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Pion loops in quenched quantum chromodynamics

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Abstract

We calculate the divergences of the generating functional of quenched Chiral Perturbation Theory to one loop for a generic number of flavours. The flavour number dependence of our result enlightens the mechanism of quark loop cancellation in the quenched effective theory for any Green function or S matrix element. We also apply our results to $\pi\pi$ scattering and evaluate the coefficient of the chiral log in the S -wave scattering lengths for the quenched case. © 1997 Elsevier Science B.V.

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Lattice simulations are at present the only available method to study in a quantitative way the nonperturbative regime of Quantum Chromodynamics. Although theoretically well founded, the method is limited by a number of technical problems, and by the computing power which is not yet enough to fully cope with QCD, despite continuous progress. One of the main technical difficulties is the implementation of an efficient algorithm to evaluate the path integral over fermions, which is at present very time consuming. The usual way to go around this problem has been the use of the “quenched” approximation, i.e. setting to one the determinant resulting from the integration over fermions. In the language of Feynman diagrams, this approximation corresponds to neglecting quark loops.

Quenching is so widely used that it has become important to understand a priori and in a quantitative way

what is the effect introduced by this approximation [1]. A very promising approach to this problem is, in our opinion, the effective Lagrangian method proposed by Bernard and Golterman [2]. This is based on the observation due to Morel (see Ref. [1]) that the quenched approximation to QCD can be described by a Lagrangian where one adds to the usual quark fields a corresponding number of ghost (complex) bosonic fields, with the same kinetic term, and the same coupling to gluons. The ghost fields exactly cancel the fermionic determinant. When the quark masses are set to zero the quenched QCD Lagrangian has a larger symmetry than the usual $U(N)_L \times U(N)_R$, and must have a special fermionic character, since it transforms bosons into fermions. This extended symmetry is described by the graded group $U(N|N)_L \times U(N|N)_R$. Following exactly the same steps that lead from the globally symmetric QCD Lagrangian in the chiral limit to Chiral Perturbation Theory (CHPT) [3,4], Bernard and Golterman have proposed an ef-

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fective Lagrangian that should describe the low energy regime of quenched QCD: quenched CHPT (often qCHPT in the following). The basic new ingredient is the presence of ghost fermion and boson fields that enter only inside loops. These are Goldstone particles of the extended symmetry, and can be viewed as bound states formed by a quark and a ghost-antiquark, or viceversa, and ghost-quark ghost-antiquark. Also for these new fields the structure of the interaction is dictated by the graded symmetry, which however leaves the coupling constants unconstrained. The main advantage of this method is that one has in principle a systematic way to study the modifications introduced by quenching: much as in CHPT, one can e.g. evaluate unambiguously the chiral logs present in any observable quantity.

Quenched CHPT has shown that the chiral logs in the two point function of the axial current to one loop disappear (i.e. both in F_π and M_π , see Ref. [2], and Sharpe in Ref. [1]), and that the anomalous η' field is ill-defined, developing a double pole in the propagator, as one would expect [2,5]. The systematics of the cancellation of chiral logs for two point functions has been further investigated in Ref. [6]. Other examples have been studied in the weak [7] and baryon sector [9]. In all these cases the success of the theory has been to automatically produce a cancellation between pion and ghost loops in such a way that the pion loops that do not contain quark loops survive it (e.g. in $\pi\pi$ scattering the graph in Fig. 1a should remain whereas that in Fig. 1b not). The aim of this letter is to demonstrate that in the strong sector this cancellation is always working properly at the one loop level, for any Green function or S matrix element.

Although at first sight this may seem difficult, there is in fact a straightforward way to do it. The idea is to develop a path integral formulation of quenched CHPT much on the same line as that of ordinary CHPT. There, Gasser and Leutwyler [4] have shown that, by using the background field method and heat kernel techniques, it is possible to derive in closed form the divergent part of the one loop generating functional. This is a chiral invariant Lagrangian at order p^4 . The calculation was done in a general $SU(N)_R \times SU(N)_L$ theory, so that the N dependence of the divergent part of the one loop generating functional is explicit. In the result of Gasser and Leutwyler four different powers of N occur: N^k with $k = -2, -1, 0, 1$. Let us disregard

for the moment the negative powers – we shall discuss them later. If we consider the case of N degenerate quarks, the interpretation of the terms linear in N and constant is rather straightforward: they come from the pion loops that contain one quark loop or none at all, respectively. This conclusion is based on the simple observation that since the strong interaction is flavour-blind, each quark loop produces a factor N . Then, if quenched CHPT is working properly, the fermionic ghost loops have to cancel only the pion loops that give rise to a divergence proportional to N . Our main aim is to verify that this is exactly what happens. This calculation will also provide the coefficients of all the chiral logs of any Green function to one loop.

For the sake of clarity we are going to ignore the presence of the $U(1)_A$ anomaly of QCD, in the following. Taking it into account would produce, as a main effect, a double pole in the η' [2]. How to cope with the diseases of this part of the theory has been extensively analyzed by Bernard and Golterman [2,8] and Sharpe [7] looking at various different observables. In the perturbative treatment of the path integral we are considering, these contributions simply add to the standard pion loop contributions, and we can safely put them aside for the moment [10].

The starting point of our calculation is the lowest order qCHPT Lagrangian, a graded symmetry generalization of the CHPT Lagrangian. This can be written as³

$$\mathcal{L}_2^{\text{qCHPT}} = \frac{F^2}{4} \text{str} (D_\mu U_s D^\mu U_s^\dagger + \chi_s^\dagger U_s + U_s^\dagger \chi_s), \quad (1)$$

where str stands for the supertrace, the generalization of the trace to graded matrices (see Ref. [2] for details), $D^\mu U_s = \partial^\mu U_s - i r_s^\mu U_s + i U_s l_s^\mu$, $\chi_s = 2B_0(s_s + i p_s)$, and the field U_s can be written as $U_s = \exp(\sqrt{2}i/F\Phi)$, where Φ is the hermitian nontraceless block-matrix

$$\Phi = \begin{pmatrix} \phi & \theta^\dagger \\ \theta & \tilde{\phi} \end{pmatrix},$$

containing the physical pseudoscalar field ϕ , the ghost field $\tilde{\phi}$, both of bosonic nature, and the hybrid fields

³ We always use the subscript s to denote the graded symmetry version of a standard CHPT quantity.

θ , $\theta^\dagger$ of fermionic nature. The scalar field s_s contains the quark mass matrix $\mathcal{M}$ through

$$s_s = \begin{pmatrix} \mathcal{M} & 0 \\ 0 & \mathcal{M} \end{pmatrix} + \delta s_s$$

and that for our purposes is taken diagonal: $\mathcal{M} = m_q \mathbf{1}$. All the Goldstone bosons will then have the same mass: $M^2 = 2B_0 m_q$. The Lagrangian in Eq. (1) contains the external fields r_s^μ , l_s^μ , s_s , p_s , which are generalizations of the standard external fields, in order to make the Lagrangian locally invariant under the group $U(N|N)_R \times U(N|N)_L$. However, since we are not interested in Green functions of the spurious fields, we directly set to zero all the corresponding spurious external fields. The generating functional will then become a function of the usual external fields only: $r_\mu = v_\mu + a_\mu$, $l_\mu = v_\mu - a_\mu$, s , p , which are all $N \times N$ matrices.

To calculate quantum corrections we expand the leading order action given by the Lagrangian in Eq. (1) in the vicinity of the classical solution which is determined by the external sources through the classical equations of motion. We define the classical solution as $\bar{U}_s \equiv u_s^2$ and describe the fluctuations around it as $U_s = u_s \exp(i\xi_s) u_s = u_s (1 + i\xi_s - 1/2\xi_s^2 + \dots) u_s$. To get the generating functional to one loop we need to expand the action in the ξ_s field up to second order. Skipping all the details [10] we end up with the following gaussian integral (in Minkowski space-time):

$$\begin{aligned} e^{iZ_{\text{one loop}}^{\text{CHPT}}} &= \int d\mu [U_s] \exp \left\{ i \int dx \frac{F^2}{4} \right. \\ &\quad \times \left[\xi_a D^{ab} \xi_b + 2\xi_a^\dagger \bar{D}^{ab} \xi_b + \tilde{\xi}_a (\square + M^2) \tilde{\xi}_a \right] \left. \right\} \\ &= \mathcal{N} \frac{\det \bar{D}}{(\det D)^{1/2}}, \end{aligned} \quad (2)$$

where we used the decomposition of the fields ξ , $\tilde{\xi}$, ζ , $\zeta^\dagger$ (quantum fluctuations corresponding to the classical fields ϕ , $\bar{\phi}$, θ , $\theta^\dagger$, respectively) in terms of the N^2 generators of each $U(N)$ flavour subgroup as $\xi = \xi^a \hat{\lambda}_a$, with $\hat{\lambda}_a = \lambda_a / \sqrt{2}$ for $a = 1, \dots, N^2 - 1$ (the λ_a 's are the usual Gell-Mann matrices of $SU(N)$), and $\hat{\lambda}_0 = \mathbf{1}/\sqrt{N}$. We remark that the scalar ghosts $\tilde{\xi}$ decouple from the physical pions and the integral over them produces only an irrelevant constant. We also stress that both differential operators D^{ab} and

$\bar{D}^{ab}$ are now functions only of the standard external fields and the ϕ field at the classical solution, since we have put to zero all the spurious external fields. The differential operator D^{ab} is defined as follows:

$$\begin{aligned} D^{ab} \xi_b &= -d_\mu d^\mu \xi^a + \hat{\sigma}^{ab} \xi_b \\ d_\mu \xi^a &= \partial_\mu \xi^a + \hat{\Gamma}_\mu^{ab} \xi_b, \end{aligned} \quad (3)$$

where

$$\begin{aligned} \hat{\Gamma}_\mu^{ab} &= -\langle \Gamma_\mu [\hat{\lambda}^a, \hat{\lambda}^b] \rangle \\ \hat{\sigma}^{ab} &= \frac{1}{4} \langle [u_\mu, \hat{\lambda}^a] [u^\mu, \hat{\lambda}^b] \rangle - \frac{1}{4} \langle \{ \hat{\lambda}^a, \hat{\lambda}^b \} \chi_+ \rangle. \end{aligned} \quad (4)$$

Let us recall some standard definitions in CHPT: $\Gamma_\mu = 1/2 ([u^\dagger, \partial_\mu u] - iu^\dagger r_\mu u - iul_\mu u^\dagger)$, $u_\mu = iu^\dagger D_\mu \bar{U} u^\dagger$, and $\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u$. The differential operator $\bar{D}^{ab}$ acting on the ghost field ζ is defined like in Eq. (3), but with barred quantities, given by

$$\begin{aligned} \bar{\Gamma}_\mu^{ab} &= -\langle \Gamma_\mu \hat{\lambda}^a \hat{\lambda}^b \rangle, \\ \bar{\sigma}^{ab} &= -\frac{1}{4} \langle (u_\mu u^\mu + \chi_+ + 4B_0 \mathcal{M}) \hat{\lambda}^a \hat{\lambda}^b \rangle, \end{aligned} \quad (5)$$

where $\mathcal{M}$ is again the quark mass matrix, which is also contained in the external scalar field $s = \mathcal{M} + \delta s$.

The divergent part of Eq. (2) can be derived in closed form by regularizing the determinants in d dimensions and using standard heat kernel techniques. The result reads

$$\begin{aligned} \frac{i}{2} \ln \det D &= \frac{-1}{(4\pi)^2(d-4)} \int dx \left\{ \frac{N}{6} \langle \Gamma_{\mu\nu} \Gamma^{\mu\nu} \rangle \right. \\ &\quad + \frac{1}{2} \left[\frac{1}{4} \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle + \frac{1}{8} \langle u_\mu u^\mu \rangle^2 \right. \\ &\quad + \frac{N}{8} \langle (u_\mu u^\mu)^2 \rangle + \frac{N}{4} \langle u_\mu u^\mu \chi_+ \rangle + \frac{1}{4} \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle \\ &\quad + \left(\frac{N}{8} - \frac{1}{2N} \right) \langle \chi_+^2 \rangle + \left(\frac{1}{8} + \frac{1}{4N^2} \right) \langle \chi_+ \rangle^2 \\ &\quad - \frac{1}{2} \langle u_\mu \rangle \langle u^\mu (u_\nu u^\nu + \chi_+) \rangle \\ &\quad \left. + \frac{1}{2N} \langle \chi_+^2 \rangle - \frac{1}{4N^2} \langle \chi_+ \rangle^2 \right] \left. \right\} + \dots \end{aligned} \quad (6)$$

$$\begin{aligned} i \ln \det \bar{D} &= \frac{-1}{(4\pi)^2(d-4)} \int dx \left[\frac{N}{6} \langle \Gamma_{\mu\nu} \Gamma^{\mu\nu} \rangle \right. \\ &\quad \left. + \frac{N}{16} \langle (u_\mu u^\mu + \chi_+ + 4B_0 \mathcal{M})^2 \rangle \right] + \dots \end{aligned} \quad (7)$$

Fig. 1. Two examples of pion loop graphs contributing to $\pi\pi$ scattering in the quark-flow diagram picture (all lines are quark lines). Diagram (a) does not contain quark loops, whereas diagram (b) does.

and their difference gives

$$\begin{aligned}
 Z_{\text{one loop}}^{\text{qCHPT}} = & \frac{-1}{(4\pi)^2(d-4)} \int dx \left\{ \frac{1}{8} \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle \right. \\
 & + \frac{1}{16} \langle u_\mu u^\mu \rangle^2 - \frac{1}{4} \langle u_\mu \rangle \langle u^\mu u_\nu u^\nu \rangle \\
 & - \frac{1}{4} \langle u_\mu \rangle \langle u^\mu \chi_+ \rangle + \frac{1}{8} \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle \\
 & + \frac{1}{16} \langle \chi_+ \rangle^2 - \frac{N}{4} M^2 \langle u_\mu u^\mu \rangle - \frac{N}{4} M^2 \langle \chi_+ \rangle \left. \right\} \\
 & + \dots
 \end{aligned} \quad (8)$$

The ellipses stand everywhere for the finite contributions to the one loop generating functional. Eq. (8) shows explicitly the flavour dependence of the qCHPT functional to one loop and the comparison with Eq. (6) provides the result we were after. We stress a few important points:

1. The last two lines of Eq. (6) contain extra contributions with respect to the $SU(N) \times SU(N)$ determinant of Gasser and Leutwyler [4], due to the presence of the singlet field. Two of those terms are proportional to $\langle u_\mu \rangle \equiv \nabla_\mu \phi_0$, and only contribute to processes with external singlet fields. The last two terms arise from loops with the singlet field running inside. These terms exactly remove the negative power dependence upon N in the rest of the expression. This shows that the presence of such terms in the $SU(N)$ theory can be understood as an effect of the $U(1)_A$ anomaly that decouples the singlet field.
2. The ζ field loop in Eq. (7) produces only contributions linear in N as expected, and cancels completely the terms linear in N of the full generating functional. We also note that the N de-

pendence in Eq. (6) is not fully explicit. In fact, since the expansion of χ_+ in powers of ϕ starts with a constant term proportional to the quark mass matrix, its trace is proportional to N in the degenerate case we are considering here. The last two terms in Eq. (8) cancel this dependence in the final expression. This result shows that quenched CHPT does what it should do, i.e. it contains only the pion loops that do not contain quark loops.

3. Each pole at $d = 4$ produces a chiral logarithm after renormalization. A different way to present our result is to say that we have calculated the chiral logs of any Green function in qCHPT to one loop. We obtained that these are nonzero in general, though certainly different from those of the ordinary CHPT case. How different has to be investigated case by case, since it is not possible to identify a general behaviour from our result.

In particular, from Eq. (8) it is easy to verify the results already obtained by Bernard and Golterman [2] about the absence of chiral logs in the chiral condensates, pion masses and decay constants. The situation is different when one considers Green functions with more than two external legs. To see what happens in one concrete example, we consider the $\pi\pi$ scattering amplitude – also because there are lattice calculations of the two S -wave scattering lengths available, Ref. [11]. As we argued above, the presence of chiral logs even in the quenched theory has to be interpreted as due to diagrams with pion loops that do not contain quark loops. For the $\pi\pi$ scattering amplitude an example is given in Fig. 1.

The complete amplitude in quenched CHPT reads

$$\begin{aligned}
A_{\pi\pi}^{\text{qCHPT}}(s, t, u) = & \frac{s - M_\pi^2}{F_\pi^2} \\
& + \frac{1}{4F_\pi^4} \left\{ -\frac{1}{2} \left(L + \frac{1}{16\pi^2} \right) [3s^2 + (t-u)^2] \right. \\
& + s^2 \bar{J}(s) + (t - 2M_\pi^2)^2 \bar{J}(t) + (u - 2M_\pi^2)^2 \bar{J}(u) \\
& + c_1 M_\pi^4 + c_2 s M_\pi^2 + c_3(\mu) s^2 + c_4(\mu) (t-u)^2 \Big\} \\
& + S_{(m_0, \alpha)}(s, t, u) + O(p^6), \quad (9)
\end{aligned}$$

where $\bar{J}(q^2) = J(q^2) - J(0)$, with

$$J(q^2) = \frac{1}{i} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(M^2 - l^2)(M^2 - (l-q)^2)}, \quad (10)$$

and $L = (16\pi^2)^{-1} \log(M_\pi^2/\mu^2)$. The polynomial part with coefficients $c_1, \dots, c_4$ comes from the counterterm Lagrangian at order p^4 . $S_{(m_0, \alpha)}(s, t, u)$ is the renormalized contribution to the amplitude of the singlet loops with one (m_0 and α) vertex insertion on the singlet propagator. The effects and diseases of $S_{(m_0, \alpha)}(s, t, u)$ have been discussed in [8,12]. F_π and M_π are the renormalized quenched values of F and M at order p^4 . While F remains unchanged to one loop, M gets a one loop correction from (m_0 and α) vertex insertion which we included in $S_{(m_0, \alpha)}(s, t, u)$. Finite contributions to both F_π and M_π from the order p^4 Lagrangian are meant to be included in the renormalized polynomial part. Let us stress that the divergent part of the amplitude before renormalization, and the corresponding chiral logs in the renormalized amplitude Eq. (9) can also be obtained from the generating functional, Eq. (8), by expanding it in powers of the external fields for $N = 2$, and taking the coefficient of the relevant term. This offers a welcome check on the full calculation.

We focus now on the S -wave scattering lengths. In full CHPT it is well known that the chiral logs dominate the one loop correction at $\mu = 1$ GeV [13]. In the quenched case, the coefficients of the chiral logs are

$$\begin{aligned}
a_0^{\text{qCHPT}} = & \frac{7M_\pi^2}{32\pi F_\pi^2} \left\{ 1 + \frac{M_\pi^2}{F_\pi^2} \left(-\frac{22}{7} L + \dots \right) \right. \\
& \left. + O(M_\pi^4) \right\}, \quad (11)
\end{aligned}$$

$$\begin{aligned}
a_0^{\text{qCHPT}} = & -\frac{M_\pi^2}{16\pi F_\pi^2} \left\{ 1 + \frac{M_\pi^2}{F_\pi^2} (2L + \dots) \right. \\
& \left. + O(M_\pi^4) \right\}, \quad (12)
\end{aligned}$$

while in full CHPT the corresponding coefficients are: $-9/2$ for $I = 0$ and $3/2$ for $I = 2$ [13]. In both cases the change in the correction due to quenching is of the order of 30%; however the one loop correction itself is roughly of this size in full CHPT (see Ref. [13]), so that the overall relative change can be estimated to be around 10%. Of course this is not the whole story: we have not included the remaining analytic contributions from the regular part of the amplitude, the constants c_i 's in Eq. (9), and the contributions from the ill part of the amplitude, $S_{(m_0, \alpha)}(s, t, u)$. As for the first point, if one assumes that quenching will not change the order of magnitude of the finite part of the $O(p^4)$ constants at $\mu = 1$ GeV (one could argue that some kind of Vector Meson Dominance should be valid also in the quenched case), one can conclude that those contributions should still produce a small correction.

The ill part of the amplitude is in principle more dangerous and requires a more careful treatment, as was done by Bernard and Golterman [8]. They have shown in fact that this part dramatically changes the very method to compute the scattering lengths on the lattice: the Lüscher's formula [14] that relates finite volume effects to the S -wave scattering lengths is modified by quenching. However, the numerical analysis of this effect, adapted to the staggered fermions calculation by Fukugita et al. [11] (the only calculation done with a pion mass small enough to justify the application of CHPT) indicates that this effect should be small. Putting the various pieces together, we can conclude that a complete analysis in qCHPT to one loop of the $\pi\pi$ scattering amplitude, seems to provide an explanation for the rather good agreement between the lattice calculation of S -wave scattering lengths done in Ref. [11] and standard CHPT, as was pointed out by one of us in Ref. [15].

Let us summarize our results: we have calculated the divergent part of the generating functional of quenched CHPT to one loop, Eq. (8), and found that, for degenerate quark masses, *it does not contain any explicit flavour number dependence*. Arguing that for a nonanomalous theory this dependence may arise only

through the presence of quark loops (become manifest here through pion loops), we conclude that quenched CHPT is working properly. In our opinion this result puts on a more solid basis quenched CHPT, the effective theory proposed by Bernard and Golterman [2] to describe the low energy physics of quenched QCD. Our calculation provides also *the chiral logs of any Green function, and shows that these are nonzero in general*. As an example we have discussed the case of the $\pi\pi$ scattering amplitude and found that the coefficient of the chiral log of the two S -wave scattering lengths has been modified by 30%, a rather modest effect.

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