Some applications of the representation theory of finite groups
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In this thesis we study the representation theory of finite simply reducible groups and more specifically the application of so-called Racah algebra. Several techniques, which are well-known for the rotation group (or its covering group SU(2)) and for SU(3), such as Clebsch-Gordan coefficients (or Wigner coefficients), Racah coefficients, irreducible tensor operators and Wigner-Eckart theorem are applied here to study some special problems in finite simply reducible groups.

In Chapter 2 the group generated by the Dirac matrices is studied. We are able to interpret the so-called Fierz transformation in terms of the Racah coefficients of this "Dirac matrix group".

The Fierz transformation is used in the theory of weak interactions. The most general form of the interaction-Hamiltonian for weak interactions between four fermions is built up from five Lorentz invariant parts viz. the scalar-scalar, vector-vector, tensor-tensor, axial vector-axial vector and pseudo scalar-pseudo scalar interactions. In calculations it sometimes turns out to be useful to rearrange the four fermion wave functions. This rearrangement can be realized by the Fierz transformation.

The method used in Chapter 2 also provides generalized Fierz transformations. Furthermore simple rules are given to calculate the corresponding Fierz invariants and anti-invariants.

The Racah coefficients which occurred in Chapter 2 are of a very special type, in the sense that four of the six representations are the same, whereas the remaining two are both one-dimensional.
In Chapter 3 we show that one can determine the values of this type of Racah coefficients and even of a more general type. In the second part of this chapter eigenvalues and eigenvectors are given of a matrix expressed in terms of Racah coefficients. These Racah coefficients can be considered as a generalization of those which play a role in Chapter 2. It appears that there exists a simple relationship between the eigenvectors and the columns of the above mentioned matrix. This relationship was already manifest from the calculations in Chapter 2.

A crucial point in our considerations on Fierz transformations was the interpretation of the T-matrices as irreducible tensor operators of the Dirac matrix group. Irreducible tensor operators have been studied extensively for the rotation group and SU(3), but for finite groups they have hardly been investigated. In Chapter 4 we make a start with an exploration of the properties of irreducible tensor operators in finite groups. In particular we derive some formulae for inner irreducible tensor operators, i.e. tensor operators which can be built from the elements of the group.

In studying the tensor operators of the Dirac matrix group there appeared to be a correspondence between the classes and the one-dimensional representations. This is a feature of the more general phenomenon of a kind of duality which seems to exist between classes and representations. This duality has also been considered by some other authors recently. Up to now, however, duality cannot be defined rigorously. Instead, it can be used as a heuristic principle to suggest new theorems.

An example of this principle is treated in Chapter 5. We show that it is possible to define a 3k-symbol for classes in analogy to the 3j-symbol for representations. The sum of the inverse squares of the dimensions of the representations can be expressed in terms of these 3k-symbols. This relation is a useful complement to Burnside's formula in calculating the dimensions of the irreducible representations of a finite group.
Representations and classes of a group are also dual to each other in the sense that problems that are formulated in terms of classes can be answered in terms of representations. In Chapter 6 we illustrate this by calculating the number of roots of certain equations in finite simply reducible groups. In this way it is possible to generalize various formulae for the number of roots which are already known in the literature. In the course of our calculations we also generalize graphical techniques in Racah algebra, which are well-known for SU(2), to arbitrary simply reducible groups.

In Chapter 7 we discuss to what extent the calculations of the previous chapter can be generalized to more general groups. We present the results of a number of calculations for multiplicity free groups. We indicate how the calculations can be done in principle for an even more general kind of group, but the calculations become much more difficult (and useless in practice if one considers groups for which the symmetry properties of the Clebsch-Gordan coefficients are no longer simple).

In this thesis it is assumed that the reader is familiar with the representation theory of groups and more specifically with the representation theory of simply reducible groups as discussed in ref. [1]. As far as our notation concerns we mostly stick to the notations of ref. [1], although for characters and representation matrices we use the more conventional notation, as in ref. [2].