A void perspective of the cosmic web
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We present a new concept, the Spine of the Cosmic Web, for the topological analysis of the Cosmic Web and the identification of its walls, filaments and cluster nodes. Based on the watershed segmentation of the cosmic density field, the SpineWeb method invokes the local adjacency properties of the boundaries between the watershed basins to trace the critical points in the density field and the separatrices defined by them. The separatrices are classified into walls and the spine, the network of filaments and nodes in the matter distribution. Testing the method with a heuristic Voronoi model yields outstanding results. Following the discussion of the test results, we apply the Cosmic Spine method to a set of cosmological N-body simulations. The latter illustrates the potential for studying the structure and dynamics of the Cosmic Web.

4.1 Introduction

The large scale distribution of matter revealed by galaxy surveys features a complex network of interconnected filamentary galaxy associations. This network, which has become known as the Cosmic Web (Bond et al. 1996), contains structures from a few megaparsecs up to tens and even hundreds of Megaparsecs of size. The weblike spatial arrangement of galaxies and mass into elongated filaments, sheetlike walls and dense compact clusters, the existence of large near-empty void regions and the hierarchical nature of this mass distribution – marked by substructure over a wide range of scales and densities – are its three major characteristics. Its appearance has been most dramatically illustrated by the recently produced maps of the nearby cosmos, the 2dFGRS, the SDSS and the 2MASS redshift surveys (e.g. Colless et al. 2003; Tegmark et al. 2004; Huchra et al. 2005).

In this paper we introduce the SpineWeb formalism for analysing the structure and topology of the Cosmic Web. It identifies the sheets and filaments in the Cosmic Web, along with the large underdense void regions, and their mutual connection into the Spine of the cosmic matter distribution. The method is based on the Watershed Transform (WST, Beucher & Meyer 1993), and is largely free of user-specific parameters and artificial smoothing scale(s). Its output will enable the study of the physical properties and dynamics of the individual morphological components, along with their topology and hierarchical characteristics.

4.1.1 the Cosmic Web

The Cosmic Web is the most salient manifestation of the anisotropic nature of gravitational collapse, the motor behind the formation of structure in the cosmos (Peebles 1980). N-body
computer simulations have profusely illustrated how a primordial field of tiny Gaussian density perturbations transforms into a pronounced and intricate filigree of filamentary features, dented by dense compact clumps at the nodes of the network (Colberg et al. 2005; Springel et al. 2005; Dolag et al. 2006). The filaments connect into the cluster nodes and act as the transport channels along which matter flows into the clusters.

Fundamental understanding of anisotropic collapse on cosmological scales came with the seminal study by Zel’Dovich (1970), who recognised the key role of the large scale tidal force field in shaping the Cosmic Web (also see Icke 1973). The collapse of a primordial cloud (dark) matter passes through successive stages, first assuming a flattened sheetlike configuration as it collapses along its shortest axis. This is followed by a rapid evolution towards an elongated filament as the intermediate axis collapses and, if collapse continues along the longest axis, may ultimately produce a dense, compact and virialized cluster or halo. The hierarchical setting of these processes, occurring simultaneously over a wide range of scales and modulated by the expansion of the Universe, complicates the picture considerably. Recent state-of-the-art computer experiments like the Millennium simulation (Springel et al. 2005) clearly show the hierarchical nature in which not only the clusters build up but also the filamentary network itself (see Aragón-Calvo 2007).

The Cosmic Web theory of Bond et al. (1996) succeeded in synthesising all relevant aspects into a coherent dynamical and evolutionary framework. Instrumental is the realisation that the outline of the cosmic web may already be recognised in the primordial density field. The statistics of the primordial tidal field explains why the large scale universe looks predominantly filamentary and why in overdense regions sheetlike membranes are only marginal features. Of key importance is the realisation that the rare high peaks which will eventually emerge as clusters are the dominant agents for generating the large scale tidal force field: it is the clusters which weave the cosmic tapestry of filaments (Bond et al. 1996; Van de Weygaert & Bertschinger 1996; van de Weygaert & Bond 2008). They cement the structural relations between the components of the Cosmic Web and themselves form the junctions at which filaments tie up. This relates the strength and prominence of the filamentary bridges to the proximity, mass, shape and mutual orientation of the generating cluster peaks: the strongest bridges are those between the richest clusters that stand closely together and point into each others direction. Through the direct relation between the Cosmic Web and the primordial tidal field one may understand why the large scale universe looks predominantly filamentary and why in overdense regions sheetlike membranes are only marginal features (Pogosyan et al. 1998).

The emerging picture is one of a primordially and hierarchically defined network whose weblike topology is imprinted over a wide spectrum of scales. Weblike patterns on ever larger scales get to dominate the density field as cosmic evolution proceeds, and as small scale structures merge into larger ones. Within the gradually emptying void regions, however, the topological outline of the early weblike patterns remains largely visible.

4.1.2 Closing in on the Cosmic Web

Despite a large variety of attempts, as yet no generally accepted descriptive framework has emerged for the objective and quantitative analysis of the Cosmic Web. Despite the multitude of elaborate qualitative descriptions it has remained a major challenge to characterise its structure, geometry and topology. The overwhelming complexity of both the individual structures as well as their connectivity, the lack of structural symmetries, its intrinsic multiscale nature and the wide range of densities that one finds in the cosmic matter distribution has prevented the use of simple and straightforward toolboxes.

Historically, the quantitative analysis of the Cosmic Web has been dominated by a description in terms of statistical measures of clustering of galaxies and matter. While correlation
functions have been the mainstay of the cosmological analysis of large scale structure, a direct interpretation in terms of the patterns and texture of the Cosmic web has largely remained elusive. Over the years a variety of heuristic measures have been forwarded to analyse specific aspects of the spatial patterns in the large scale Universe, but only in recent years there have been attempts towards developing solid and complete descriptors of the intricate spatial
patterns that define the Cosmic Web. Nearly without exception these methods borrow extensively from other branches of science such as image processing, mathematical morphology, computational geometry and medical imaging.

Noteworthy examples include filament detection with the help of the Candy model (Stoica et al. 2005) and wavelet analysis of the Cosmic Web (Martínez et al. 2005). Several methods seek to relate morphological features to singularities in the density field, usually invoking information on the gradient and Hessian of the density field, or of the tidal field, see Sousbie et al. (2008); Aragón-Calvo et al. (2007); Aragón-Calvo (2007); Hahn et al. (2007); Bond et al. (2009); Hahn et al. (2007). An elaborate classification scheme on the basis of the manifolds in the tidal field – involving all morphological features in the cosmic matter distribution – has been forwarded by Hahn et al. (2007); Forero-Romero et al. (2009); Hahn et al. (2007). It has the great virtue of referring to the structure of the tidal field, which links it directly to our theoretical understanding of the formation and shaping of the Cosmic Web. However, its success may depend strongly on the correct choice of the smoothing scale. Another concept addressing the gradient and Hessian of the density field is that of the skeleton analysis, see Novikov et al. (2006); Sousbie et al. (2008); Sousbie et al. (2008), based on Morse theory (see Colombi et al. 2000; Pogosyan et al. 2009). It formed the basis for the development of an elegant and mathematically rigorous tool for filament identification, which in the meantime has also been extended towards tracing a range of morphological features (Sousbie et al. 2009). Its present implementation refers to only one specific scale. The multiscale nature of the cosmic matter distribution is explicitly addressed by the Multiscale Morphology Filter, which is based on a multiscale analysis of the Hessian of the density field Aragón-Calvo et al. (2007); Aragón-Calvo (2007) to identify cluster, filaments and sheets on a scale where they are locally most prominent.

4.1.3 Watershed and Cosmic Spine

One technique that implicitly addresses the topological structure of the Cosmic Web is an application of the Watershed Transform. The Watershed Void Finder (Chapter 2) detects and traces the outline of voids in the matter distribution. The watershed transform is a key instrument for the segmentation of a density field, and as such is also ideally suited for tracing the boundaries between the identified segments.

With this in mind we have embedded the watershed transform into a wider context as a framework for studying both identity and topology of the cosmic web and its various constituents. The result is the SpineWeb method, a complete framework for the identification of voids, walls and filaments. Via the practical role of the watershed transform in computing the Morse complex it is intimately related to Morse theory, in which it finds a solid mathematical foundation.

An important aspect of our method is that it is an intrinsically scale-free method, starting from a scale-free reconstruction of the density field. We use the DTFE method of Schaap & van de Weygaert (2000), which guarantees an optimal and unbiased representation of the hierarchical nature and anisotropic morphology of cosmic structure (see van de Weygaert & Schaap 2007, for an extensive description). Having guaranteed the capability of a scale-free representation of cosmic structures, our watershed procedure not only traces the outline of filaments and sheets, but may also be extended towards doing so over a range of scales, i.e. in Scale-space, in order to address their hierarchical structure.

In this paper we will focus specifically on the description of the basic SpineWeb formalism, confined to a density field sampled on a regular grid. In a follow-up paper, this will be followed up by a more elaborate implementation in which we extend the method towards the analysis of the multiscale nature of the cosmic web. In the third step of this program the formalism will be evaluated directly on the Delaunay tessellation of the generating particle or
Watershed Segmentation of the Cosmic Web

4.2 Watershed Segmentation of the Cosmic Web

When studying the topological and morphological structure of the cosmic matter distribution in the Cosmic Web, it is convenient to draw the analogy with a landscape (see fig. 4.1, top row). Valleys represent the large underdense voids that define the cells of the Cosmic Web. Their boundaries are sheets and ridges, defining the network of walls, filaments and clusters that defines the Cosmic Web (cf. top panels fig. 4.1).

4.2.1 Watershed Transform

A common used method in Image Analysis is the Watershed Transform (WST). It is a concept defined within the context of mathematical morphology, and was first introduced by Beucher & Lantuejoul (1979). It is widely used for segmenting images into distinct patches and features. The basic idea behind the WST stems from geophysics, where it is used to delineate the boundaries of separate domains, i.e. basins into which yields of e.g. rainfall will collect.

The word *watershed* finds its origin in the analogy of the procedure with that of a landscape being flooded by a rising level of water. Suppose we have a surface in the shape of a landscape (cf. top right panel, fig. 4.1). The surface is pierced at the location of each of the minima. As the water-level rises a growing fraction of the landscape will be flooded by the water in the expanding basins. Ultimately basins will meet at the ridges defined by saddle-points and maxima in the density field. The final result of the completely immersed landscape is a division of the landscape into individual cells, separated by the ridge dams (see left bottom panel fig. 4.1).

4.2.2 A watershed search for voids

The watershed transform was first introduced in a cosmological context as an objective technique to identify and outline voids in the cosmic matter and galaxy distribution (see Chapter 2). Following the density field-landscape analogy, the Watershed Void Finder (WVF) method identifies the underdense void patches in the cosmic matter distribution with the watershed basins. The method is parameter free in case there is no noise in the data.

A major advantage of the WVF method is its independence of assumptions on the shape and size of voids (see the Introduction and for further analysis see Colberg et al. 2008). Sharing this virtue with a similar tessellation-based void finding method, ZOBOV (Neyrinck 2008) and WVF are particularly suited for the analysis of the hierarchical void distribution expected in the commonly accepted cosmological scenarios. A particular illustrative example of its use concerns the study of the mutual alignment of voids on scales up to $\approx 30h^{-1}$Mpc in a $\Lambda$CDM dark matter N-body simulations (see Chapter 3, Platen et al. 2008). Enabled by the ability to extract the shape and orientation of voids, the study succeeded in demonstrating the decisive influence of the large scale tidal field in defining their large scale configuration.

4.2.3 Morse theory

Extrapolating its application to other areas of interest, the implementation of the watershed transform may also be seen as a practical instrument for the segmentation of surfaces and volumes on the basis of the topological structure of the “landscape” $F(x)$. This can be directly appreciated from the fact that the watershed transform segments the landscape $F$ into regions...
of uniform local gradient behaviour: the watershed basin \( j \) consists of the collection of points \( x \) that are closer in topographic distance \( T(x, y_j)\),

\[
T(x, y_j) \equiv \inf \int_{\gamma} |\nabla F(\gamma(s))| ds .
\]

(4.1)

to the defining minimum \( y_j \) of the basin than to any of the other minima. The watershed itself consists of the ridge lines that delimit the boundaries between basins in the terrain. In this
definition the integral is the pathlength along the integral line, the line along whose path the
tangent at each point is parallel to the local gradient \( \nabla F \).

The vast majority of applications of the watershed transform concern the interior of the segmented regions. However, it is straightforward to extend its focus to other morphological components of the Cosmic Web, towards the delineation of the network of overdense ridges and walls which form the boundary manifolds of the cosmic density landscape. This can be directly appreciated by noting the close relation between the definition of the watershed transform and the more formal concept of the Morse complex (of \( \partial F \), see subsect. 4.2.3). Even though there are some differences between the two (see e.g. Gyulassy 2008), the close similarity indicates that the computation of the watershed transform may be used as an efficient means of computing the various structural elements in a landscape dissected along the lines of Morse theory (Morse 1996; Milnor et al. 1963). Morse theory is the mathematical framework for the analysis of the topological structure of manifolds, by relating it to smooth, \( C^2 \)-differentiable, functions defined on those spaces. The location and nature of the critical points – minima, maxima and saddle points – and their mutual connection via the gradient-based integral lines define the morphological features of the functional surface.

**Gradient Field and Integral Lines**

The gradient \( \nabla f \) is a direct way of tracing the topological structure of a field \( f(x) \). The gradient delineates a smooth vector field, which vanishes at critical points,

\[
\nabla f(x_k) = 0 ,
\]

(4.2)

Important for the context of this paper is that cosmic density fields in general behave like a proper Morse function \( f(x) \). These are functions whose critical points \( x_k \) are isolated and non-degenerate, determined by the non-singularity of the Hessian matrix at their location,

\[
\left| \frac{\partial^2 f}{\partial x_i \partial x_j} \right| \neq 0 .
\]

(4.3)

The integral lines or slope lines represent the flow along the gradient field \( \nabla f \) between the critical points. On the basis of these connections one may infer a variety of spatial segmentations, see Cayley (1859); Maxwell (1870); Eberly et al. (1994); Furst & Pizer (2002); Edelsbrunner et al. (2003); Edelsbrunner et al. (2003); Danovaro et al. (2003); Gyulassy et al. (2005).

An illustration of the close link between the gradient field and structural features in the Universe is offered by the right-hand panel of fig. 4.2. The image shows that the integral lines that define the boundaries of adjacent valleys are in fact the watersheds. It also reveals the intimate relationship between the critical points in the flow field and the nodes, filaments and voids in the landscape: maxima are found at nodes of the weblike network of watershed ridges, minima at the centres of the void cells, while saddle points are to be found at key locations along the ridges. Following this view, we see that the watershed lines are the set of slope lines emanating from saddle points and connecting to a local maximum or minimum.
Within this framework, saddle points have the crucial function of defining the sheets and filaments in the density field through their connection to the maxima via the integral lines.

Note that because the image in fig. 4.2 is a slice through a three-dimensional field, the identification between the structural elements and the critical points in the image is not entirely unequivocal (see below). Nonetheless, the principal observation is that the resulting weblike segmentation of space, and the corresponding boundary manifolds, contain the full information on its topological structure marked by sheets, filaments and nodes.

Figure 4.2– Left: A slice of the density field shown as a shaded landscape with the watershed lines superimposed as black lines. Right: The zoomed area in the blue square of the left panel showing the slope lines (white lines) superimposed on the density field (grey background). The contour of the watershed transform is delineated by the thick black lines.

Saddle Points and Structural Features
While the definition of minima and maxima is universal, the same for any dimension \( d \) of a manifold, the identity of saddle points is more complex in higher-dimensional spaces. In 2-dimensional space there is only one class of saddle point, while in 3-dimensional space we have to distinguish between two different classes.

In two-dimensional space, the saddles are the key points in the ridges surrounding watershed segment: the ridges are the separatrices between two segments, defined by the connection of the saddle and the maxima. To appreciate the more complex three-dimensional structure, we may first note that in terms of the eigenvalues of the Hessian, minima and maxima are critical points with a \((+ + +)\) (minimum) and \((- - -)\) (maximum) signature. We may then distinguish two classes of saddle points: the ones with a \((+ - -)\) signature and the ones with a \((+ + -)\) signature. In turn, this is responsible for a richer topological structure, in which the different saddles and their mutual connections define sheets or ridges. The \((+ + -)\) saddles are central to the definition of a sheet in the density field, while \((+ - -)\) saddles will be instrumental to the definition of the ridges that straddle the sheets.

In our study we concentrate specifically on the ascending manifolds surrounding the minima, ie. the set of points belong to integral lines whose origin is a minimum of \( f(x) \). The corresponding descending manifolds correspond to the inflow regions surrounding the maxima.
in the field. The complex of all descending manifolds of \( f \) is its Morse complex. The watershed segmentation relates to the Morse complex of \(-f\), the complex of all ascending manifolds of \( f \).

4.2.4 the Cosmic Spine

The Cosmic Web is an interconnected system of dense compact clusters, elongated filaments and tenuous sheetlike walls. Visible through the galaxies, gas and dark matter populating these structural features, the Cosmic Web theory (Bond et al. 1996) teaches us that its topological outline was already present in the primordial perturbation field out of which all structure arose.

A crucial observation, which can be most readily appreciated by studying high resolution N-body simulations (e.g. Springel et al. 2005), is that all elements of the Cosmic Web are interconnected. Otherwise seemingly isolated objects usually turn out to be connected to less massive features which become visible when assessing the mass distribution at a higher resolution. The most illustrative representatives of such objects are galaxies in voids, lined up along low-density dark matter filaments (see e.g. Zitrin & Brosch 2008; Park & Lee 2009; Stanonik et al. 2009).

Filaments are suspended between clusters or, dependent on scale, massive halo clumps. Their prominence and density may vary substantially, dependent on the mass, distance and alignment of the generating dark matter haloes. However, the sheer presence of two matter clumps is already sufficient for the corresponding tidal force field to guarantee the topological presence of a filamentary bridge. Tenuous membranes permeate the space between adjacent filaments, and are part of the large wall which defines the boundary between two underdense voids. The wall boundary is outlined by various filaments, connecting each other at the cluster nodes.

The analogy between the watershed transform defining the boundary between underdense basins and the topology of the cosmic matter distribution, described above, is in itself one of the major justifications of the SpineWeb method presented in this study.

In fact, the mathematical foundation within the context of Morse Theory has already found application in the skeleton formalism. Following the application of Morse theory in the analysis of the CMB by Colombi et al. (2000), the skeleton formalism has been gradually developed for the morphological analysis of the Megaparsec Cosmic Web, in redshift surveys like SDSS as well as in N-body simulations, in a sequence of studies (Novikov et al. 2006; Sousbie et al. 2008, 2009). While our watershed-based formalism ultimately has the same mathematical basis, its computational ease and efficiency in combination with the intrinsically scale-free DTFE density field underlying the watershed procedure also paves the path towards an extension of the SpineWeb algorithm that will involve the intrinsic hierarchical nature of the components of the Cosmic Web (see Aragón-Calvo et al. 2009).

4.3 The SpineWeb Procedure

The watershed transform, i.e. the ridges and sheets surrounding the watershed basins, represents a subset of all the critical points in the density field. The SpineWeb procedure exploits the intrinsic topological information contained in the watershed transform to delineate the Cosmic Spine, the topological network of nodes, filaments and sheets along which the cosmic matter distribution on large Megaparsec scales has assembled (see fig. 4.1). In other words, the watershed transform defines the Cosmic Spine.

For the computation of the Cosmic Spine by means of the Watershed Transform we need to consider a few issues of practical importance.
4.3.1 The Discrete Watershed Transform

The implementation of the watershed transform in a large variety of scientific applications has to address a few important practical issues. A typical characteristic of most scientific images is their discrete nature. The discreteness concerns two aspects: the spatial discreteness, i.e., the discrete number of intervals at which the image has been sampled (pixels/voxels), and the discrete intensity levels at which the image has been sampled.

Image discreteness creates a few complications for an accurate calculation of the watershed transform. It renders it difficult to identify the existence and exact location of saddle points on the basis of a discretized local neighbourhood. For the same reason, it is difficult to accurately extract slope lines. On the other hand, the discretization also involves some advantageous aspects. A major asset of the discretization is that it helps to remove faint features, and therefore also removes artifacts without the need of pre- or postprocessing. Perhaps the greatest advantage is that discrete images allow the use of highly efficient algorithms.

Several methods for the extraction of critical points have been developed in an attempt to alleviate the limitations imposed by the discreteness of images. Among these, the discrete watershed transform algorithm (Beucher & Meyer 1993) represents a simple and elegant formalism for identifying the watershed separatrices. Important is the demonstration by Najman & Schmitt (1994) that it converges to the continuous case. The procedure emulates the flooding of valleys or catchment basins in a (discrete) image representing a landscape. The points where two or more lakes converge are marked, and the algorithm continues until all the pixels in the image have been flooded. At the end of the process the image will be segmented into individual regions sharing a local minima, with the points that were marked as the dividing boundaries between two or more valleys defining the watershed transform.

In the case of images with continuous (floating point) values one retains the option of computing the watershed transform directly from the continuous intensity image, in addition to the option of discretising the intensity. On the basis of the continuous image, the watershed transform would delineate the topology more accurately than would be feasible on the basis of the the discrete-level representation.

4.3.2 Watershed Implementation

Our discrete watershed transform code is an implementation and adaptation of the immersion and topographical distance algorithms for floating point intensity values (see Roerdink & Meijster 2000, for a review). The code assumes a density map which is sampled on a regular grid. Our C code* computes the watershed transform from a double-precision $512^3$ grid in just a couple of minutes.

In a first step, the code starts by finding and labelling the local minima in the density map, by identifying the voxels with the lowest density value among all their 26 neighbours. These local minima are the seeds of the void valleys to be identified by the watershed transform.

In a second step, we follow the topographical distance algorithm in order to obtain a fast segmentation of the space into locally connected underdense regions. For each voxel we identify the voxel among its 26 neighbours which has the lowest density. The maximum gradient paths are traced by iteratively connecting the voxels to their lowest density neighbour until the path reaches a local minimum. Subsequently, we assign the label of the corresponding minimum to the path.

In the third step we extract the watershed transform itself, i.e., the boundaries between the void regions. The pixels in the watershed transform itself are identified by means of a local immersion algorithm. First, we identify all the voxels that lie at the boundaries between

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*The code will be publicly released in an upcoming article. In the meantime it can be provided upon request.
two or more regions. Subsequently, this subset of voxels is sorted in density and the standard immersion algorithm is applied. By following this two-step procedure, we avoid what would be the most expensive component of the algorithm. Instead of having to sort the complete density field, the sorting evaluations are restricted to the points in the watershed boundaries, a minor fraction of the complete volume.

In a final step, each of the pixels in the watershed boundary is assigned a morphological label, following its identification as void, wall or filament element, according to the criterion expressed in equation 4.4 and as illustrated in fig. 4.3.

### 4.3.3 from Watershed to Spineweb

From the analogy between the Cosmic Web and the watershed transform one can define, on the basis of the discrete watershed transform of a cosmic density field, a set of simple but effective criteria to identify voids, walls and filaments.

The criteria are based on the properties of the local neighbourhood of all the points that comprise the discrete watershed transform. Instead of computing at any given point the local eigenvalues of the Hessian of the density field, one may simply resort to the entirely equivalent evaluation of the identity of the surrounding 26 neighbour pixels (for the three-dimensional situation). By counting the number of adjacent watershed basins (voids) amongst these, it is straightforward to discriminate between voxels which belong to a void, a wall or a filament by means of the following set of rules:

\[
N_{\text{voids}} = \begin{cases} 
1, & \text{void} \\
2, & \text{wall} \\
3, & \text{spine} \\
\geq 3, & \text{filament + node}
\end{cases}
\]  

(4.4)

Note that in the present implementation we do not discriminate between filament or cluster node, and instead consider them to be part of the same spinal structure. In a future implementation we will include a density maximum criterion which would allow us to find the cluster nodes amongst the spine voxels.

The above criterion is a purely and solely a topological one. By definition, walls are the regions between two adjacent voids. Filaments are to be found at the confluence of three watershed basins, at the intersection of 3 walls. The success of these criteria can be appreciated from the 3-D surface maps in fig. 4.9 and the comparison between density and spine maps in fig. 4.8.

### Resolution Artifacts

A detailed inspection of the structure of a Cosmic Spine realisation reveals a variety of tenuous and small filamentary features which have not been identified as a spinal filament. In
most cases these are to be found within the interior area of spinal walls. These features are real, and they are only missed by the spineweb procedure because of the limited resolution. As a result, the discrete density grid lacks sufficient resolution for the representation of the necessary topological details.

In the second study of this series (Aragón-Calvo et al. 2009), we will describe the extension of the spine formalism into a genuine multiscale formalism. With respect to the density field, this is not unlike the scale-space approach followed by the Multiscale Morphology Filter technique (Aragón-Calvo 2007).

The first multiscale spineweb results do demonstrate that seemingly unrelated substructures do show up in the spine of higher resolution realisations of the density field. This is of course a direct reflection of the presence of the cosmic web’s topological outline in the primordial density field, and assures the fact that filaments always lie at the intersection of walls and form the edges of underdense void cells.

4.3.4 Image Grid Representation

While a regular grid facilitates the computation of the watershed transform and the subsequent topological identification of the various boundary pixels (see fig. 4.3), its simplicity may also involve a few possibly worrisome artifacts.

On an orthogonal regular grid the neighbourhood of each voxel consist of 26 voxels. While more or less fulfilling the requirement of uniformity, a regular gridlike neighbourhood does not constitute a proper isotropic sampling of the density field. Each voxel has 6 neighbouring voxels at a distance $d = 1$ (in voxel units), 16 neighbours at $d = \sqrt{2}$ and 8 neighbours at $d = \sqrt{3}$. It also involves an angular neighbour distribution deviating substantially from uniformity.

A possible alternative would be to limit the neighbourhood evaluation to the 6 most direct
neighbours. However, the poor angular sampling might lead to a considerable risk of missing important topological information. For two-dimensional images the solution would be more straightforward. The use of a hexagonal grid would involve equal distance for all neighbour pairs and a perfectly uniform angular distribution. Unfortunately, an equivalent perfect grid for the three-dimensional situation does not exist. However, the use of Centroidal Voronoi Tessellations (CVT, Du et al. (1999) see further Chapter 5) would certainly help to alleviate the main artifacts.

Extrapolating this line of reasoning, one may foresee that a procedure which does not involve a representation on a regular grid would be a more natural choice. In a forthcoming paper, we will present the extension of our formalism to that of a watershed procedure directly defined on a linear interpolation density field representation on an (irregular) Delaunay triangulation (see e.g. Schaap 2007; van de Weygaert & Schaap 2007). The feasibility and virtues of such an approach in the context of Morse theory were first pointed out by Edelsbrunner et al. (2003), and spurred considerable activity in improving and extending this to numerous issues of interest in computational topology (Edelsbrunner et al. 2003; Danovaro et al. 2003; Gyulassy et al. 2005; Magillo et al. 2007).

4.4 Voronoi Clustering models

In order to test and quantify in an objective way the identification of walls and filaments with our method we applied it to an heuristic model of the large scale matter distribution. This model creates a cellular distribution of matter based on the Voronoi tessellation of a set of seeds defining the centres of Voronoi cells (Van de Weygaert & Icke 1989; van de Weygaert 2002). The Voronoi clustering model confines a point in space to one of the distinct structural components of a Voronoi tessellation:

- **Void**: regions located in the interior of Voronoi cells.
- **Wall**: regions within and around the Voronoi cell faces.
- **Filament**: regions within and around the Voronoi cell edges.
- **Clusters**: regions within and around the Voronoi cell vertices.

We generated two Voronoi point distributions. One in which all points lie exactly at the edges and faces of the Voronoi cells (clean model) and another distribution in which each point has a small Gaussian dispersion around its parent Voronoi edge or cell (noisy model). We will discuss first the clean model and subsequently the noisy one.

4.4.1 Clean Voronoi clustering model

In order to keep our test as simple and clear as possible we constructed a density field directly from the void seeds defining the Voronoi distribution instead of performing a full density field reconstruction from the point distribution. We use the seed points to compute the distance field of the corresponding Voronoi tessellation. The distance field is defined as the euclidean distance from each point in the field to its closest Voronoi seed. Regions close to the cell centres have low values while regions in the planes and edges of the cell have large values. The definition of the Voronoi Tessellation ensures that the value of the density field converges
at the boundaries of the Voronoi cells. It is possible to normalise the distance field by dividing the distance to the closest Voronoi seed by the distance to the second closest Voronoi seed. This field gives a value of 1 at the walls of the Voronoi cell. We opted to use the distance field with no normalisation since it gives a better representation of the large dynamical range of densities characteristic of Cosmic Web. With this approach small cells will be less dense than larger cells emulating the range of densities in the cosmic voids.

In practice we use the inverse of the distance field. This is maximum along the planes and edges of the Voronoi cell and it decreases in the direction of the Voronoi cell centres. This distance field is topologically equivalent to any generic field whose value is a monotonically decreasing function of the distance to the watershed transform, akin to the density field in the cosmic web.

It is important to note that in order to test the validity of the SpineWeb method we only need a density field that is topologically equivalent to the Cosmic Web. The details in the determination of a density field from observed data or computer simulations are irrelevant to the test of our main idea, i.e. the identification of morphological structures based on their topology.

The distance field was computed by first defining a set of random points as Voronoi seeds. Next we created a cubic grid with 512 pixels per dimension and assigned to each pixel its closest Euclidean distance from the set of nucleus defining the Voronoi cells (see fig. 4.4).

4.4.2 Spine identification

We start by smoothing the distance field with a Gaussian function of $\sigma = 2$ voxels in order to remove small-scale spurious variations. From the smoothed distance field we proceed to compute the watershed transform and subsequently the Spine. Figure 4.5 shows the result of our method applied to the Voronoi distribution. Visual inspection gives a good initial indication of the good performance of the method. Both the walls and filaments are identified and extracted with remarkable accuracy. The recovered distribution of particles in walls (panel e) and filaments panel (d) looks extremely similar to the original distributions (panels a and b). In fact, even the particles erroneously identified as filaments (panel f) closely delineate the original filaments, indicating that this is an artifact resulted from the discrete resolution of the grid. The reconstructed spine (panel c) gives an outstanding impression of the topological nature of the SpineWeb method. Walls form a watertight network of intersecting planes with filaments running across those intersections. This idealised example based on a Voronoi model illustrates the power of our method in identifying structures solely on the basis of the topology of the density field. No density threshold or other free parameter is required for the morphological segmentation in this noise-free application.

The discrete nature of the voxels in the density field gives an artificial thickness to filaments and walls making them look pixelated or jagged (see fig. 4.5, panel c). It may occur that a particle inside a filament is not identified simply as a result of the jagged nature of the voxels. In the limit of an infinitesimal thin voxel the discrete watershed converges to the continuous case. Since filaments and walls are not infinitesimally thin structures one must define a thickness to them in order to identify the particles the enclose. We account for this by applying the dilation morphological operator to the voxels labelled as filament and wall. The process increases the thickness of filaments and walls by one voxel and this procedure can be performed iteratively to further increase the thickness. In our particular case a single iteration with a $3 \times 3 \times 3$ kernel provides a good result without excessively fattening the structures. The dilation operator was applied first to voxels labelled as wall and subsequently to pixels labelled as filaments following the number of degrees of freedom in the local variation of the density field, i.e. first walls and subsequently filaments (Aragón-Calvo et al. 2007).
Figure 4.5— The SpineWeb method applied to a Voronoi distribution. a) original particles lying at the edges of the Voronoi cells (filaments). b) original particles lying at the faces of the Voronoi cells (walls). c) Pixels inside filaments (red) and walls (blue) identified with the SpineWeb method. d) recovered particles lying at the edges of the Voronoi cells (filaments). e) recovered particles lying at the faces of the Voronoi cells (walls). f) particles erroneously identified as particles in filaments. The box shown here contains 1/64 of the original box volume.

Table 4.1— Recovered particles per morphology

<table>
<thead>
<tr>
<th>Structure</th>
<th>$R_{\text{real}}$</th>
<th>$R_{\text{false}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walls_DIS</td>
<td>0.93</td>
<td>0.15</td>
</tr>
<tr>
<td>Spine_DIS</td>
<td>0.91</td>
<td>0.03</td>
</tr>
<tr>
<td>Walls_DEN</td>
<td>0.76</td>
<td>0.32</td>
</tr>
<tr>
<td>Spine_DEN</td>
<td>0.87</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 4.2— Ratio of real and false recovery rates per morphology. The top half corresponds to the results from the distance field (dis), while the bottom half concerns the results for the DTFE reconstructed density field (den). For definition of $R_{\text{real}}$ and $R_{\text{false}}$ see eqn. 4.5 and 4.6

4.4.3 Real vs. false detections

We measured the success of our method in identifying and recovering points located inside filaments and walls by computing the ratio of real vs. false detections as follows:
where $N_{\text{real}}$ is the number of original particles labelled as wall or filament located inside reconstructed walls or filaments and $N_{\text{original}}$ is the number of particles originally labelled a wall or filament in the Voronoi model. In a similar way we measure the contamination rate as follows:

$$R_{\text{false}} = \frac{N_{\text{false}}}{N_{\text{original}}}$$

where $N_{\text{false}}$ is the number of particles identified in walls of filaments but having a different morphological label. Our method is able to recover 93% of particles in walls and 91% of particles in filaments with a 15% and 3% of contamination respectively.

Figure 4.6– Volume rendering of a thick slice of the density field reconstructed from the particle distribution (left panel) and the density field inside an isosurface at $\delta = 1$.

### 4.4.4 Voronoi Clustering models with density reconstruction

We performed the same test as in the previous section but this time we used the density field instead of the distance field. The density field was reconstructed from a point distribution using the DTFE method (Schaap & van de Weygaert 2000; Schap 2007; van de Weygaert & Schaap 2007). We used a slightly different Voronoi model that contains particles inside voids, walls and filaments. The point distribution inside each morphology is randomly placed according to a user defined Gaussian dispersion of $\sigma = 1.0h^{-1}\text{Mpc}$. This pseudo-random Voronoi point distribution emulates the density field in the Cosmic Web more closely than the Voronoi models used to construct the distance field in the previous section (Although the topological properties of both models are equally valid as a model of the Cosmic Web).

The particles that lie within a radius of $2\sigma$ from a wall, filaments of the Voronoi tessellation are given their corresponding wall and filament classification. We compare this correct classification with classification from the reconstructed watershed segmentation (using the same $2\sigma$ distance). Like the Voronoi classification a wall-particle is identified when within a distance of
2σ there are two watershed cells. Similarly using equation 1 we classify the void and filament particles according to the number of cells in the neighbourhood.

Figure 4.4 shows the reconstructed density field from the Voronoi point distribution as well as the watershed transform and the particles found in filaments and walls. The watershed transform is able to identify most of the voids and their original boundaries. The identified structures closely follow the general location of the original Voronoi edges and faces, corresponding to filaments and walls respectively. However, at small scales some deviations can be appreciated. This is the result of Poisson noise from the point distribution and the discrete spatial nature of the sampled density field. Since our filaments and walls are one-voxel thick by construction it may happen that we miss the particles inside a given structure because of small-scale variations in the density field that translate into variations in the watershed transform. This effect can be seen in the misclassified particles (blue) in Figure 4.4. They are clustered around the boundaries between walls and filaments. This can happen even though we are able to reconstruct the general location of the filament. Particles in walls are identified to 76% with 32% of contamination while we are able to identify 87% of particles inside filaments with 13% of misclassifications. Our results are summarized in the lower half of table 4.1.

4.5 Single-scale ΛCDM Spine

In order to test the SpineWeb method in a more realistic and challenging situation we applied it to a cosmological N-body simulation. It concerns a ΛCDM universe inside a box of 200 h⁻¹ Mpc, restricted to the dark matter particles. Initial conditions were generated on a 512³ grid with Ωₘ₀ = 0.3, Ωₐ₀ = 0.7, σₖ₀ = 0.9 and h = 0.73. We used the transfer function of Bardeen et al. (1986), with the shape parameter defined by Sugiyama (1995).

Lower-resolution versions of 256³, 128³ and 64³ particles were generated from the same initial conditions, following the “averaging” prescription described in (Klypin et al. 2001). For the single-scale analysis in this paper, which focuses on the largest filaments and walls in the particle distribution, it is sufficient to analyse the low-resolution 64³ dataset. This lower resolution corresponds to a cut-off scale of roughly 3 h⁻¹Mpc in the initial conditions, which makes it suitable for large scale structure analysis. The higher resolution datasets are used for visualisation purposes. After having set up the initial conditions, we follow the subsequent gravitational evolution to the present time using the public N-body code Gadget2 (Springel et al. 2005).

4.5.1 Density field estimation

From the final particle distribution we compute the density field inside a cubic grid of 512 voxels per dimension using a recent implementation of the DTFE method (Schaap & van de Weygaert 2000; Schaap 2007; van de Weygaert & Schaap 2007). This particular implementation is based on the publicly available CGAL library⁣[⁣ which makes it highly efficient and fast. The DTFE procedure produces a self-adaptive volume-filling density field on the basis of the Delaunay tessellation of the point distribution. DTFE is based on the assumption that the density at the position of each particle is proportional to the inverse of the total volume of the adjacent Delaunay tetrahedra of the point, i.e. its contiguous Voronoi cell. The density field values at each voxel of a rectangular grid are subsequently determined by linear interpolation within the Delaunay tetrahedron containing the centre of the voxel. DTFE density (and velocity) fields have been found to optimally trace a hierarchical matter distribution at any resolution level represented by the point sample, while at the same time resolving the

⁣[⁣www.cgal.org
local anisotropies in the matter distribution. This makes DTFE ideally suited for studying the complex patterns of the Cosmic Web.

4.5.2 Density field morphology

Figure 4.6 depicts a volume rendering of the (DTFE) density field in a thick slice through the simulation box (left panel). It shows that DTFE manages to follow the intricacies of the web-like structures in great detail, over a range of scales and by reproducing the correct geometry of the various features.

An interesting example of structural complexities in the displayed region is the cluster at the left-hand side of the slice. A full 3D visualisation of the system shows that the filaments entering the cluster define several semi-planar structures, all sharing the cluster as their common node. Lower isodensity contours reveal even more of the tenuous walls. However, at such low levels the image gets easily confused by spurious interloping features.

An approach frequently encountered for delineating structural features such as filaments and walls is by assigning a particular density range to the features. Filaments or walls can be singled out by selecting the corresponding density range. In the right-hand panel of fig. 4.6 we show the density field inside an isodensity surface at \( \delta = 1 \), for clarity superimposed on top of a white background. The density value \( \delta = 1 \) is roughly comparable to typical values encountered in filaments and walls (see e.g. Aragón-Calvo et al. 2007). These contours roughly define the boundaries of the filaments and the clusters embedded within their realm, along with the tenuous walls suspended between the filaments.

While the isodensity surfaces provide good insight into the overall distribution of matter, one immediate observation is that the attempted pure density selection of filaments in the top-right panel is not very successful. As was pointed out by Aragón-Calvo et al. (2007) (also see e.g. Hahn et al. 2007) filaments and walls involve a rather broad range of densities. The cluster, filamentary, wall and field density distributions are also partially overlapping (also see Aragón-Calvo et al. 2009). Purely on the basis of a density criterion one would therefore not be able to decide whether a certain location belongs to a cluster node, filament or wall, which may be directly appreciated from the truncated density map in fig. 4.6. Although the image shows a substantial degree of filamentary structure, comparison with the full density field shows that it unjustifiably discards the wealthy pattern of lower density filaments. Also, it does not manage to disentangle the highly concentrated agglomeration of filaments near the massive cluster at the central left-hand side of the box. Moreover, throughout the whole volume it is rather difficult to see which locations would belong to a filament and which ones to a wall. In summary, density maps are rather limited with respect to identifying the true morphological and dynamical environment.

4.5.3 Cellular Morphology

Following the computation of the density field, we compute its watershed transform. The resulting segmentation of the density field into its watershed basins is illustrated in Figure 4.7. These basins are to be identified with the void regions. To get a better idea of its spatial structure and the connections between the various structural components, we slice through the watershed field at regular intervals along the \( x \)-axis to yield a sample of \( yz \) slices. The correspondence between the white watershed segmentation lines and the underlying density contour levels (blue-green background) can be easily appreciated by inspecting the 5 depicted slices.

The spatial segmentation defined by the watershed transform emphasises the characteristic cellular nature of the Cosmic Web as well as the close relation between voids, walls and filaments. One can immediately recognise cosmological voids as large empty cells in the wa-
Figure 4.7– Three-dimensional watershed transform of the density field. The cube shows the pixels that compose the watershed transform, from which the Spine is extracted. Several slices cut across the simulation box show the watershed (white lines) delineating the density field (blue-green background). The three dimensional nature of the watershed network is evident.

tershed transform. The boundaries between two adjacent watershed cells define the walls in the cosmic matter distribution, while the intersection of walls allocates its filaments. The watershed lines in the $yz$ slices reveal the considerable variety of sizes and shapes of the voids. Even though on stereological grounds this is somewhat exaggerated in the 2-D slices, the comparison with the void basins in the 3-D box does confirm the impression of the diversity of the void population. It is a reflection of the complexity of the dynamical processes which are forming and shaping the voids (Sheth & van de Weygaert 2004, and Chapter 3).
The 2-D \( yz \) slices do show the strong correlation between the intensity of the density field and the watershed transform. We also note that the highest density regions contain more cells than more moderate or underdense regions. This immediately translates into a more complex local wall and filament network. The opposite effect occurs in underdense regions. These are mainly characterised by large symmetrical voids, surrounded by relatively simple wall-filament environments.
4.5.4 Cosmic Spine: Filaments and Walls

In the next step of our procedure we invoke the watershed transform to identify and label the voxels that correspond to the filaments and walls, following the SpineWeb criteria specified in equation 4.4. The resulting morphological segmentation is presented in Figure 4.8, and compared to the corresponding density field. The two top panels are images of the density field (see discussion sect. 4.5.2), while the bottom panels present the density field within morphologically segregated regions: the bottom left panel encloses the wall features in the density field, the bottom right one the filamentary features.

The two frames at the bottom show the density field inside semi-transparent surfaces enclosing the walls (bottom left) and the filaments (bottom right). The appearance of a uniform width, at places considerably in excess of the local width of the density field contours, is a result of our choice to show, for visualisation purposes, a uniform smoothed outline. The plotted isosurface of both walls and filaments is obtained by filtering the mask defined by all wall voxels with a Gaussian kernel of $\sigma = 2$ voxels. The Gaussian smoothing radius corresponds roughly to the average width of $\approx 2h^{-1}\text{Mpc}$ of filaments and walls, as we found in a previous study (Aragón-Calvo et al. 2007, 2009). In order to appreciate the varying density and width along the spinal structures, we have superimposed the density field colour contour maps within their smoothed boundaries.

While the first superficial impression might be that the isocontour map of the density field is richer in detail and structure than the filamentary and sheetlike morphologies in the bottom frames, we need to make a few important observations. First, the SpineWeb analysis in the present study is restricted to one single Gaussian smoothing scale. To elucidate the topological workings of the SpineWeb procedure we pay less attention to the hierarchical nature of the different morphological components of the cosmic matter distribution. Moreover, instead of seeking to describe the full three-dimensional matter distribution, the SpineWeb procedure is meant to segregate the information contained in the density map into different morphological elements. On the basis of the resulting morphological selections, we hope to be able to relate the matter and galaxy distribution in a more meaningful fashion to underlying dynamical evolution. One particular aspect of this dynamically motivated disentanglement of structure is the attempt to identify various evolutionary stages of the tidally induced anisotropic collapse of structure in the Universe.

We immediately notice that the contours of the filamentary and sheetlike features defined by the SpineWeb procedure delineate an interconnected three-dimensional network. There are no isolated structures in the simulation box. It forms a telling contrast to the discontinuous nature of the isodensity contour map (top panels, and fig. 4.6). Particularly outstanding structures are filaments close to the infall region of clusters. As we may observe from the massive cluster concentration at the left-side of the panels, the SpineWeb procedure manages to resolve the filamentary extensions connecting to the cluster. It would be very challenging for traditional density-based filament detection techniques to trace filaments near cluster-filament interfaces. Because the density in the infall region towards the cluster tends to increase dramatically, a density-based criterion would leave the local morphology undetermined (Aragón-Calvo et al. 2007).

An even more insightful impression of the intricacy of the full three-dimensional network of filaments and walls is offered by fig. 4.9. The top two frames show the wall-like (blue) and filamentary (red) regions separately. Both show the percolating nature of these structural entities. Revealing is the rather complex shape of the sheets. Instead of a regular “planar” geometry, on small scales the walls have a curved appearance marked by an irregular surface. To a considerable extent this reflects their inhomogeneous internal mass distribution, itself a result of their hierarchical buildup (see Aragón-Calvo et al. 2009). The irregular convoluted shapes are found on all scales, although the walls do have a slightly more regular semi-planar
An analysis of wall and filament sample

4.6 Analysis Wall & Filament Sample

In this section we present a few quantitative measures of the voids, walls and filaments extracted by the SpineWeb technique. The results concern the single-scale analysis of our simulation containing $64^3$ particles.

4.6.1 Density distribution

The density field of the $64^3$ simulation was computed on a regular grid of 512 voxels per dimension, no smoothing was applied to the filament and wall masks. The pdf’s of the density distribution in the voids, walls, filaments as well as for the entire mass distribution are shown in fig. 4.11. The curves shown in the Figure are the normalised pdf, i.e. normalised with respect to the volume occupied by the diverse morphologies. If we wish to scale these functions to that for the entire simulation volume, we have to take into account the volume fractions listed in table 4.3.
Figure 4.9– Surfaces enclosing the voxels which are identified as belonging to walls (blue, top left) and filaments (red, top right) within a cubic region of $50h^{-1}$ Mpc. The bottom frame shows how in the same region both morphological components are connected and intertwined. The latter forms a nice illustration of the intimate relationship between filaments and walls. For visualisation purposes the surfaces are smoothed with a Gaussian kernel of $\sigma = 2$ voxels.

Figure 4.10– Zoom-in onto the cosmic spine in a subregion of the $50h^{-1}$ Mpc, highlighting the intricate connections between wall surfaces (blue), filamentary edges and cluster nodes (red).
The density distributions $p(\delta)$ in the different morphologies have the same shape, but differ in the range over which it runs. Overall, it turns out that the pdf’s can be reasonably well approximated by a lognormal distribution. Numerous studies have shown that a gravitationally evolving matter distribution, starting from Gaussian initial conditions, tends to attain a lognormal density distribution towards more advanced quasi-linear stages (Coles & Jones 1991, see also Chapter 7). Apparently, this remains true within each of the distinct morphologies.

The one distinguishing feature between the pdf’s in voids, filaments and walls are the density at which they peak. Voids have the lowest densities followed by walls and filaments respectively. It is important to note that the spinal network found by the Spineweb method contains also the clusters at the nodes of the weblike network. This evidently affects the high density tail of the distribution. For this reason, the median tends to be a more representative measure for the peak value (see column 1 and 2 of table 4.3).

Finally, we immediately note the considerable overlap between the pdf’s of the different morphologies. Perhaps most remarkable is the sizable overlap between densities in the void fields and those in filaments.

<table>
<thead>
<tr>
<th>Structure</th>
<th>mean</th>
<th>median</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>voids</td>
<td>0.91</td>
<td>0.92</td>
<td>0.87</td>
</tr>
<tr>
<td>walls</td>
<td>2.71</td>
<td>1.38</td>
<td>0.11</td>
</tr>
<tr>
<td>spine</td>
<td>6.65</td>
<td>2.27</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 4.4– Mean and median of the density $\rho/\rho_n = 1 + \delta$, for the mass in voids, filaments and walls. The volume occupancy by these structures is listed in the final column.

The density distribution inside voids is almost identical to the overall density distribution. This is not surprising given the fact that even though voids are extremely underdense they occupy most of the space in the Cosmic Web. The difference between both distributions occurs at the high density tails where the clusters lie. More significant are the differences between density distributions inside filaments and walls. Both distributions peak at clearly different
places. However, there is a large overlap between all density distributions. This degeneracy in the density distributions between morphologies explains why a given isocontour will pick one morphology but will invariably also contain the others.

### 4.6.2 Minkowski-Bouligand dimension

We performed a preliminary scaling analysis of the identified filamentary and wall-like networks in the analysed ΛCDM N-body simulation. To this end, we have determined for each of these networks the Minkowski-Bouligand dimension $D_{MB}$ – or box counting dimension – of the filamentary spine (black dashed line) and walls (black solid line) of the Cosmic Web. For comparison we show the curves for ideal one and two-dimensional objects (gray dashed and gray solid lines respectively).

![Figure 4.12](image1)

**Figure 4.12**– Box-counting dimension of the filamentary spine (black dashed line) and walls (black solid line) of the Cosmic Web. For comparison we show the curves for ideal one and two-dimensional objects (gray dashed and gray solid lines respectively).

![Figure 4.13](image2)

**Figure 4.13**– Number of cells labelled as wall (left panel) or filament (right panel) inside boxes of $8\,h^{-1}\text{Mpc}$ of side as a function of the mean overdensity inside the $8\,h^{-1}\text{Mpc}$ box. The number of cells is normalised with the mean count of all the $8\,h^{-1}\text{Mpc}$ boxes.
formally defined as:

\[ D_{MB} = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log(1/\epsilon)}. \]  

(4.7)

In this expression, we count the number \( N(\epsilon) \) of boxes of (infinitesimal) size \( \epsilon \) required to fill or cover the set of points belonging to the filamentary or wall-like web. In practice, we divide the simulation box into subboxes of size \( s \) and count the number \( N(s) \) of subboxes that contain at least one voxel labelled as filament or wall. By repeating this evaluation for several box sizes, and determining the scaling index \( N(s) \propto s^D \), we obtain an estimate of the Minkowski-Bouligand dimension. One may visualise this by plotting the count \( N(s) \) versus the size \( s \) if the boxes, preferentially in a logarithmic diagram. If indeed characterised by a single fractal dimension, the resulting curve would be characterised by one slope. In practice, the structural patterns tend to be more complex, manifesting itself in scaling curves that cannot be characterised by a single uniform slope.

Figure 4.12 shows the Minkowski-Bouligand dimension computed for the wall and filament networks. We also show two lines with slope -1 and -2 as a reference indicating the cases of a pure one and two-dimensional objects. The slope of the curves for filaments and walls differ considerably at small scales. Filaments behave like one dimensional lines up to scales of \( 3 - 4 \ h^{-1}\text{Mpc} \) after which point the absolute magnitude of the slope of the curve increases from -1 to -3 at scales of approximately \( 10 \ h^{-1}\text{Mpc} \). In the case of the wall network we see a similar behaviour with walls having a clear two-dimensional nature at scales smaller than \( 3 - 4 \ h^{-1}\text{Mpc} \). This departure of the pure one dimensional case in filaments seem to reflect the dependency between the length of filaments and its geometrical properties. Long filaments are located between distant clusters where the local tidal shear is less pronounced while short filaments are formed between close pairs of clusters and tend to be more straight and dense than long filaments. The transition point in the curve of Figure 4.12 provides a good indication of the scale at which filaments and walls start joining each other forming an interconnected network. At this point their dimension is no longer 1 (filaments) or 2 (walls) but a higher value reflecting the complexity of the network of filaments and walls that form the Cosmic Web.

### 4.6.3 LSS complexity and local density

Another measure of the local complexity of the network of filaments and walls is presented in Figure 4.13 where we show the mean number of cells labelled as filament or wall inside boxes of \( 8 \ h^{-1}\text{Mpc} \) of size as a function of the mean density inside the boxes. Low values indicate very simple local configurations while large values reflect complex environments. At first glance this may seem straightforward as increasing excursions sets of the density field have a similar behaviour. However, the filaments and walls we identify are one-voxel thick so their voxel count correlates with their length and surface area respectively. So larger counts indicate more intricate filament and wall systems for a given fixed volume. We find a trend between the density and the complexity of the environment. Highly dense boxes tend to contain more structures than underdense boxes. The regions in the vicinity of massive clusters are a good example of complex neighbourhoods defined in a locally overdense regions while the large voids define relatively simple wall and filament structures. The trend is not very strong as a result of the restriction in the scale of our analysis since we count faint structures together with more significant ones. In a following paper we will show that this trend is stronger once we take into account all the scales.
4.7 Conclusion and future work

The Spine of the Cosmic Web is the cosmic web’s framework, consisting of the network of filaments and their connections at the clusters nodes. In this study we present a topological technique based on the discrete watershed transform of the cosmic density field for the identification and characterisation of voids, walls and filaments. Our method is closely related to a variety of concepts from computational topology, and has a strong mathematical underpinning in Morse theory of singularities.

The SpineWeb method is ideally suited for morphological and dynamic studies of the Large Scale Structure. Amongst others, it will allow a better insight into the formation and dynamics of the anisotropic filamentary and wall-like structures in the Large Scale Universe. Another immediate application is in addressing the question whether and which influences the large scale environment has on the haloes and galaxies that are forming within their realm.

As a first test of its viability, we applied our method on a set of heuristic Voronoi clustering models. The SpineWeb procedure succeeded excellently in reconstructing the original properties of the cellular galaxy distribution. In the implementation presented in this work, we effectively restrict ourselves to a single spatial scale, that determined by the voxel scale of the regular grid on which the density field is sampled. In a forthcoming paper we will discuss the effect of the multiscale nature of the matter distribution. The scale-space formulation of the SpineWeb method has enabled us to identify fainter features in the density field and establish their connections with other objects into a truly hierarchical weblike pattern. In other words, it provides an effective way towards characterising the hierarchy of structures in the Cosmic Web.

A crucial aspect of the watershed transform and of our method is the definition of local neighbourhood. In the case of regular grids the immediate neighbourhood of 26 pixels is arguably the best option. However, for unmeshed data such as galaxy surveys and N-body simulations, other neighbourhood definitions offer a better choice. Among these, the Voronoi contiguous cell defined by the Delaunay tessellation of the point distribution represents a promising option. In the third paper of this series we will present the result of a Delaunay implementation of the Spine method.

4.8 Acknowledgements

We would like to thank Bernard Jones for inspiring discussions and insightful comments. This research was funded by the Gordon and Betty Moore foundation.