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Published in:
Optics Communications

DOI:
10.1016/0030-4018(93)90280-I

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Document Version
Publisher's PDF, also known as Version of record

Publication date:
1993

Link to publication in University of Groningen/UMCG research database

Citation for published version (APA):

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Download date: 27-12-2018
Numerical calculations of the optical propagation properties of the interconnections of symmetric multimode slab waveguides

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Received 26 May 1992; revised manuscript received 9 September 1992

The implementation of a program to compute the optical properties of the interconnection region of two multimode waveguides is discussed. The interconnection problem is described by a scalar boundary value problem defined for the TE polarized component of the electric field. Because of the particular design of the boundary value problem, the conditions at the open boundaries of the incoming and outgoing waveguides are relatively simple. A discretization is obtained using a hybrid system of finite element and boundary element methods. The method is illustrated by calculations on two interconnection configurations. Estimates of the accuracy of the results are given.

1. Introduction

This paper describes the authors' investigations on a numerical method to calculate the optical fields as well as the optical propagation properties of the interconnection region of two ideal semi-infinite slab waveguides. Especially, the attention is directed to the interconnections of multimode waveguides with a transition region, having geometrically non-uniform cross sections and a spatially variable refractive index. It is required that the method is applicable to a wide range of complex interconnection configurations. In particular, a transition region with large differences of the refractive index has to be allowed and optical reflections may be essentially present.

The theoretical approach consists of the derivation of a boundary value problem in terms of an appropriate electric field component. This component satisfies the corresponding Helmholtz equation in the various parts of the problem. Numerical solutions are obtained by the application of a combination of a boundary integral equation method in sections with a constant refractive index and a finite element method in sections with a varying refractive index.

Many authors [1–6] reported studies about more or less complicated interconnection problems. Broadly outlined the concepts of the mathematical models and the numerical methods in these papers are very similar and agree with the approaches just given. As will be indicated, differences between the various approaches occur in the final practical mathematical design and the numerical implementation.

The authors previously published papers [4–6] on the research subject. In ref. [4] they proposed the basic ideas of the mathematical and numerical model. A description of the method in more details together with the numerical results of some waveguide junctions are given in ref. [5]. As a particular feature of the model in refs. [4,5], the reflection and transmission coefficients of the bound modes are explicitly defined as independent unknowns to be calculated. In order to make the computer programming as easy as possible, the corresponding numerical model given in ref. [5], is described with the use of an equidistant net. Hence, only relatively simple configurations are allowed. For instance, ref. [5] shows an application of a configuration in which both waveguides and the interconnection region have a core with a uniform width for each cross section. In the case of monomode waveguides, the model of refs. [4,5] has been implemented in a reasonably effective computer program to handle a range of interconnection problems as illustrated in the applications of ref. [5].
Two versions of a model, that is useful for the generation of a flexible computer program and can be applied to the interconnection of multimode waveguides, have been discussed in the theoretical paper [6]. The first version corresponds with the one given in the literature [1-3]. In these studies, generally the optical propagation coefficients are implicitly defined using an expansion of the optical field in terms of the complete orthogonal set of guided, radiation and evanescent local modes [7] at suitably chosen transverse boundary planes. The resulting boundary conditions at these planes consist of integral equations in which the electric field component and its normal derivatives at the whole infinite transverse plane are unknown. In the second version, applied by the authors in this paper, the transverse boundary planes are at alternative positions. In that model, they chose the transverse boundary planes at positions sufficiently distant from the interconnection region, where the radiation and evanescent modes are assumed to be vanishingly small. Hence, at these boundaries the electrical field component can be expanded exclusively in terms of the guided modes. Using these expansions in the expressions for the reflection and transmission coefficients of the bound modes, much simpler integral equation boundary conditions can be derived. The paths of integration and consequently the region of definition of the unknown electric field component and its normal derivatives can be limited to the finite parts of the transverse boundary planes that coincide with the core of the waveguides.

Based on the theoretical model of the authors' version of ref. [6], this paper describes a computer program for evaluating interconnection problems. Applying standard finite element techniques with a non uniform net, complex geometries, such as tapers and step discontinuities, can be considered. Specific numerical aspects are emphasized. For some selected case studies, the accuracy of the computational results is discussed. A more advanced example of a complicated interconnection multimode system shows the reflection and transmission behaviour of each of the bound modes in the corresponding waveguides.

2. Statement of the problem

The optical model to be considered consists of the following. Two semi-infinite lossless symmetric multimode dielectric slab waveguides are interconnected in a direction along the z-axis. It is assumed that there are no structural variations in the y-direction. In this case, the model can be reduced to a two-dimensional problem defined in the (z, x)-plane. Each waveguide has a uniform width and a core with a constant refractive index. The interconnection region around the junction, which is also symmetric in the x-direction, may have an arbitrary geometrical shape in the z-direction and a spatially dependent refractive index. The whole system is embedded in a cladding extending to infinity with a constant refractive index. Figure 1 shows a sketch of the two-dimensional symmetric model in the upper (z, x)-plane.

Exciting the incoming waveguide coherently at infinity, a light beam consisting of a combination of incident bound modes with different intensities appears in that waveguide. Next the light beam continues the path by passing the interconnection region into the outgoing waveguide. The propagation properties of the light beam strongly depend on the design properties of the actual configuration. They are considered as a function of the geometries and refractive indices of the interconnection region and the incoming and outgoing waveguides, respectively. In particular, the reflectance of the bound modes in the incoming waveguide, the transmittance of the bound modes in the outgoing waveguide and the radiation losses are determined.

3. The boundary value problem

Mathematically the interconnection problem will
be described by a two-dimensional boundary value problem. Limiting the electric field to the case of even TE polarisation, this boundary value problem can be defined for the transverse $y$-component $E(z, x)$. As indicated in fig. 2, the closed boundary includes parts of the $z$-axis, the transverse lines in the $x$-direction at the positions $z = L_1$ and $z = L_2$ and is closed by a large circle in the surrounding medium. Within this defining region, subregions with different refractive indices can be distinguished. Assuming that the electric field has a harmonic time dependence $\exp(-i\omega t)$, the $y$-component $E(z, x)$ satisfies the Helmholtz equation
\[
\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial x^2} + k^2 n^2(z, x) E = 0. \tag{3.1}
\]
The refractive index is given by $n(z, x)$, while $\omega$ and $k$ are the radian frequency and the free space wavenumber of the excited light beam, respectively. At the interfaces of the subregions, $E(z, x)$ and $\frac{\partial E}{\partial n}$ are continuous, as follows from the conditions of continuity of the tangential parts of the electric and magnetic field.

Looking in more detail at the separate subregions, a transition region $\Omega_1$ bounded by the $z$-axis, the transverse lines at $z = L_3$ and $z = L_4$ and a suitably chosen line parallel with the $z$-axis, is observed. This subregion includes completely the interconnection region with inhomogeneous properties. In $\Omega_1$, $E(z, x)$ will be described by eq. (3.1), $n(z, x)$ being the known refractive index.

The subregion $\Omega_2$, consisting of the core region of the incoming waveguide and bounded by $z = L_1$ and $z = L_3$, has the constant refractive index $n_1$. Instead of using eq. (3.1), for the particular case of a constant refractive index, $E(z, x)$ will be alternatively represented in $\Omega_2$. Referring to ref. [9] for a detailed description, starting with a weighted residual formulation, $E(z, x)$ at each point $(z, x)$ of the enclosing boundary $\Gamma_2$ of $\Omega_2$ can be expressed in terms of a boundary integral along that boundary $\Gamma_2$ of the type
\[
\frac{1}{2}E(z, x) = \int_{\Gamma_2} (G \frac{\partial E}{\partial n} - E \frac{\partial G}{\partial n}) \, dz. \tag{3.2}
\]
The function $G$ has been chosen as the weighting function and is defined by the free space Green function of the Helmholtz equation for the corresponding constant refractive index $n_1$, satisfying the Sommerfeld radiation condition at infinity. $G$ can be expressed as
\[
G(kn_1 r) = -0.25 \left[ N_0(kn_1 r) + iJ_0(kn_1 r) \right], \tag{3.3}
\]
where $r$ is the distance between $(z, x)$ and the point on the path of integration of $\Gamma_2$ being considered and $J_0$ and $N_0$ are the Bessel functions of the first and sec-

Fig. 2. The boundary value problem for $E(z, x)$. The different subregions with the corresponding refractive indices are shown.
The core region $\Omega_3$ of the outgoing waveguide between $z=L_4$ and $z=L_2$ has the constant refractive index $n_2$. Similarly as above, for this region $\Omega_3$ in each point $(z, x)$ of the enclosing boundary $\Gamma_3$ of $\Omega_3$, $E(z, x)$ can be expressed in terms of a boundary integral of the type (3.2) along $\Gamma_3$ with the refractive index $n_2$.

The remaining subregion $\Omega_4$ of the cladding with the constant refractive index $n_3$ will also be treated by a boundary integral equation of the type (3.2). The closed boundary $\Gamma_4$ of that cladding region has to be taken as integration path and the refractive index occurring in $G$ of eq. (3.3) is equal to $n_3$.

Finally the boundary conditions along the enclosing boundary of the whole boundary value problem have to be considered. The electric field $E(z, x)$ is assumed to be even, which implies that along the $z$-axis the condition $\partial E(z, 0)/\partial x=0$ has to be valid.

At large distances from the interconnection junction, the condition of a vanishing electric field $E(z, x)$ and the Sommerfeld radiation condition are imposed. These conditions express mathematically the physical requirement that, apart of the contributions of the incident bound modes in the incoming waveguide, the flow of energy is directed towards infinity. Hence, $E(z, x)$ and $\partial E/\partial n$ are vanishingly small at the large circle, if tending to infinity.

In order to facilitate the formulation of reasonably simple conditions at the open boundaries of the waveguides at $z=L_1$ and $z=L_2$, additional requirements are imposed at these locations. The particular choices of $z=L_1$ and $z=L_2$ sufficiently far from the interconnection region, warrant that at these locations, the field $E(z, x)$ exclusively consists of the guided modes of the corresponding waveguides. Because of the Sommerfeld radiation condition the radiation and evanescent modes are negligibly small at these locations. The development of an appropriate boundary condition at $z=L_1$ has been discussed already in ref. [6] in the case of exciting a monomode incoming waveguide. For the sake of clearity the reasoning will be repeated in more detail. At $z=L_1$ the electric field and its normal derivative can be written as

$$E(z, x) = A f(x) \exp(-i\beta z) + R A f(x) \exp(i\beta z)$$

(3.4)

and

$$\partial E(z, x)/\partial z = -i\beta A f(x) \exp(-i\beta z) + i\beta RA f(x) \exp(i\beta z),$$

(3.5)

where $A$ and $\beta$ are the amplitude and the wave number of the bound mode, respectively. The function $f(x)$ represents the transverse shape of the bound mode and has been normalized with respect to the time-averaged power carried by that bound mode. Explicit expressions of $f(x)$ are given in e.g. refs. [5,8]. $R$ is the reflection coefficient of the reflected bound mode in the incoming waveguide. Applying a procedure given in ref. [7] for guided modes only, multiplying both sides of eq. (3.5) with $f(x)$, integrating in the $x$-direction from zero to infinity next, lead to the reflection coefficient

$$R = \exp(-2i\beta L_1) + \exp(-i\beta L_1)$$

$$\times \int_0^\infty [f(x) \partial E(L_1, x)/\partial z] \, dx$$

$$\times \left( i\beta A \int_0^\infty f^2(x) \, dx \right)^{-1}. \quad (3.6)$$

A crucial consequence of the particular choice of $z=L_1$ is that the integration paths of the integrals occurring in eq. (3.6), can be limited to the core part of the incoming waveguide. Having substituted $\partial E/\partial z$ by the right hand side of eq. (3.5) in the cladding part of the first integral of eq. (3.6), the integration with respect to that cladding part can be performed. With the use of the latter contribution depending on $R$, and a reordering of some terms, the expression (3.6) can be straightforwardly reduced to

$$R = \exp(-2i\beta L_1) + \exp(-i\beta L_1)$$

$$\times \int_0^D [f(x) \partial E(L_1, x)/\partial z] \, dx$$

$$\times \left( i\beta A \int_0^D f^2(x) \, dx \right)^{-1}, \quad (3.7)$$
in which \( D \) is the width of the waveguide. The substitution of eq. (3.7) into eq. (3.4), gives the final boundary condition at \( z=L_1 \) as
\[
E(L_1, x) = 2A f(x) \exp(-i\beta L_1) + \frac{f(x)}{D} \int_0^D \left[ f(x) \frac{\partial E(L_1, x)}{\partial z} \right] dx \\
\times \left( i\beta \int_0^D f^2(x) \, dx \right)^{-1}.
\]

(3.8)

With these results, \( R \) and the electric field \( E(z, x) \) at any point of \( z=L_1 \) can be expressed in terms of the normal derivatives of \( E(z, x) \) of the core region of \( z=L_1 \) only. Applying the same procedure starting from eq. (3.4), an alternative expression for \( R \) and an expression for \( \frac{\partial E(L_1, x)}{\partial z} \) both involving \( E(L_1, x) \) at points of the core region can be derived. For reference later, the latter reads
\[
\frac{\partial E(L_1, x)}{\partial z} = -2i\beta A f(x) \exp(-i\beta L_1) + i\beta f(x) \int_0^D \left[ f(x) E(L_1, x) \right] \, dx \left( \int_0^D f^2(x) \, dx \right)^{-1}.
\]

(3.9)

At \( z=L_2 \) for the case that the outgoing waveguide is monomode, similarly the transmission coefficient \( T \) and a boundary condition for \( E(L_2, x) \) and their alternative expressions, can be obtained. These various quantities can be expressed in terms of either \( \frac{\partial E(L_2, x)}{\partial z} \) or \( E(L_2, x) \) in points of the core region of the outgoing waveguide at \( z=L_2 \).

In the case of multimode waveguides using the orthonormality of the bound modes, analogously expressions for the reflection and transmission coefficients of the bound modes and boundary conditions of the kind of eqs. (3.8) and (3.9) can be calculated straightforwardly. In order to save space, these more complicated relations are not given.

4. The numerical solution of the interconnection problem

As already mentioned the boundary problem for \( E(z, x) \) has been solved using a mixture of discretization methods. In the transition region, the Helmholtz equation (3.1) is approximated by a finite element method (FEM), while a boundary element method (BEM) is applied to the boundary integral equations of the type (3.2) for the closed boundaries of the various subregions. At first, the transition region and the pertinent closed boundaries are covered with net points as shown in fig. 3. The final approximate solution of the electric field consists of the determination of \( E(z, x) \) in the discrete net points of the transition region and \( E(z, x) \) and the normal derivative \( \frac{\partial E}{\partial n} \) in the discrete net points of the closed boundaries. In the following numerical model \( E(z, x) \) and \( \frac{\partial E}{\partial n} \) in the discrete net points will be considered as independent variables.

Next, a more detailed treatment for each subregion will be given. A skillful choice of the net points in the transition region depends strongly on the actual configuration. For the sake of accuracy of the solution and easy programming, net points should preferably be chosen as equidistant as possible and the net should be at least a topologically rectangular net. Due to the particularly chosen rectangular geometry of the transition region, the latter choice is assured. At the same time, net points have to be taken at positions where the refractive index changes discontinuously. Examples of the last positions are on the interface of the core and the cladding parts within the transition region. Having chosen the net points, the transition region is divided into triangular elements with the net points as angular points by systematically connecting these net points, see fig. 3. In each element the Helmholtz equation (3.1) is valid. A discretization of eq. (3.1) is obtained by the application of a weighted residual technique [9] of a Bubnov–Galerkin kind [10] on the triangular net. To this end, in each triangular element both the electric field \( E(z, x) \) and the spatially dependent refractive index \( n(z, x) \) are linearly approximated. Generally in any internal net point of the transition region there results an algebraic equation in terms of the value of \( E(z, x) \) in that net point and the values of \( E(z, x) \) in the net points of the neighbouring triangular elements. Apart from the values of the coefficients, these equations are of the same types as has been obtained in ref. [5] by a finite difference method in the particular case of a regular net. The algebraic equations corresponding with net points on
the boundary of the transition region also depend on the values of the normal derivatives $\partial E/\partial n$ of some neighbouring net points on that boundary. More details are described in literature [5,10] and are not given in this paper.

Also of the following subjects the main lines of thought will be repeated, referring to, in particular ref. [5], for extended details. A discretization of the boundary integral equations of the type (3.2) representing the electric field in subregions with a constant refractive index, is obtained by the application of a boundary element approximation along the corresponding closed boundaries. Again a Galerkin method analogously to the one described in ref. [10] has been used. In these cases the mesh points on the boundaries are also preferably equidistant, see fig. 3. In each line element between two neighbouring net points both $E(z, x)$ and $\partial E/\partial n$ are linearly approximated. The discretization procedure applied to the boundary integral equations in the subregions of the core parts of the waveguides provides for each net point of the closed boundary a relation in terms of $E(z, x)$ and $\partial E/\partial n$ in all net points of that boundary.

The boundary integral equation representing the cladding part, see fig. 2, requires a particular treatment. Due to the assumptions of the Sommerfeld radiation condition and the vanishing electric field at infinity, the radiation modes can be neglected at $z=L_1$ and $z=L_2$ and at the large circle within the cladding. Hence, $E(z, x)$ and $\partial E/\partial n$ are vanishingly small at the large circle and at $z=L_1$, they can be represented by eqs. (3.8) and (3.9), respectively and at $z=L_2$ by corresponding relations. As a result the integration of the boundary integral equation in eq. (3.2) can be reduced to the integration along the interface between the the cladding on one side and the core parts of the waveguides and the transition region on the other. Substituting eqs. (3.8) and (3.9), the integration along the cladding part of $z=L_1$ can be performed. The contribution of that part transforms into an expression in which $E(z, x)$ and $\partial E/\partial n$ of the core region at $z=L_1$, occurring in eqs. (3.8) and (3.9), remain to be the unknowns. Similarly, the integral along the cladding part of $z=L_2$ can be replaced by an expression with $E(z, x)$ and $\partial E/\partial n$ of the core region of $z=L_2$ as unknowns. The application of the discretization procedure to the reduced integral equation, gives for every net point on the interface of the cladding part and the core and transition parts a relation of $E(z, x)$ and $\partial E/\partial n$ in all net points of that interface and in net points of the core regions of $z=L_1$ and $z=L_2$.

Taking into account the symmetry condition $\partial E(z, 0)/\partial x=0$ on $x=0$, the validity of eqs. (3.8) or (3.9) on the core part of $z=L_1$, and similar equations on the core part of $z=L_2$, the method of solution of the boundary value problem can be summarized as follows. The determination of the electric field of the interconnection problem has been reduced to the derivation and solution of a set of linear algebraic equations in a number of discrete net points. With each internal net point of the transition region there corresponds one equation which relates the unknown values of $E(z, x)$ in neighbouring net points. For each of the remaining net points of the interfaces between different subregions and on the core parts of $z=L_1$ and $z=L_2$, two different equations defining relations between the unknown $E(z, x)$ and $\partial E/\partial n$ of the net points, are found. The large set of linear algebraic, generally complex, equations has a locally
sparse, non-symmetric, non-singular and indefinite matrix. A useful method to solve the set of equations has been found by the application of Craigh’s variant of a conjugate gradient method [11].

5. Applications

It is the intention to illustrate the method of solution with a few examples in which several aspects come up for discussion. Besides the calculation of the reflection coefficient $R$ and the transmission coefficient $T$ for each bound mode, the reflected and the transmitted power are of interest. Referring to ref. [5], for each bound mode these last quantities relatively taken with respect to the incident power, can be expressed as

$$P_{\text{refl}} = RR^* = R_R + R_I$$
$$P_{\text{trans}} = TT^* = T_R + T_I$$, \hspace{1cm} (5.1)

the asterisks denoting the complex conjugate and the indices $R$ and $I$ indicating the real and imaginary part of the corresponding complex quantity. The radiation losses $P_{\text{rad}}$ are determined by the difference between the total incident power and the total contribution of the transmitted and reflected power of each bound mode.

An obvious test case is the calculation of the optical properties of a simple configuration with an analytically known electric field. In the case of exciting an infinite duomode waveguide of a fixed width having a core with a constant refractive index, as shown in fig. 4, the electric field in the whole configuration is equal to the sum of the two bound modes of the kind of eq. (3.4), given as

$$E(z, x) = A_1 f_1(x) \exp(-i\beta_1 z) + A_2 f_2(x) \exp(-i\beta_2 z).$$ \hspace{1cm} (5.2)

As a result of an imaginary interconnection at $z=0$, theoretically the generally complex transmission and reflection coefficients of both bound modes are equal to one and zero, respectively. The numerical procedure is applied to the example above. In order to get insight in the accuracy of the numerical results in particular also of actual, more complicated interconnection problems, the numerical configuration to be designed in the testcase will be chosen in agreement with the one as required in the case of an actual problem. The accuracy depends on the net width and the number of net points. Hence, choosing for both numerical models the shapes, the net width and the number of net points of the covering net of the same order, this accuracy may have a comparable behaviour as well. Calculations of the testcase are made for two configurations of the problem, indicated as the small configuration and the large configuration. The geometrical shapes are indicated in fig. 4. The end-planes at $z=L_1$ and $z=L_2$ are defined by the choices of $L_1/D = -17$ and $L_2/D = 17$. At these positions the radiation modes are assumed to have vanished in this kind of problems as has been discussed in ref. [5]. The transition regions differ in size. Having chosen the heights equal to $D_2 = 8D/3$ in both configurations, the lengths of the small and the large one are determined by $L_3/D = L_4/D = 1$ and $L_3/D = L_4/D = 3$, respectively.

With the choices of the refractive indices as $n_1 = 2.165$ and $n_3 = 1.25$, the width $D$ equal to $1 \mu$m, the waveguide configuration will operate as a duomode waveguide by the excitation of a light beam of

Fig. 4. The testcase model of the homogeneous duomode waveguide, showing the different sizes of the small and the large configuration.
a wavelength in vacuum of $\lambda_i = \pi \mu m$. For that particular case and the assumption that the amplitudes $A_1$ and $A_2$ of both incident bound modes are equal to one, the expression (5.2) can be specified \cite{8}. To that end, the following expressions and data are used. The transverse behaviour of the bound modes $f_j(x)$, $j=1, 2$, can be given as

$$f_j(x) = \left[ \frac{\mu_0}{\beta_j} \right]^{1/2} \cos(\kappa_j x),$$

$$0 < x < D,$$  

$$f_j(x) = \left[ \frac{\mu_0}{\beta_j} \right]^{1/2} \cos(\kappa_j D) \times \exp[-\delta_j(x-D)], \quad x > D, \quad (5.3)$$

$\mu_0$ being the magnetic permeability of vacuum. The characteristic parameters of the two bound modes read as follows

$$\beta_1 D = 4.155, \quad \kappa_1 D = 1.219, \quad \delta_1 D = 3.319, \quad (5.5)$$

$$\beta_2 D = 2.670, \quad \kappa_2 D = 3.409, \quad \delta_2 D = 0.937. \quad (5.6)$$

Choosing an equidistant covering net, calculations are performed for successively a net width of $h/D = 1/6$, $h/D = 1/9$ and occasionally $h/D = 1/12$. The results of the complex transmission coefficients for the two bound modes are plotted in figs. 5 and 6. For the small as well as for the large configuration for each mesh width considered, the results substantially differ from the theoretical values equal to one. As has been pointed out in ref. \cite{5} more accurate results are obtained using an extrapolation to $h/D=0$. As the transmission coefficient $T$ behaves quadratically on the net width $h$, this quantity can be improved to the approximation $T_0$ by

$$T_0 = \frac{(h_2^2T_2 - h_1^2T_1)}{(h_2^2 - h_1^2)}. \quad (5.7)$$

$T_1$ and $T_2$ are the inaccurate results, obtained for two different net widths $h_1$ and $h_2$, respectively. Analogously, the reflection coefficient $R$ can be improved applying an approximation of the kind of eq. (5.7) using the corresponding quantities. Looking first at the extrapolation to $h/D=0$ from the results for $h/D=1/6$ and $h/D=1/9$, the transmission coefficients converge to the theoretical values equal to one with an absolute deviation of the order of 2% for the first bound mode (fig. 5) and of 5% for the second one (fig. 6) in the case of the small configuration. In the case of the large configuration the approximations are still poor with deviations of 7% and 13%, respectively. In the last case much better results are plotted by the extrapolation from the calculations for $h/D=1/9$ and $h/D=1/12$, giving deviations of 2% and 5%, respectively. Obviously as particularly shown in fig. 6, the deviations of the final transmission coefficients are rather related to phase shifts than to moduli. The transmitted power, eq. (5.1), is proportional to the square of the moduli. Referring to the plots of some lines of constant transmitted power in figs. 5 and 6, it is found that the accuracy of the transmitted power of both bound modes are certainly within a percentage of 3% for the most favourable extrapolations.

For the second example, an interconnection configuration has been chosen in which the covering net essentially requires an unequal net width. This sec-
Fig. 7. A representative model of the tapered configuration. The design principles of the covering net are indicated.

Fig. 8. The relative power quantities of the two bound modes and the radiation losses for the various tapered configurations.
points and the net width of the covering model are of the same order. Hence, as the plotted results are assumed to have the same order of accuracy, at least the behaviour of the optical properties can be concluded from fig. 8 by varying the tapered configurations. The number of net points and the net width of the covering net are also comparable with the ones applied to the testcase given as a first example. With reference to the calculation of that testcase and considering the accuracy obtained in that configuration as representative, the accuracy of the plots of the transmitted and reflected power shown in fig. 8, are within an estimate of about 0.03. This order of estimate has been confirmed by numerical experiments on different complicated configurations (not given in this paper), using the results of the calculations of refined values of the net width.

6. Additional remarks and conclusions

This paper treats a numerical method to determine the optical propagation properties arising from the interconnection of two semi-infinite dielectric multimode waveguides. To this end the authors describe the interconnection problem by an appropriate boundary value problem. In the literature [1–3], it is usual to formulate the boundary value problem somewhat differently in some parts. The possible advantages or disadvantages of the two versions are discussed in ref. [6] and are not repeated in this paper.

The boundary value problem is discretized using both finite element and boundary element methods. For the sake of easy programming in each finite element the electric field is linearly approximated. Consequently, a large number of net points is required for accurate calculations. Possibly as has been done in the literature [2,3], results of the same accuracy may be obtained with a smaller number of net points by using higher order finite element approximations. The last types of approximation are not considered. Based upon calculations on the same interconnection configuration with different values of the net width of the covering net, the accuracy of the optical propagating properties can successfully be improved by the application of the technique of extrapolation to a net width zero. Through the application of the numerical method to a configuration with analytically known properties and the application to a more complicated one, estimates of the accuracy obtained for the various optical quantities are discussed.

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