. Typical examples of such pairs are currents and voltages in electrical circuits or forces and velocities in mechanical systems. The system’s interconnection structure is captured in the \(n \times n\) skew-symmetric matrix \(J^T(x) = -J(x)\) and \(g(x)\), while \(R(x) = R^T(x) \geq 0\) represents the dissipation structure. All the matrices \(J(x), R(x), g(x)\) have entries depending smoothly on \(x\). A PHSD given by the equations (1.1)-(1.2) can be shown as a generalization of the well
known Hamiltonian equations. In chapter 2, we start with the classical Euler-Lagrange equations and show how the equations (1.1)-(1.2) can be derived from them.

Our main focus will be on mechanical and electro-mechanical systems and we now provide a motivation by presenting some physical examples.

### 1.1.1 Mechanical Systems

A general \( n \)-degree of freedom mechanical system (without constraints) is modeled as

\[
\begin{pmatrix}
\dot{q} \\
\dot{p}
\end{pmatrix} = \begin{bmatrix}
0 & I_n \\
-I_n & 0
\end{bmatrix} \begin{bmatrix}
\nabla_q H(q,p) \\
\nabla_p H(q,p)
\end{bmatrix} + \begin{bmatrix}
0 \\
G(q)
\end{bmatrix} u, 
\]

\( y = G^T(q)\nabla_p H(q,p), \) \hspace{1cm} (1.3)

where \( q \in \mathbb{R}^n, p \in \mathbb{R}^n \) are the generalized positions and momentum respectively with \( p = M^{-1}(q)\dot{q}, u \in \mathbb{R}^m \) is the input, \( G \) is an \( n \times m \) full rank matrix with \( m \leq n \). Further, the Hamiltonian function \( H(q,p) \) is the total energy of the system and is given as

\[
H(q,p) = \frac{1}{2} p^T M^{-1}(q)p + U(q), 
\]

where \( M = M^T > 0 \) is the mass matrix and \( U \) is the potential energy function. By choosing the output as \( y = G^T(q)\nabla_p H(q,p) \), the system (1.3)-(1.4) is clearly of the form (1.1)-(1.2) with the Hamiltonian satisfying \( \dot{H} = u^T y \).

If \( m < n \), the system is said to be underactuated, that is, the number of control inputs is less than the number of independent degrees of freedom. Underactuated systems form an interesting class for study as they are (in general) difficult to control.

**Example 1.1.** We take a look at the benchmark nonlinear system which consists of an inverted pendulum on a cart [24], [81] (also refer to the recent reference [1]), as shown in figure 1.1. The inertia matrix \( M(q) \), input matrix \( G(q) \) and the potential energy \( U(q) \) of the system are given as,

\[
M(q) = \begin{bmatrix}
1 & b \cos q_1 \\
b \cos q_1 & m_3
\end{bmatrix}, \quad G(q) = \begin{bmatrix}
0 \\
1
\end{bmatrix}, \quad U(q) = a \cos(q_1),
\]

where \( q_1 \) is the angle made by the pendulum with the vertical axis and \( q_2 \) refers to the horizontal position of the cart. The actuation appears only at the \( q_2 \) coordinate and the control problem is to stabilize the pendulum in its vertically upward position. Reference [90] considers some other interesting examples of underactuated systems in the context of observer design.
1.1 Port-Hamiltonian systems

![Inverted pendulum on cart system](image)

**Constrained Mechanical Systems:** Another interesting class of mechanical systems are those with kinematic constraints (refer to [15] for a detailed description). The kinematic constraints are modeled as

\[ Z^\top(q)\dot{q} = 0 \]  

(1.7)

where \( Z : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times k}, \text{rank}(Z) = k \). Usually, two types of kinematic constraints arise in (time-invariant) mechanical systems, namely the holonomic and the nonholonomic constraints. A constraint is called holonomic if the equation (1.7) is integrable and can be expressed exclusively as a function of the system’s position coordinates. Thus, a holonomic constraint restricts the configuration space of the system and, as a consequence, also constrains the velocities. On the other hand, kinematic constraints of the form (1.7) which are not integrable are called nonholonomic constraints. Thus, a nonholonomic constraint does not restrict the achievable positions of the system, that is, the system configuration space remains the same, but they constrain the velocities of the system. Mechanical systems with nonholonomic constraints have been extensively studied in the context of path planning. Reference [88] shows how such systems can be modeled as a PHSD which we shall now briefly explain. The mechanical system (1.3)-(1.4) with the nonholonomic constraint (1.7) when restricted to the constrained space

\[ X_c = \{(q, \dot{q})|Z^\top(q)\dot{q} = 0\}, \]  

(1.8)

takes the form

\[
\begin{bmatrix}
\dot{q} \\
\dot{\tilde{p}}
\end{bmatrix} = 
\begin{bmatrix}
0 & \tilde{S}(q) \\
-\tilde{S}^\top(q) & \tilde{J}(q, \tilde{p})
\end{bmatrix}
\begin{bmatrix}
\nabla_q H_c(q, \tilde{p}) \\
\nabla_{\tilde{p}} H_c(q, \tilde{p})
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
G_c(q)
\end{bmatrix}
u,
\]  

(1.9)
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with

\[ H_c(q, \tilde{p}) = \frac{1}{2} p^\top \tilde{M}^{-1}(q) \tilde{p} + U(q), \]  

where

\[ \tilde{p} = \tilde{S}^\top(q)p, \]

are the pseudo momenta with \( \tilde{S} : \mathbb{R}^n \to \mathbb{R}^{n\times(n-k)} \) being the full–rank right annihilator of \( Z^\top(q) \), that is, \( Z^\top(q)\tilde{S}(q) = 0 \) and \( G_c : \mathbb{R}^n \to \mathbb{R}^{(n-k)\times m} \). The \((i,j)\)-th element of \( \tilde{J} : \mathbb{R}^n \times \mathbb{R}^{n-k} \to \mathbb{R}^{(n-k)\times(n-k)} \) is given by

\[ \tilde{J}_{ij}(q, \tilde{p}) = -p^\top [\tilde{S}_i, \tilde{S}_j], \]

with \( \tilde{S}_i \) being the \( i \)-th column of \( \tilde{S} \), and \([\tilde{S}_i, \tilde{S}_j]\) is the standard Lie bracket of the vectors \( \tilde{S}_i \) and \( \tilde{S}_j \). We recall that the Lie bracket (refer to [58]) of two vector fields \( T_i, T_j \) is defined as

\[ [T_i, T_j] := \frac{\partial T_j}{\partial q} T_i - \frac{\partial T_i}{\partial q} T_j. \]

Using this, we obtain that

\[ [\tilde{S}_i, \tilde{S}_j] = -[\tilde{S}_j, \tilde{S}_i] \]

and thereby we conclude that \( \tilde{J} \) is skew–symmetric.

Example 1.2. Consider the well known example of the Chaplygin Sleigh [56] (also refer to [15]) that consists of a rigid body in the plane, supported at three points, two of which slide freely without friction while the third is a knife edge—a constraint that allows no motion perpendicular to its edge. The motion is described by the position coordinates \( q = [x, y, \theta] \), as shown in Fig. 1.2, where \( x \) and \( y \) denote the Cartesian coordinates of the point of contact of the knife edge with the ground and \( \theta \) denotes the orientation. The matrices \( \tilde{M}(q) \) and \( \tilde{S}(q) \) are given by

\[ \tilde{M} = \begin{bmatrix} m & 0 \\ 0 & I_0 + ma^2 \end{bmatrix}, \quad \tilde{S} = \begin{bmatrix} \cos(q_3) & 0 \\ \sin(q_3) & 0 \\ 0 & 1 \end{bmatrix} \]

where \( m \) is the mass of the rigid body, \( I_0 \) is the moment of inertia of the rigid body about its center of mass and \( a \) denotes the fixed distance between the knife edge and the center of mass. Correspondingly, the pseudo momentum coordinates \( \tilde{p} \) and the matrix \( \tilde{J} \) are obtained as

\[ \begin{bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \end{bmatrix} = \begin{bmatrix} p_1 \cos(q_3) + p_2 \sin(q_3) \\ p_3 \end{bmatrix}, \quad \tilde{J} = \begin{bmatrix} 0 & \frac{ma}{I_0 + ma^2} & \frac{ma}{I_0 + ma^2} \\ -\frac{ma}{I_0 + ma^2} & 0 \\ \frac{ma}{I_0 + ma^2} & 0 \end{bmatrix}. \]

We assume the body to be moving on a horizontal plane, that is, \( U(q) = 0 \). The control objective for this example would be to steer the system to a desired position on the horizontal plane by strategically planning its path. Few other interesting examples of nonholonomic mechanical systems can be found in [56], [15], [30].
1.1 Port-Hamiltonian systems

1.1.2 Electromechanical Systems

Electrical and electromechanical systems can also be written down in the form of a PHSD (refer to [87] for more examples). It was shown in [55] that every controlled $LC$-circuit can be modeled as a port-hamiltonian system. We now look at two well known examples of electromechanical systems, the capacitor microphone and the magnetic levitation system.

**Example 1.3.** We consider the capacitor microphone shown in Figure 1.3. The system is described in [56] (also refer to [87]). The capacitance $C(q)$ is varying as a function of the displacement $q$ of the right plate (with mass $m$), which is attached to a spring (with spring constant $k > 0$) and a damper (with constant $c > 0$) and is affected by a mechanical force $F$ (air pressure arising from sound). Furthermore, $E$ is a voltage source. The dynamical equations of motion can be written as the PHSD

\[
\begin{bmatrix}
\dot{q} \\
\dot{p} \\
\dot{Q}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-1 & -c & 0 \\
0 & 0 & -1/R
\end{bmatrix}
\begin{bmatrix}
\nabla_q H \\
\nabla_p H \\
\nabla_Q H
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 1/R
\end{bmatrix}
\begin{bmatrix}
F \\
E
\end{bmatrix}
\]

with $p$ the momentum, $R$ the resistance of the resistor, $I_v$ the current through the voltage source and the Hamiltonian given by

\[
H(q, p, Q) = \frac{1}{2m} p^2 + \frac{1}{2} k(q - \tilde{q})^2 + \frac{q}{2A\epsilon} Q^2,
\]
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Figure 1.3: Capacitor microphone

where $A$ is the plate area and $\epsilon$ the permittivity in the gap. Note that $Fq$ is the mechanical power and $EI$ is the electrical power applied to the system. In the application as a microphone the voltage over the resistor will be used (after amplification) as a measure for the mechanical force $F$. One control problem could be to stabilize the system in such a manner that the desired equilibrium position $q = q^*$ is attained.

Example 1.4. The magnetic levitation system is shown in the figure 1.4. The system is described in [69] (also refer to [7]) and consists of an iron ball in a vertical magnetic field created by a single electromagnet, described by the model

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
-R_2 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
\nabla x_1 H \\
\nabla x_2 H \\
\nabla x_3 H
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} u,
$$

(1.12)

$$
y = \nabla x_1 H
$$

(1.13)

where $x_1 = \lambda$ corresponds to the flux, $x_2 = \theta$ corresponds to the difference between the position of the center of the ball and its nominal position, $x_3 = m\dot{\theta}$ corresponds to the momentum and the system’s total energy is given as $H(x_1, x_2, x_3) = \frac{1}{2m}x_3^2 + mgx_2 + \frac{1}{2k}x_1^2(1 - x_2)$ with $m$ being the mass of the ball and $k$ is some positive constant that depends on the number of coil turns. The control goal will be to stabilize the ball at some desired position $\theta^*$. 

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1.2 Control of port-Hamiltonian systems with dissipation

We now go back to the equations (1.1)-(1.2). By computing the derivative of the Hamiltonian $H$ along the system trajectory we obtain that

$$\dot{H} = -[\nabla H(x)^T R(x) \nabla H(x)] + u^Ty, \quad (1.14)$$

where the first term on the right-hand side, which is non-positive, represents the dissipation due to the resistive elements in the system, and the second term denotes the external power supplied. We note from (1.14) that the energy function satisfies $\dot{H} \leq u^Ty$. Hence, if the total energy function $H(x)$ is bounded from below, then the PHSD given by (1.1)-(1.2) represents a passive system [87, Chapter 2] with respect to the input $u$ and the output $y$, with storage function being $H(x)$.

It follows that the energy of the uncontrolled system, i.e with $u(t) \equiv 0$, is non-increasing along the system dynamics. In the presence of enough dissipation, the system would eventually stop at a point of minimum energy. However, the point at which the open-loop energy function has a minimum is not always the equilibrium of practical interest and thus in such a case, control is implemented to operate the system around some desired equilibrium point $x = x^*$.

We now take a brief look at two alternative approaches for controlling a PHSD.
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1.2.1 Passivity based control

The term passivity-based control (PBC) was first introduced in [61] where the control design methodology aims to achieve asymptotic stabilization of a system by passivation of the closed-loop dynamics. More precisely, the goal is to design a control law to make the closed-loop system passive with a storage function which has a minimum at the desired equilibrium point. Further, if the new passive output is detectable, then applying damping injection of that output ensures asymptotic stability of the closed-loop system. Also refer to the well known paper [83] which presents the idea of developing algorithms for position control of a robot based on shaping the closed loop energy function. Later on this idea was extended to a large class of systems (eg: Euler-Lagrange, port-Hamiltonian etc). See the references below.

Passivity based control design techniques can be broadly classified into two categories. The first is the so-called classical PBC where the desired storage function is selected a priori and then the control law is designed in order to make that function non-increasing. This approach has been applied to mechanical, electrical and electro-mechanical systems (refer to [60]) which can be described using the Euler-Lagrange equations of motion. In the second method, the closed-loop storage function is not fixed but rather the desired structure of the closed-loop dynamics is selected, for example, Lagrangian or the port-Hamiltonian system with dissipation. Subsequently, all the assignable energy functions that are compatible with this structure are characterized and are given as the solution of a partial differential equation (PDE). Notable examples of this approach are the controlled-Lagrangian [16, 18] method, energy balancing passivity based control (EB-PBC) [69] and the interconnection and damping assignment passivity based control (IDA-PBC) [64] methods. We now briefly review the EB-PBC and IDA-PBC methodologies for port-Hamiltonian systems with dissipation.

In the context of port-Hamiltonian systems with dissipation, the problem of stabilizing the system at a desired equilibrium point by using a passivity based control design technique is posed as follows: Given a PHSD (1.1)-(1.2) and a desired equilibrium point \( x^* \), find a smooth state feedback law \( \rho(x) \in \mathbb{R}^m \), \( n \times n \) matrices \( J_d(x), R_d(x) \) depending smoothly on \( x \) with \( J_d^T = -J_d, \ R_d^T = R_d \geq 0 \), and a function \( H_d(x) \) having a strict minimum at \( x^* \) such that upon choosing

\[
u = \rho(x) + v,
\]

the system (1.1) in closed loop with the control action \( u \) in (1.15) becomes

\[
\dot{x}(t) = (J_d(x) - R_d(x)) \nabla H_d(x) + g(x)v.
\]

Differentiating \( H_d \) along (1.16) yields

\[
\dot{H}_d(x) = - (\nabla H_d)^T(x) R_d(x) \nabla H_d(x) + \bar{y}^T v,
\]
where the new output is $\tilde{y} = g^T(x) \nabla H_d(x)$ and hence the closed loop system satisfies $\dot{H}_d(x) \leq \tilde{y}^Tv$. It is clear that the energy of the closed loop system with $v \equiv 0$ is non-increasing and if the system (1.16) is zero-state detectable\(^1\) with respect to the output $\tilde{y}$, then we can establish by invoking La Salle’s invariance principle that (1.16) is asymptotically stable at the desired equilibrium point $x^*$.

In energy balancing passivity based control (EB-PBC), the closed loop energy $H_d$ is chosen as the sum of the plant energy $H$ minus the energy supplied to the plant system by the controller system. That is,

$$H_d(x) = H(x) - \int_0^t \rho^T(s)y(s)ds,$$

and moreover the interconnection and damping matrices remain the same, that is, $J_d = J$ and $R_d = R$. The EB-PBC methodology suffers from the dissipation obstacle according to which those coordinates in which dissipation is present, cannot be shaped. References [59], [69] (also refer to [64]) give a tutorial account of this method.

Another version of passivity based control is the so-called Interconnection and Damping Assignment Passivity Based Control (IDA-PBC). In the basic version of IDA-PBC, only the Hamiltonian function is shaped while in the actual IDA-PBC method, along with the Hamiltonian, the interconnection and damping matrices are also shaped, thus resulting in the closed loop structure (1.16). In this method, a set of matching equations that involves partial differential equations, given by

$$[J(x) - R(x)] \nabla H(x) + g(x) \rho(x) = [J_d(x) - R_d(x)] \nabla H_d(x), \quad (1.18)$$

need to be solved to obtain $\rho(x)$. As can be seen, this method provides extra degrees of freedom in the matrices $J_d, R_d$ whose form is apriori chosen so as to simplify the computation of the control $u$ in (1.18). The IDA-PBC scheme (unlike EB-PBC) does not suffer from the dissipation obstacle. The IDA-PBC methodology was introduced in [64] for general port-Hamiltonian systems with dissipation, reference [59] provides a detail survey of this method, references [1, 93, 62] apply the IDA-PBC technique for underactuated mechanical systems and [72] considers IDA-PBC for electromechanical systems. Also refer to [40], [67] and the more recent work [29], [63] and the references included there.

### 1.2.2 Control by Interconnection

We have seen that the input $u$ and the output $y$ appearing in the equations (1.1)-(1.2) are conjugate variables, that is, their scalar product defines the

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\(^1\)Zero-state detectability of the system (1.16) with respect to the output $\tilde{y}$ implies asymptotic stability of the system at $x = x^*$ subject to the condition $\tilde{y} = v = 0$ (refer to [87]).
power flows exchanged between the system and the environment. This can also be seen from the examples discussed where for the mechanical case, the input and output are the force and velocity while for the electromechanical system (refer to the Capacitor Microphone example), the inputs are force and voltage while the outputs are velocity and current respectively. Now, a power conserving interconnection between two physical systems is defined such that there is no gain or loss of energy in the interconnection. It is well known that a power conserving interconnection of two passive systems yields another passive system with the total energy being the sum of energies of the individual systems. Since a PHSD is passive, its power conserving interconnection with another PHSD yields a passive system whose Hamiltonian is the sum of the individual Hamiltonians. It has been shown in [23, 28] that the power-conserving interconnection of a number of PHSD is again a PHSD. Further, an alternative viewpoint of passivity based control as power conserving interconnection of dynamical systems instead of a state feedback action, can be found in [69]. Also refer to [87] and the more recent paper [63].

At this point, we recall what is called a Casimir function. A Casimir function \( C : X \to \mathbb{R} \) for a PHSD satisfies the property that its time derivative along the solutions of the PHSD is zero irrespective of the Hamiltonian function \( H \), when the input \( u \) is put equal to zero. For the system (1.1), this means

\[
\nabla^\top C(x) [J(x) - R(x)] = 0.
\]

(1.19)

whenever \( u = 0 \). In the control by interconnection method, we interconnect, using a power conserving relationship, the plant PHSD with a controller PHSD to yield a closed-loop PHSD. For this, consider the following controller system

\[
\begin{align*}
\dot{x}_C &= [J_C(x_C) - R_C(x_C)] \nabla H_C(x_C) + g_C(x_C)u_C, \\
y_C &= g_C^\top(x_C) \nabla H_C(x_C),
\end{align*}
\]

(1.20)

(1.21)

where \( x_C \in X_C \) with \( X_C \) being a \( d \) dimensional manifold and \( u_C, y_C \in \mathbb{R}^m \). Then, the standard feedback interconnection given by the power conserving relationship

\[
u = -y_C + e, \quad u_C = y + e_C,
\]

(1.22)

with the system (1.1)-(1.2) yields

\[
\begin{align*}
\begin{pmatrix} \dot{x} \\ \dot{x}_C \end{pmatrix} &= \begin{bmatrix} J(x) - R(x) & -g(x)g_C^\top(x_C) \\ g_C(x_C)g^\top(x) & J_C(x_C) - R_C(x_C) \end{bmatrix} \begin{pmatrix} \nabla H(x) \\ \nabla H_C(x_C) \end{pmatrix} \\
&\quad + \begin{bmatrix} g(x) & 0 \\ 0 & g_C(x_C) \end{bmatrix} \begin{pmatrix} e \\ e_C \end{pmatrix}, \\
\begin{pmatrix} y \\ y_C \end{pmatrix} &= \begin{bmatrix} g^\top(x) & 0 \\ 0 & g_C^\top(x_C) \end{bmatrix} \begin{pmatrix} \nabla H(x) \\ \nabla H_C(\zeta) \end{pmatrix}.
\end{align*}
\]

(1.23)

(1.24)
1.3 Observers and alternate passive input-output pairs for PHSD

From a control perspective, it is of interest to investigate the achievable Casimirs for the closed loop system (1.23). If we can find Casimirs $C_i(x, x_C)$, $i = 1, ..., r$ relating the plant states to the controller states, then by the energy Casimir method [53], we can replace the closed loop Hamiltonian $H(x) + H_C(x_C)$ by $\dot{H}(x, x_C) = H(x) + H_C(x_C) + H_0(C_1, ..., C_r)$, thus making $\dot{H}(x, x_C)$ as a possible Lyapunov function candidate for the closed loop system, provided it has the minimum at the desired equilibrium point $(x^*, x_C^*)$ where $x^*$ is the desired plant equilibrium point.

Further, if the Casimirs are of the form $C_i(x, x_C) = x_{C_i} - \tilde{C}_i(x_i)$ where $i = 1, 2, ..., r \leq d$, then by restricting the closed-loop dynamics of (1.24) to the multi-level set $L_C = \{(x, x_C) \mid x_{C_i} = \tilde{C}_i(x_i) + c_i, i = 1, 2, ..., r\}$ and eliminating $x_C$, we can obtain the dynamics on $L_C$ as

$$\dot{x} = [J(x) - R(x)] \frac{\partial H_s}{\partial x}(x),$$

(1.25)

where $H_s(x) := H(x) + H_C(\tilde{C}_1(x_1) + c_1, ..., \tilde{C}_r(x_r) + c_r)$. Thus, the standard feedback interconnection of the plant PHSD (1.1) with the controller PHSD (1.20)-(1.21) results in the PHSD (1.25) on the multi-level set $L_C$. As can be seen, the PHSD (1.25) has the same interconnection and damping structure as (1.1) but has a shaped Hamiltonian.

Please refer to [69], [87, Chapter 4] for a comprehensive description of the CbI methodology. References [23, 66] characterize the set of all achievable Casimirs for a PHSD by looking at its underlying geometric structure known as the Dirac structure. This approach is interesting as it investigates the achievable Casimirs by only looking at the plant system, without bringing the controller into the picture. We shall be speaking more on Dirac structures in Chapter 2.

### 1.3 Observers and alternate passive input-output pairs for port-Hamiltonian systems with dissipation

As seen from the previous section, the passivity-based control design technique for stabilization of port-Hamiltonian systems with dissipation is based on the availability of the measurements of state variables. Even if the desired equilibrium is already stable, the detectable output $\bar{y}$ would be required for damping injection to make the equilibrium asymptotically stable. But, there might occur situations where we may not have the accurate measurement of $\bar{y}$ or even the state variables. This might happen for instance in mechanical systems, where the quality of the velocity measurements (which is the usual port-Hamiltonian output as in (1.4)) might be very poor and thus we
might choose to measure the position instead of the velocity. Hence, the port-
Hamiltonian output may not always be the output which is measurable and
in many cases, it is not, which stymies the application of the state feedback
passivity based control method. Moreover, since this output is the standard
passive output, it also plays a pivotal role in the control by interconnection
method as we had seen. The above discussion provides a motivation to de-
sign observers for estimating the state variables of a PHSD assuming that
only the output (which may or may not equal the standard port-Hamiltonian
output (1.2)) is available for measurement.

In the context of observer design for general nonlinear systems, the first
attempts were to identify necessary and sufficient conditions on the nonlinear
system for converting it into a simpler form (like linear or bilinear system
up to an output injection term) using a change of coordinates or by perform-
ing a system immersion. References [13], [14], [35], [37], [47], [49], [51], [95]
are based on these ideas. Reference [46, Chapter 14] proposed the so-called
High Gain observer where the state dependent nonlinearities do not cancel
in the error dynamics but are in fact dominated by using high gain linear
terms and therefore caters to a larger class of nonlinear systems. Another
class of nonlinear systems that was studied consisted of those in which the
state-dependent nonlinearities satisfied certain conditions, like being globally
Lipschitz as studied by [32], [70], [85], [86], being a monotonic function of a
linear combination of the states as in [4], [31] or having a bounded slope [5].
The observer design for such systems was performed by employing quadratic
Lyapunov functions. The recent interesting work [45] attempts at providing a
systematic observer design procedure for nonlinear systems by generalizing
the early ideas of the classical Luenberger observer to the nonlinear case. The
observer problem is translated into the problem of solving a system of linear
partial differential equations which admits a unique and locally analytic
solution, according to the so-called Lyapunov’s auxiliary theorem. Refer to
[2, 48] for further extensions of this result. More recently, the references [7],
[42] proposed the so-called “Immersion and Invariance Observer” in which the
observer design was studied as a problem of rendering a selected manifold
in the extended state-space of the plant and observer as positively invariant
and globally attractive. Reference [42] in particular allows for non-monotonic
nonlinearities to appear in the unmeasured state dynamics and proposes a
reduced-order observer design for such systems. Reference [77] provides a
different viewpoint to observer design by invoking passivity based concepts.
The underlying idea is to make the augmented system consisting of the plant
and the observer dynamics strictly passive with respect to an invariant set in
which the state estimation error is zero. In order to establish passivity, a new
input and output is defined on the extended state space and, under some as-
sumptions on the plant and the observer, it is proved that passivation can be
achieved.

For mechanical systems, the problem of velocity reconstruction and posi-
1.4 Observers and alternate passive input-output pairs for PHSD

tion feedback stabilization has been extensively treated in the literature for special classes of systems—the reader is referred to the books [6, 26, 57, 60] for an exhaustive list of references. Chapters 3 and 4 of the thesis are focused on the construction of full and reduced order observers for special classes of mechanical systems, where the position is assumed to be measurable and the velocity is unmeasurable and hence needs to be estimated. We consider in Chapter 5, mechanical systems with nonholonomic constraints and construct full-order observers for such systems. The observer design in these chapters are based on the ideas in [7, 42, 77]. An important issue that arises with nonlinear observer design is the absence of the so called separation principle which otherwise holds for linear systems. For our observers, we prove under some mild boundedness assumptions that the observer can be used in conjunction with an asymptotically stabilizing, full state feedback, passivity based controller ensuring global asymptotic stability of the closed loop system.

We next recall the control by interconnection (CbI) method discussed in the previous section. As we had seen, the PHSD (1.1)-(1.2) is passive with respect to the input $u$ and the output $y$ and in the CbI method, this passive input-output pair is used for interconnection with the controller. Reference [40] showed that for certain electrical systems, it is possible to construct an alternate passive output (different from (1.2)) by the “swapping the damping” approach. It was also shown that this passive output helps to overcome the dissipation obstacle which otherwise exists when we use the original output (1.2) for interconnection with the controller. In the presence of the dissipation obstacle, the energy of the system (represented by the Hamiltonian) cannot be shaped in those plant variables $x$ in which the natural damping $R$ enters directly. On the other hand, in many examples it so happens that the energy needs to be shaped in these state variables which are affected by the dissipation obstacle. More recently, reference [91] proposed a general class of new passive input-output pairs for a PHSD and based on the similar lines of [23, 66], the achievable Casimirs were investigated for the PHSD when this new passive input-output ports are used for interconnection. A particular case was considered when the original input is retained and only the passive output is changed. The “swapping the damping” approach was shown to be a special case of this approach. Chapter 6 contains a detailed discussion of all these issues and presents some new interesting results in this direction.

1.4 Outline of the thesis

- In chapter 2, we provide the basic mathematical background for the forthcoming chapters. We start with a succinct introduction to the classical Euler-Lagrange equations by outlining some of their main properties which we shall need later in chapters 3 and 5. We continue and show how the PHSD in (1.1)-(1.2) can be obtained as a generalization of
the well known Hamiltonian equations. We next explore the underlying geometric structure of a PHSD which is given as the composition of a Dirac and a resistive structure and present from the existing literature some basic results on different representations of Dirac structures. We shall be referring to these results in chapter 6. Towards the end of the chapter, we give a brief synopsis of the Immersion and Invariance principle [6, 7] for observer design on which the results of chapters 3 and 5 are based.

• In chapter 3, we characterize the class of mechanical systems that can be rendered linear in the velocity via a partial change of coordinates. We show that such a class is characterized by the solvability of a set of partial differential equations (PDEs) and strictly contains the class studied in the existing literature on linearization for velocity observation or control. We then propose a reduced order globally exponentially stable observer to estimate the velocity, constructed using the immersion and invariance methodology. The design requires the solution of another set of PDEs, which are shown to be solvable in several practical examples. Finally, we prove that the observer can be used in conjunction with an asymptotically stabilizing full state-feedback interconnection and damping assignment passivity-based controller preserving asymptotic stability. The results in this chapter are based on the paper [90].

• In chapter 4, we present a different viewpoint on observer design. We consider a special class of PHSD and propose a design methodology for constructing globally exponentially stable full-order observers using a passivity-based approach. The essential idea is to make the augmented system consisting of the plant and the observer dynamics to become strictly passive with respect to an invariant manifold defined on the extended state space, on which the state estimation error is zero. We first introduce the concept of passivity of a system with respect to a manifold by defining a new input and output on the extended state space and then perform a partial state feedback passivation which leads to the construction of the observer. We then illustrate this observer design procedure for some well known mechanical and electromechanical systems, modeled in PHSD form. We also prove under some additional assumptions the separation principle for the proposed observer, when employed in closed-loop with a passivity based control (PBC) state feedback law, by using concepts from nonlinear cascaded systems theory. The results in this chapter are based on the paper [92].

• In chapter 5, we consider the problem of velocity estimation for a general $n$ degrees-of-freedom mechanical system with nonholonomic constraints. For unconstrained mechanical systems, many partial solutions have been reported in the literature. However, even in this case, the
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basic question of whether it is possible to design a globally convergent velocity observer remains open. In this chapter, we give an affirmative answer to the question for general $n$ degree of freedom mechanical systems with $k$ nonholonomic constraints, by proving the existence of a $3n - 2k + 1$-dimensional globally exponentially convergent velocity observer. An observer for unconstrained mechanical systems is obtained as a particular case of this general result. For the construction of the velocity observer, we use the Immersion and Invariance technique with dynamic scaling, recently proposed in [44] for a special class of nonlinear systems and later extended in [8], [9] for general class of mechanical systems with kinematic constraints. The results of this chapter are based on the papers [8] and [9].

• In chapter 6, we consider port-Hamiltonian systems with dissipation whose underlying geometric structure is represented as the composition of a Dirac and a resistive structure. We show how the choice of a new passive input-output pair for a PHSD is reflected in a new Dirac structure. We define a general class of new passive input-output pairs for a PHSD and subsequently compute (in a constructive manner) the resulting new Dirac structure and examine the achievable Casimirs for this new Dirac structure. We focus on the special case where only the passive output is changed (while retaining the original input) and subsequently define a general class of new passive outputs for the PHSD. We then identify (on the basis of the achievable Casimirs) the precise form of the so-called dissipation obstacle, and how this obstacle may be removed by changing the passive output. We also review the “swapping the damping” procedure for computing a new passive output, and show how this can be obtained as a special case within our approach. We consider the examples of the RLC-circuit and MEMS optical switch to investigate the role played by the new class of passive outputs in shaping the system’s energy. In the last section of the chapter, we explore the possibility of generating new passive outputs for a PHSD via modifying as well its Hamiltonian function. The results of this chapter are based on the paper [91].
1 Introduction