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Cortese, S.; Pallante, E.; Petronzio, R.

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Higgs mass dependence of electroweak radiative corrections and the triviality bound

S. Cortese

INFN, Sezione di Roma II, Tor Vergata, Via della ricerca scientifica, I-00133 Rome, Italy

E. Pallante and R. Petronzio

Dipartimento di Fisica, Università di Tor Vergata, Via della ricerca scientifica, I-00133, Roma, Italy

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We discuss how the existence of a triviality bound for the Higgs sector of the standard model does influence the dependence of radiative corrections upon the Higgs mass. The lowering of the Landau pole with increasing Higgs mass implies that the sensitivity to Higgs mass values beyond ≈ 500 GeV is lost due to the sizeable cut-off dependence of the theory

In some cases the emergence of new physics at an energy scale μ^2 can be explored at a scale $q^2 \ll \mu^2$ through the contributions that virtual particles give to physical processes. This is what happens at LEP, where high precision measurements can test both the correctness of the standard model and the feasibility of a wider theory of which SM is a low-energy limit. However, before searching for effects of a truly new physics, such as technicolor or supersymmetric models, we must search for those effects coming from the missing particles which are needed for the SM to work, namely the top and the Higgs. For the latter present mass limits extend up to very high values: in this letter, we want to show how the known “triviality bound” on the Higgs mass implies a loss of sensitivity of radiative corrections to Higgs mass values beyond five hundred GeV.

The Higgs mass dependence is concentrated in the so-called “oblique” corrections [1–3], arising from loops in vacuum polarization diagrams of gauge bosons $\Pi_{ij}^{\mu\nu}(q^2)$, where i and j stand for $W^\pm$, Z , γ . In the general expression

$$\Pi_{ij}^{\mu\nu}(q^2) = -ig^{\mu\nu}[A_{ij} + q^2 F_{ij}(q^2)] + q^\mu q^\nu \text{ terms} \quad (1)$$

at $q^2 \leq m_Z^2$ only A_{ij} and the $F_{ij}(0)$ are retained, moreover $A_{\gamma\gamma} = A_{\gamma Z} = 0$ as required from the Ward identities, so that loop corrections are contained in the six quantities $A_{WW}, A_{ZZ}, F_{WW}, F_{ZZ}, F_{\gamma Z}, F_{\gamma\gamma}$. Renormalization reduces the independent ones to three; let us briefly recall how it works.

The $\Pi_{ij}^{\mu\nu}(q^2)$ are in principle logarithmically divergent quantities in the UV region, so that the first step is to regularize loop integrals introducing a momentum cut-off Λ , while the target theory will be defined in the limit $\Lambda \rightarrow \infty$. The cut-off dependent quantities have the general expression (we write it for A_{WW} but is just the same for the others)

$$A_{WW} = b_{WW} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \sum_{n=0}^{\infty} c_{WW}^{(n)} \left(\frac{\mu^2}{\Lambda^2}\right)^n. \quad (2)$$

The coefficient b_{WW} is independent from the regularization procedure, while the $c_{WW}^{(n)}$ depend on the scheme choice; in the limit $\Lambda \rightarrow \infty$ eq. (2) reduces to

$$A_{WW} = b_{WW} \log\left(\frac{\Lambda^2}{\mu^2}\right) + c_{WW}^{(0)}. \quad (3)$$

In the SM, a renormalizable theory, the logarithmically divergent cut-off dependence can be reabsorbed in a redefinition of physical constants and it is easy to derive that three independent combinations of our original six quantities enter the definition of α , G_F , m_Z and there are only three free parameters.

A common choice for these finite and scheme independent parameters is

$$\begin{aligned} \varepsilon_1 &= \frac{A_{ZZ}}{m_Z^2} - \frac{A_{WW}}{m_W^2}, \\ \varepsilon_2 &= F_{WW} - F_{33}, \\ \varepsilon_3 &= \frac{c}{s} F_{30}, \end{aligned} \quad (4)$$

where indices 3 and 0 stands for $W_3 = cZ + s\gamma$, $W_0 = c\gamma - sZ$ (c and s are the cosine and the sine of the Weinberg angle).

One loop calculations show that the dependence on the top mass is quadratic for ε_1 and logarithmic for ε_2 and ε_3 , while the dependence on the Higgs mass is absent for ε_2 and logarithmic for ε_1 and ε_3 .

The explicit expressions are

$$\begin{aligned} \varepsilon_1 &= \frac{3}{4} \frac{G_F m_t^2}{\sqrt{2}\pi^2} + \frac{3}{4} \frac{G_F m_W^2 s^2}{2\sqrt{2}\pi^2 c^2} \log\left(\frac{m_W^2}{m_H^2}\right), \\ \varepsilon_2 &= \frac{1}{2} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \log\left(\frac{m_W^2}{m_t^2}\right), \\ \varepsilon_3 &= \frac{1}{6} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \log\left(\frac{m_W^2}{m_t^2}\right) - \frac{1}{12} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \log\left(\frac{m_W^2}{m_H^2}\right). \end{aligned} \quad (5)$$

We now turn to the Higgs sector of the theory; the renormalization of the Higgs self-coupling constants γ yields, at one loop level,

$$\frac{1}{\lambda(v)} = \frac{1}{\lambda(\Lambda_{SM})} + \frac{3}{8\pi^2} \log\left(\frac{\Lambda_{SM}}{v}\right), \quad (6)$$

which in turn implies

$$m_H^2 \leq \frac{8\pi^2 v^2}{3 \log(\Lambda_{SM}/v)}, \quad \Lambda_{SM} \leq v \exp\left(\frac{8\pi^2 v^2}{3m_H^2}\right). \quad (7,8)$$

This upper bound on Λ_{SM} can be thought of as the energy scale at which the SM has no more meaning, and new physics must be revealed. Of course as m_H gets higher, λ gets higher too, and we cannot completely trust one loop perturbation theory, but lattice calculations show that, at least down to $\Lambda_{SM} \approx 2m_H$, these relations are numerically valid.

What we want to point out is that the existence of an intrinsic energy upper limit for the SM implies that the infinite cutoff limit of eq. (2) cannot be safely taken and that we should expect corrections of order μ/Λ_{SM} to affect sizeably radiative corrections when μ is replaced by the heavy Higgs mass.

Of course the relation between Λ_{SM} and m_H is such that if $m_H \approx 200$ GeV then $\Lambda_{SM} \approx 10^{11}$ GeV and no visible effect is possible, but as m_H gets higher Λ_{SM} gets lower to the point where, according to eq. (8), $\Lambda_{SM} \approx m_H$ for $m_H \approx 740$ GeV, so that corrections of order m_H/Λ_{SM} become of order one. Of course we expect corrections of

order m_i/Λ_{SM} too, but as showed $\Lambda_{\text{SM}} > 700$ GeV, so that we might as well ignore them.

To simulate the effect of new physics emergence at a scale Λ_{SM} the momentum cut-off Λ is simply replaced by the upper limit of eq. (8):

$$\bar{\Lambda} = v \exp\left(\frac{8\pi^2 v^2}{3m_H^2}\right). \quad (9)$$

Thus we expect

$$\begin{aligned} \varepsilon_1 &= \frac{3}{4} \frac{G_F m_i^2}{2\sqrt{2}\pi^2} + \frac{3}{4} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \frac{s^2}{c^2} \log\left(\frac{m_W^2}{m_H^2}\right) + g_1 \left(\frac{m_H^2}{\Lambda^2}\right), \\ \varepsilon_3 &= \frac{1}{6} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \log\left(\frac{m_W^2}{m_i^2}\right) - \frac{1}{12} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \log\left(\frac{m_W^2}{m_H^2}\right) + g_3 \left(\frac{m_H^2}{\Lambda^2}\right), \\ \lim_{\Lambda \rightarrow \infty} g_i \left(\frac{m_H^2}{\Lambda^2}\right) &= \gamma_i = \text{constant}. \end{aligned} \quad (10)$$

As far as the Higgs mass dependence is concerned we only need to calculate the Feynman graphs of fig. 1 (tadpoles do not contribute to ε_i).

The explicit expressions are

$$\begin{aligned} \Pi_{ZZ}^{\mu\nu} &= \frac{G_F m_W^2}{2\pi^4 \sqrt{2} c^2} \int d^4 k \frac{-g^{\mu\nu} m_Z^2 + k^\mu k^\nu}{(k^2 - m_Z^2) [(k+q)^2 - m_i^2]}, \\ \Pi_{WW}^{\mu\nu} &= \frac{G_F m_W^2}{2\pi^4 \sqrt{2}} \int d^4 k \frac{-g^{\mu\nu} m_W^2 + k^\mu k^\nu}{(k^2 - m_W^2) [(k+q)^2 - m_H^2]}. \end{aligned} \quad (11)$$

As a matter of fact the combinations of these integrals entering the definitions of ε_i represent only a subset of the full calculation and are still logarithmically divergent; in order to avoid all the renormalization procedure (i.e. calculating all the diagrams) we evaluate

$$\frac{d\varepsilon_i}{dm_H^2} = b_i \frac{1}{m_H^2} + \frac{1}{\Lambda^2} g'_i \left(\frac{m_H^2}{\Lambda^2}\right) \quad (12)$$

and reconstruct the full ε_i through

$$\varepsilon_i(m_H^2) = b_i \log(m_H^2) + \int_0^{m_H^2} \frac{1}{\Lambda^2} g'_i \left(\frac{m_H^2}{\Lambda^2}\right) - b_i \log(m_W^2) + \gamma_i, \quad (13)$$

where the constant is added to match the value of ε_i calculated with all diagrams in the limit $\Lambda \rightarrow \infty$. The cut-off dependence is introduced replacing Λ with $\bar{\Lambda}$ of eq. (9) in the functions g_i .

When the theory becomes sensitive to cut-off effects, it becomes also regularization scheme dependent. We are going to show such a dependence by evaluating the functions g_i in eq. (10) with different schemes.

The first regularization scheme used introduces the cut-off as an upper limit on momentum integration by replacing

Fig. 1. Feynman diagrams contributing to the Higgs mass dependence of ε_1 and ε_3 .

$$\int d^4k \rightarrow \pi^2 \int_0^\infty dk^2 k^2 \rightarrow \int_0^{\Lambda^2} dk^2 k^2. \quad (14)$$

While the substitution is straightforward as far as ε_1 is concerned (we may work at $q=0$), in the calculation of ε_3 we first use the Feynman parameters

$$\frac{d\varepsilon_3}{dm_H^2} = \frac{1}{8\pi^4} \frac{G_F m_W^2}{\sqrt{2}} \frac{d}{dq^2} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \int d^4k \frac{2(-4m_Z^2 + k^2)}{\{k^2 - (1-\alpha)m_Z^2 + \alpha[q^2 + 2(qk) - m_H^2]\}^3}, \quad (15)$$

shift $k \rightarrow k + \alpha q$ letting $q=0$ in the numerator (it would have given terms proportional to $q^\mu q^\nu$), take the derivative with respect to q^2 and eventually apply eq. (14).

The final result reads

$$g_1 = \frac{3}{4} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \frac{s^2}{c^2} \log\left(\frac{\Lambda^2 + m_H^2}{\Lambda^2}\right) + \gamma_1, \\ g_3 = -\frac{1}{12} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \left[\log\left(\frac{\Lambda^2 + m_H^2}{\Lambda^2}\right) - 2 \frac{\Lambda^6}{m_H^6} \log\left(\frac{\Lambda^2 + m_H^2}{\Lambda^2}\right) + 2 \frac{\Lambda^4}{m_H^4} - \frac{\Lambda^2}{m_H^2} + \frac{1}{3} \right] + \gamma_3. \quad (16)$$

The second regularization employs the exponential representation of euclidean propagators:

$$\frac{1}{k^2 + m_\nu^2} = \int_{-\infty}^0 d\alpha \exp[\alpha(m_\nu^2 + k^2)], \quad (17)$$

so that

$$\frac{d\varepsilon_1}{dm_H^2} = -\frac{3}{4} \frac{G_F m_W^2}{2\pi^4} \frac{s^2}{\sqrt{2} c^2} \int_{-\infty}^0 d\alpha \int_{-\infty}^0 d\beta \int_{-\infty}^0 d\gamma \int d^4k \exp[\alpha(k^2 + m_Z^2) + (\beta + \gamma)(k^2 + m_H^2)], \\ \frac{d\varepsilon_3}{dm_H^2} = -\frac{G_F m_W^2}{2\pi^4 \sqrt{2}} \int_{-\infty}^0 d\alpha \int_{-\infty}^0 d\beta \int_{-\infty}^0 d\gamma \int d^4k \frac{\alpha(\beta + \gamma)k^2}{\alpha + \beta + \gamma} \exp[\alpha(k^2 + m_Z^2) + (\beta + \gamma)(k^2 + m_H^2)]. \quad (18)$$

The integrals are regularized through the substitution (just in the α integration)

$$\int_{-\infty}^0 d\alpha \rightarrow \int_{-\infty}^{-1/\Lambda^2} d\alpha \quad (19)$$

and the values for the g are

$$g_1 = -\frac{3}{4} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \frac{s^2}{c^2} \left[\exp(m_H^2/\Lambda^2) \text{Ei}\left(\frac{-m_H^2}{\Lambda^2}\right) - \log\left(\frac{m_H^2}{\Lambda^2}\right) \right] + \gamma_1, \\ g_3 = \frac{1}{12} \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \left[\exp(m_H^2/\Lambda^2) \text{Ei}\left(\frac{-2m_H^2}{\Lambda^2}\right) \left(1 - \frac{m_H^2}{\Lambda^2} + \frac{m_H^4}{\Lambda^4}\right) - \frac{m_H^2}{\Lambda^2} - \log\left(\frac{m_H^2}{\Lambda^2}\right) \right] + \gamma_3. \quad (20)$$

The last scheme used is Pauli-Villars regularization, every propagator is replaced by

$$\frac{1}{(k^2 - m^2)^2} \rightarrow \frac{1}{(k^2 - m^2)^2} - \frac{1}{(k^2 - \Lambda^2)^2}, \quad (21)$$

and in this case

Fig. 2. Higgs mass dependence of ϵ_1 , the four curves represent the value at infinite cut-off and with finite cut-off correction in the three schemes analyzed.

Fig. 3. The same as fig. 2 but for ϵ_3 .

$$g_1 = \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \frac{s^2}{c^2} \left[\frac{m_H^2}{\Lambda^2 - m_H^2} \log\left(\frac{\Lambda^2}{m_H^2}\right) \right] + \gamma_1,$$

$$g_3 = \frac{G_F m_W^2}{2\sqrt{2}\pi^2} \left\{ \frac{3\Lambda^6}{(\Lambda^2 - m_H^2)^3} \left[-\frac{m_H^4}{\Lambda^4} \log\left(\frac{\Lambda^2}{m_H^2}\right) + \frac{m_H^6}{3\Lambda^6} \log\left(\frac{\Lambda^2}{m_H^2}\right) \right] - 2 \frac{m_H^2 \Lambda^2}{(\Lambda^2 - m_H^2)^2} \right\} + \gamma_3. \quad (22)$$

In order to isolate the dependence of ϵ_1 and ϵ_3 on the Higgs mass we use the quantity

$$\tilde{\epsilon}(m_H) = \epsilon(m_H) - \epsilon(200 \text{ GeV}), \quad (23)$$

getting rid of the still unknown constants γ_i . In absence of cutoff effects, such a quantity is scheme independent. For the Higgs mass a reference value of 200 GeV has been chosen where the corrections of order m_H/Λ are totally negligible.

Results for $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_3$ in the different schemes, together with the infinite Λ result, are shown in figs. 2 and 3; for a Higgs mass higher than ≈ 500 GeV the four curves definitely split, and one cannot trust the predictions of the standard model any more. In other words, these one loop processes for $m_H \geq 500$ GeV should receive a significant contribution from “unknown physics” and calculations performed in the SM may be totally unreliable.

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