Competing interactions in multiferroics and low-dimensional systems
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Chapter 4

Origin of the elliptic spiral phase in $RMnO_3$

Rare-earth manganites $RMnO_3$ ($R$ is rare earth ion) with orthorhombically distorted perovskite structure have been attracting enormous attention since the discovery of polarization in the spiral phase of these materials.[1] Particularly striking is the magnetic field induced flop of the electric polarization from $P \parallel c$ to $P \parallel a$ observed in TbMnO$_3$, DyMnO$_3$ and from $P \parallel a$ to $P \parallel c$ in Eu$_{1-x}$Y$_x$MnO$_3$, which shows the strong correlation between dielectric and magnetic properties in these compounds.[2] Here we study the magnetic excitation spectrum of $RMnO_3$ using linear spin-wave theory and show that the combined effect of quantum fluctuations and anisotropy gives rise to ellipticity of the spiral ordering.

4.1 Introduction

The perovskite multiferroics $RMnO_3$ exhibit a variety of magnetic and electric phases, when the radius of the rare earth ion, which controls the GdFeO$_3$-type distortion, is varied.[1; 3] Intervening between the collinear A-type and E-type antiferromagnetic phases, the spiral spin order emerges in the moderately distorted materials. In TbMnO$_3$, for instance, a collinear sinusoidal order of Mn spins sets in first below the Néel temperature $T_N \approx 41$ K [4; 5]. In this phase, the Mn spins are aligned along the $b$-axis with an incommensurate propagation wave vector $Q_b = (0, 0.28, 0)$ along the crystallographic $b$ direction (where $Q_b$ is measured in units of $2\pi / b$). At $T_C \approx 28$ K, a magnetic transition

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takes place into a phase characterized by spiral ordering where Mn spins rotate in the \(bc\)-plane. The average magnetic moment \(\mathbf{M}_i\) on the \(i\)-th Mn site is given by

\[
\mathbf{M}_i = \mathbf{m}_b \cos \left( \frac{2\pi}{b} \mathbf{Q}_b \cdot \mathbf{R}_i \right) + \mathbf{m}_c \sin \left( \frac{2\pi}{b} \mathbf{Q}_b \cdot \mathbf{R}_i \right),
\]

(4.1)

where \(\mathbf{R}_i\) is the position of the \(i\)-th Mn spin. Experimentally, it was found that \(\mathbf{m}_b = (0, 3.9, 0) \mu_B\) and \(\mathbf{m}_c = (0, 0, 2.8) \mu_B\) for TbMnO\(_3\) at 15 K.

Thus the trajectory of the Mn spins is not a circle but an ellipsoid. Furthermore, the spiral arrangement of spins breaks spatial inversion symmetry and gives rise to electric polarization.[6; 8; 9] In such non-collinear magnetic structures the antisymmetric Dzyaloshinsky-Moria interaction implies a polar displacement of ions with a ferroelectric polarization given by

\[
P \propto \mathbf{r}_{ij} \times (\mathbf{S}_i \times \mathbf{S}_j),
\]

(4.2)

for a pair of neighboring spins \(\mathbf{S}_i\) and \(\mathbf{S}_j\), where \(\mathbf{r}_{ij}\) is the vector connecting the two spins. A spiral structure with a finite angle between the spin rotation axis \(\mathbf{S}_i \times \mathbf{S}_j\) and the propagation vector \(\mathbf{Q}\), therefore, induces a finite electric polarization. In TbMnO\(_3\), for instance, the spiral order in the \(bc\)-plane and \(\mathbf{Q} \parallel b\) imply a spontaneous electric polarization \(P \parallel c\), which is experimentally observed below \(T_C\).[6]

The magnetoelectric coupling Eq. (4.2) implies a coupling between polar phonons and magnetic excitations, which results in mixed phonon-magnon excitations. The existence of such elementary excitations in magnetoelectric coupled systems, called electromagnons, was already predicted long ago.[10] More recently, it was noted that the mechanism Eq.(4.2) which leads to polarization in spiral magnets also makes it possible to excite magnons by an oscillating electric field of light.[11] Here, the uniform lattice displacement is coupled to a magnetic excitation which corresponds to the rotation of both the spin plane and the direction of the polarization around the spiral propagation axis (\(\mathbf{Q}\)-axis). Consequently, the electromagnon excitation via the DM-coupling is constrained by the criterium that the electric field \(e\) of the light should be normal to the spiral plane. Evidence for electromagnon excitations was obtained in infrared optical spectroscopy measurements on \(RMnO_3\)\((R = \text{Gd,Tb,Dy,Eu}_{1-x}Y_x)\).[12; 13; 14; 15] However, the selection rule for the electromagnon polarization resulting from this coupling does not agree with experimental data.[14; 16; 17; 18] For instance in DyMnO\(_3\) and Eu\(_{0.75}\)Y\(_{0.25}\)MnO\(_3\) with the spiral ordering in the \(ab\)-plane (\(\mathbf{Q} \parallel b\)) electromagnon peaks were observed for \(e \parallel a\). Furthermore, experiments on TbMnO\(_3\) show the \(e /|a\) selection rule for electromagnon peaks at 20 and 60 cm\(^{-1}\) even if the system undergoes
a magnetic field-induced spin-flop transition from an \( bc \)-plane to an \( ab \)-plane spiral.\[20\] These observations clearly contradict the predictions that rely on the DM-mechanism and suggest that the selection rule is tied to the crystal lattice. In addition, the inverse Dzyaloshinskii-Moriya mechanism of relativistic nature is too weak to explain the strength of the electromagnon peaks in \( \text{RMnO}_3 \).

Moreover, for all ions \( R \), two electromagnons are always observed in the low temperature spiral spin state. Recently, the origin of one of these electromagnons was explained in Ref. \[20\], where it was shown that the high frequency electromagnon (at \( \sim 60 \text{ cm}^{-1} \)) corresponds to a zone-edge magnon activated by pure magnetostriction. However, the origin of the low frequency electromagnon (at \( \sim 20 \text{ cm}^{-1} \)) remains controversial.

This chapter is organized as follows. In Section 4.2, we introduce a simple microscopic model for \( \text{RMnO}_3 \), based on isotropic Heisenberg exchange interactions between spins. The ground state of the corresponding classical model is discussed in Section 4.3. In Section 4.4, the spin-wave spectrum of \( \text{RMnO}_3 \), obtained using linear spin-wave theory, is presented and the origin of electromagnons is reviewed. The effect of quantum fluctuations on the spiral ground state is discussed in Section 4.5 where we show that the combined effect of fluctuations and single-ion anisotropy can give rise to ellipticity of the spiral magnetic structure. The main conclusions are summarized and discussed in Section 4.6.

### 4.2 Spin model

Based on the previous discussion we propose a simplified microscopic model which accounts for the observed spiral spin state and magnetic excitations in \( \text{RMnO}_3 \) compounds. There are four Mn ions in the unit cell. The spins 1 and 2 lie in the \((z = 0)\) \( ab \)-plane, while the spins 3 and 4 belong to the \( z = c/2 \) plane (see Fig. 4.1). Our model describes the \( \text{Mn}^{3+}(S = 2) \) spins and includes the relevant magnetic interactions and single-ion anisotropy:

\[
H = H_{ex} + H_{an},
\]

where

\[
H_{ex} = \sum_i J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j
\]
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Figure 4.1 (a) Orthorhombic perovskite structure of $\text{RMnO}_3$ and (b) schematic representation of the staggered orbital ordering and superexchange interactions. Our model includes ferromagnetic exchange $J_0$ between nearest neighbor Mn spins, antiferromagnetic exchange constants $J_a$ and $J_b$ on the Mn-Mn bonds along, respectively, the orthorhombic $a$ and $b$ axes, and antiferromagnetic exchange $J_c$. Here, blue circles represent Mn ions and red circles correspond to oxygen ions.Rare earth ions are not shown.

The first term represents superexchange interactions. This term contains ferromagnetic exchange $J_0$ between the nearest-neighbor Mn ions in the $ab$-plane, antiferromagnetic exchange constants $J_a$ and $J_b$ along the corresponding crystal axes of the orthorhombic unit cell coupling next-nearest-neighbor Mn atoms, and antiferromagnetic exchange $J_c$ along the $c$-axis (see Fig. 4.1).

In perovskites with a relatively small orthorhombic distortion, such as for instance LaMnO$_3$, the strength of the second-neighbor exchange $J_b$ is small compared to the nearest-neighbor exchange $J_0$. In contrast, $J_b$ and $J_0$ are comparable in more distorted rare earth manganites, where they give rise to frustration of the magnetic ordering and stabilize spiral spin states. The relatively weak ferromagnetic exchange $J_0$ in $\text{RMnO}_3$ can be attributed to the combined effect of the electronic structure of Mn$^{3+}$ with $t_{2g}^3 e_g^1$ electron configuration and the staggered arrangement of the $e_g$ orbitals (see Fig 4.1). Furthermore, antiferromagnetic $J_b$ arising from an exchange path in the $b$-direction via two intermediate oxygen ions increases with the GdFeO$_3$-type distortion, while $J_0$ decreases with the distortion.
The second term in Eq.(4.3) describes single-ion anisotropies. In TbMnO$_3$, the $b$ axis is the easy magnetization axis ($K_b < 0$) and $a$ is the hard axis ($K_a > 0$). Throughout this chapter we assume $K_c = -K_b \equiv K$ and $K_a > K_c$, so that the $bc$-plane is the easy plane for Mn spins. The antisymmetric Dzyaloshinskii-Moriya exchange in addition to other types of anisotropic exchange interactions are not taken into account.

4.3 Spiral ground state

To obtain the spin excitation spectrum for the spiral state, we should first find the lowest-energy spin configuration of the related classical model where spins are assumed to be classical vectors of fixed length and the spin-spin interaction energy is minimized with respect to the angles that define the orientation of each spin. While a numerical solution is required (and provided below) for the ground state of the frustrated model with anisotropies described by Eq.(4.3-4.5), it is constructive to consider first an isotropic model ($K_i = 0$). Furthermore, since $J_a$ is typically small in $RMnO_3$, we first neglect this interactions and add it later. Under these assumptions, the classical ground state of the model can be easily found as a function of the ratio $J_b/J_0$. In the ground state the spins lie in the $bc$-plane forming a circular spiral:

$$S = S(\sin \theta_i, 0, \cos \theta_i). \quad (4.6)$$

Here, the angles $\theta_i = Q \cdot r_i$ define the relative orientation of the spins for given wave vector $Q$. Minimization of the classical energy with respect to $Q$ gives the so-called A-type antiferromagnetic order for $J_b/J_0 < 1/2$: Mn$^{3+}$ spins order ferromagnetically within ab-planes and antiferromagnetically between the planes. If, however, $J_b/|J_0| > 1/2$ the minimal energy configuration is a circular spiral with

$$Q_b = \frac{2}{b} \arccos \left[ \frac{|J_0|}{2J_b} \right]. \quad (4.7)$$

These results are of direct relevance to the magnetic phases found in the $RMnO_3$. For example in LaMnO$_3$ the GdFeO$_3$-type distortion is small and the low temperature magnetic ground state exhibits the A-type antiferromagnetic order. On the other hand larger rotations of octahedra in TbMnO$_3$, yield a sizeable overlap between the $e_g$ orbitals of next-nearest neighbor sites along $b$. The resulting strong antiferromagnetic interaction $J_b$ frustrates the magnetic ordering and gives rise to spiral magnetic order with $Q_b = 0.28$ in units of $\frac{2\pi}{b}$ below 28 K. A finite antiferromagnetic exchange $J_a$ will affect the spiral ground
Figure 4.2 Phase diagram of the classical spin model Eq.(4.3) with $K = 0$ containing the A-type antiferromagnetic (A-AFM) state for $J_b, J_a < 1/2$, the E-type antiferromagnetic state (E-AFM) and incommensurate states (IC) with spiral spin ordering ($J_c > 0$). For $J_a > J_b/2$ and $J_a < J_b^2/4 J_b$, the ground state is a coplanar spiral with $Q_b$ given by Eq.(4.7).

state and, in particular, give rise to a nonzero $Q_a$. However, for $J_a < J_b^2/4 J_b$, which is the case for RMnO$_3$ compounds, one can easily show that the uniform solution Eq. (4.6) with $Q_b$ given by Eq.(4.7) corresponds to the minimum of the classical energy. In Fig. 4.2 we show the phase diagram of the classical spin model described by Eq. (4.4), taken also into account the antiferromagnetic interlayer exchange $J_c$.

If an easy-axis anisotropy is present, the spiral Ansatz Eq.(4.6) with $\theta_i = Q \cdot r_i$ no longer describes the minimal energy spin arrangement because of the admixture of 3rd, 5th and higher order harmonics. The problem simplifies significantly if the period of the of the spin configuration is commensurate with the lattice. To show this we consider the effect of anisotropy on a commensurate spiral with $J_0/J_b = \sqrt{2}$ in which case $Q_b = 0.25$ and the magnetic unit cell contains 16 spins (2 layers each containing 8 spins). Without anisotropy, the angle between neighboring spins is constant, $\phi_{ij} = \phi_0$. However, easy-axis anisotropy tends to align the spins along the easy axis, thereby distorting the original perfect spiral into one with modulated rotation angles $\phi_{ij} = \phi_0 + \Delta \phi_{ij}$ where $\Delta \phi_{ij} \propto K J_0^2$ (see Fig. 4.3). Small anisotropy, however, does not affect the period of the commensurate spiral but rather locks it in (if the period would be non-commensurate anisotropy would change it). Thus finding the ground state reduces to minimizing the energy with respect to 16 angles between spins. In figure 4.3 we show a perfect spiral ($K = 0$) with $Q_b = 0.25$,
Figure 4.3 (a) One magnetic unit cell of the ab-cycloidal state with $Q_b = 0.25$. As in Fig 4.1, blue (red) circles correspond to Mn$^{3+}$ (O$^{2-}$) ions. Spins in successive layers are antiparallel (not shown) due to the antiferromagnetic exchange $J_c$. (b) Spin directions in the ab-cycloidal state with uniform rotation angles in the absence of easy axis anisotropy (left), and with modulated rotation angles in the presence of the easy axis anisotropy $K = 0.35J_0$ (right). The other couplings used to obtain this spin configuration are: $J_b = \frac{J_0}{\sqrt{2}}$, $J_c > 0$, $J_a < \frac{J_0^2}{4J_b}$ and $K_a > K$.

and the distorted spiral for $K = 0.35J_0$ with the same period.

Next we examine the zero temperature form factor

$$F^\mu(q) = \frac{1}{N} \sum_{ij} \langle S^\mu_i \rangle e^{iqr_i},$$

where $\langle S^\mu_i \rangle$ is the $\mu$ component of the average spin on the $i$'th site. Due to single ion anisotropy, spins prefer to point along to easy magnetization axis ($b$-axis) and avoid to point along the hard axes. Therefore, $F^b(Q)$ and $F^a(Q)$ are not equal in magnitude. We find, $\frac{F^c(Q)}{F^e(Q)} = 0.88$ for the spin configuration with $Q_b = 0.25$ obtained for $K = 0.35J_0$. This result should be compared with experimental measurements. For instance the spiral spin structure of TbMnO$_3$ at $T = 15$ K is well described by Eq.(4.1) with $Q_b = 0.28$, $m_b = 3.9\mu_B$ and
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$m_a = 2.7\mu_B$ such that \( \frac{F^c(Q)}{F^b(Q)} = \frac{m_c}{m_b} = 0.72 \).\cite{6,7} We note that classical spins have the same length on all sites. Therefore, \(|F^b(Q)| > |F^c(Q)|\) and the inequivalence between $F^b(Q)$ and $F^c(Q)$ can be ascribed to the fact that spins prefer to point along the easy axis, which gives rise to higher order harmonics.\cite{19}

Within the classical treatment the calculated ratio between the form factors is underestimated. The sublattice magnetizations at zero temperature are smaller than their classical values, owing to zero-point quantum fluctuations of spins. In Section 4.5 we discuss these effects and show how fluctuations further reduce the ratio \( \left| \frac{F^c(Q)}{F^b(Q)} \right| \).

4.4 Linear spin wave theory

In this section the dispersion of spin-waves in the spin model described by Eqs.\(\text{(4.3-4.5)}\) is discussed using linear spin-wave theory in the leading \(\frac{1}{\sqrt{S}}\) order. The reference state for the spin-wave analysis is the commensurate spin state with $Q_b = 0.25$. The magnetic unit cell (see Fig. 4.3) contains 16 spins. It is, however, possible to reduce the number of spins to 8 due to the symmetry property of the magnetic ground state: $S_i = -S_{i+2b}$. The magnon excitation spectrum can be calculated in the co-rotating spin frame introduced in Section 3.5. In this way we find the 8 magnon branches depicted in Fig. 4.3. It should be mentioned at this point that not all branches are visible in inelastic neutron scattering experiments (see the discussion below).

Next we discuss essential features of magnetic excitations in the spiral state. The $\Gamma_1$ mode at co-rotating wave vector $q = 0$ (filled blue circle in Fig. 4.4) corresponds to the sliding mode of the spiral called phason. The $\Gamma_2$ magnon with $k_b = Q_b = 0.25$ (denoted by a filled red square) corresponds to a rotation of both the spiral plane and the electric polarization around $Q||b$. Katsura, Nagaosa, and Balatsky showed that the magnetoelectric coupling based on Eq.\(\text{(4.2)}\) couples this magnetic excitation to the electric field $e$ of a light wave normal to the spiral plane.\cite{11} At the same wave vector but at a somewhat higher energy, a mode corresponding to a rotation of the spiral plane ($bc$-plane) along the $c$ axis is present. This mode (denoted by a filled red circle in Fig 4.4) can be excited by the magnetic field $h||c$ of a light wave, i.e. it is an antiferromagnetic resonance (AFMR). Optical absorption measurements show strong features at $\sim 20 \text{ cm}^{-1}$ and $\sim 60 \text{ cm}^{-1}$ for $e||a$.\cite{12,20} The high-frequency electromagnon at 60 cm$^{-1}$ was recently identified with the zone edge magnon ($k_0 = (0, 1, 0)$ in the simplified model without orthorombic distortions) which is coupled to the electric field by exchange striction.\cite{20} To
understand this we consider the coupling of spins to the electric field $e||a$,

$$H_{me} = -gE_a \sum_j \left[ (S_{1,j} - S_{1,j+b}) \cdot (S_{2,j} - S_{2,j+a}) 
+ (S_{3,j} - S_{3,j+b}) \cdot (S_{4,j} - S_{4,j+a}) \right],$$

(4.9)

which is invariant under the $Pbnm$ space group symmetry operations. Here, the sum runs over all unit cells and $g$ is the magnetoelectric coupling. Due to the GdFeO$_3$-type distortion in $RMnO_3$, the oxygen ions mediating the superexchange between neighboring Mn spins in the $ab$-plane are displaced from the straight line connecting the Mn spins. The magnetoelectric coupling Eq. (4.9) describes how a uniform electric field with $e||a$ that shifts the oxygen ions along the $a$-direction, gives rise to an alternation of the nearest neighbor exchange constants along $b$: $J = J_0 + gE_a \cos(k_0 \cdot r)$. The zone-edge magnon with $k = k_0$ corresponding to staggered rotations of the spins as indicated in Fig. 4.5, induces an electric dipole moment along the $a$-direction, irrespective of the orientation of the spiral spin plane.

Next, we shall calculate inelastic neutron scattering spectra. In order to obtain these, we have to rotate the spin coordinates back to the original laboratory system (see Section 3.5). Figure 4.6, shows the model calculations of the dynamical structure factor along with the neutron-scattering data for
Figure 4.5 Sketch of the polarization schemes of the different magnon excitations of the cycloid structure as described in the text. (a) The sliding mode which can be regarded as a rotation of the spin plane around \(a\). (b) The mode resulting in a rotation of the spiral plane around the \(c\) axis. (c) The mode corresponding to a rotation of the spiral plane along the \(b\) axis. This mode can be excited by the electric field of light via the coupling Eq.4.2. (d) Schematic representation of the zone-edge electromagnon with \(k = (0, 1, 0)\). The red arrows correspond to the Mn spins and the black arrows correspond to their oscillations.

\(\text{TbMnO}_3\) in the low-temperature ordered phase with \(Q_b = 0.28\).[21] Clearly the Heisenberg model with magnetic anisotropy provides a good description of the spin dynamics. Best fits are obtained with nearest neighbor exchange \(J_0 = 0.45\) meV, \(J_c = 0.6\) meV, next-nearest exchanges \(J_a = 0.05\) meV and \(J_b = 0.32\) meV, and single ion anisotropies \(K_1 = 0.18\) meV, and \(K = 0.16\) meV. The magnon linewidth is due to a finite decay rate \(\Gamma = 0.45\) meV. At the spiral wave vector \(Q\) three modes dominate the scattering intensity: (i) the phason mode, (ii) the KBN electromagnon and (iii) the mode corresponding to a rotation of the spiral plane around the \(a\) axis. Note that within our model the later one is always higher in energy as compared to the KBN electromagnon.

4.5 Quantum fluctuations

The sublattice magnetizations at zero temperature are smaller than their classical values due to the zero-point quantum fluctuations of the spins. In this section we discuss the effect of quantum fluctuations on the spiral ground state.

The magnetization on site \(i\) is defined as the average spin component on
Figure 4.6 Maps of the dynamic structure factor obtained from model calculations on a ground state with $Q_b = 0.25$. Open circles correspond to inelastic neutron scattering peaks of TbMnO$_3$ in the incommensurate phase with $Q_b = 0.28$ obtained by Senff et al. [21] Note that measurements along $k_a$ and $k_c$ where performed at 23 K while data along $k_b$ was obtained at 17 K. Spin wave dispersion curves and structure factors were calculated using the Heisenberg Hamiltonian Eq.(4.3) with $J_0 = 0.45$, $J_a = 0.05$, $J_b = 0.32$, $J_c = 0.6$, $K = 0.16$ and $K_a = 0.18$ meV, The magnon linewidth is due to a finite decay rate $\Gamma = 0.45$ meV.

that site along its quantization axis. In the spin co-rotating reference frame the local quantization axis points along the $\tilde{z}$ axis. Using the Holstein-Primakoff transformation (see Section 3.5) the sublattice magnetization can be expressed as

$$\tilde{S}_i^z = S - \langle a_i^\dagger a_i \rangle.$$

In the above equation, $S = 2$ is the classical value of the spin, the second term, $\langle a_i^\dagger a_i \rangle = \delta S_i$, is the reduction from the classical value of the spin due to the quantum and thermal fluctuation, and $\langle Q \rangle$ is the expectation value of the operator $Q$ in the ground state. $\delta S_i$ can be calculated within the linear spin wave theory using the Bogoliubov transformation introduced in Section 3.5.

As mentioned before, due to canting of Mn spin towards the easy axis ($b$-axis) the ratio $m_a/m_b = F_c^Q/F_b^Q$ differs from unity for classical spins. In Fig. 4.7 we show the effect of quantum fluctuations. In the classical ground state with $Q_b = 0.25$ half of the Mn spins (denoted by $A$ in Fig 4.7) make an angle $\phi$ with respect to the local easy axis ($b$ axis), while the other half of the spins (denoted by $B$ in Fig. 4.7) make an angle $\theta > \phi$ with the easy axis. We find that the reduction of magnetic moments of $B$ spins due to quantum fluctuations is
Figure 4.7 (a) The ellipticity $m_c/m_b = \sqrt{F_c(Q)/F_b(Q)}$ as a function of the anisotropy $K$ for the ab-spiral spin state with $J_a = 0.17J_0$, $J_b = J_0/\sqrt{2}$, $J_c = 1.3J_0$, $K_a = 0.4J_0$, $K_b = 0.35J_0$ and $T = 15$ K (and the other couplings as defined in the caption of Fig 4.7) we find $\delta S_A = 0.11$, $\delta S_B = 0.19$ and $m_c/m_b = 0.84$. (b) Spin structure in the ab-cycloidal state in the classical limit (left) and including fluctuations (right). Note that the orthorhombic unit cell contains alternatively stacked MnO planes.

larger than the reduction of the magnetic moments of A-spins. Therefore, quantum fluctuations further reduce the ratio $F_c(Q)/F_b(Q)$ between form factors. In particular, for $K = 0.35J_0$ and $T = 15$ K (and the other couplings as defined in the caption of Fig 4.7) we find $\delta S_A = 0.11$, $\delta S_B = 0.19$ and $m_c/m_b = 0.84$. We note that the correction to the sublattice magnetization is very small in comparison with the classical value of the spin $S = 2$. This indicates that the spin wave theory is a good approximation for the present model.

4.6 Concluding remarks

By investigating the detailed structures of the cycloidal spin states, we have found that the spiral rotation angles are not uniformly the same but are significantly modulated due to the single-ion anisotropies. In Ref. [19] the experimentally observed direction-dependent magnitude of the spin-correlation peak was ascribed to this rotation-angle distribution. However, we have shown that
quantum and thermal fluctuations also contribute to elliptical modulation of the spin cycloid.

The observed ellipticity in TbMnO$_3$ at 15 K is smaller than the results of the model calculations for realistic values of the couplings and anisotropies. This difference might be due to covalent mixing: the magnetic moment spreads out to neighboring anions where it gets cancelled by contributions arising from other Mn ions. This effect is not included into our model calculations.

To summarize, we showed that the combined effect of single ion anisotropy and spin fluctuations gives rise to ellipticity of the spiral magnetic ordering. Furthermore, we have discussed the magnetic excitation spectrum of the $R$MnO$_3$ family in the cycloidal phase. Due to anisotropies the excitation energy of the antiferromagnetic resonance is higher in comparison to the KBN electromagnon, which corresponds to a rotation of the spiral plane around the wave vector $Q$ of the spiral.
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