The M5-brane and non-commutative loop space

E Bergshoeff, D Berman, J P van der Schaar and P Sundell

Institute for Theoretical Physics, Nijenborgh 4, 9747 AG Groningen, The Netherlands

Received 4 October 2000
Published 1 August 2001
Online at stacks.iop.org/CQG/18/3265

Abstract
We investigate, in a certain decoupling limit, the effect of having a constant C-field on the M-theory 5-brane using an open-membrane probe. We define an open-membrane metric for the 5-brane that remains non-degenerate in the limit. The canonical quantization of the open-membrane boundary leads to a non-commutative loop space which is a functional analogue of the non-commutative geometry that occurs for D-branes.

PACS number: 1125

1. Introduction
Before considering the M2/M5 system it is instructive to first briefly review the relation between D-branes and non-commutative geometry [1]. Consider a fundamental string F1 ending on a Dp-brane via a point or 0-brane. The effective tensions \( \tau \) of the string and the Dp-brane behave like \( \tau_{F1} \sim 1 \), \( \tau_{Dp} \sim 1/\alpha' \). Therefore, for small \( \alpha' \), the string is much lighter than the Dp-brane and can be treated as a test string probing the Dp-brane. Furthermore, the effective gravitational couplings \( G_N \tau \) (Newton’s constant times tension) behave like \( G_N \tau_{F1} \sim g_s^2 \), \( G_N \tau_{Dp} \sim g_s \) and therefore we can assume that the spacetime background is approximately flat. The open-string action reads

\[
S = \frac{1}{2\alpha'} \int_{M^2} d^2\sigma \sqrt{-\mathcal{G}} \partial X^\mu \partial X^\nu \eta_{\mu\nu} + \frac{1}{2} \int_{0^2} d\tau \mathcal{F}_{\mu\nu} X^\mu \dot{X}^\nu, \tag{1}
\]

where \( \mathcal{F}_{\mu\nu} \) is the constant background field strength on the Dp-brane. We assume that the only non-vanishing components of \( \mathcal{F} \) are \( \mathcal{F}_{r r'} \), where we have decomposed the worldvolume index \( \mu \) as \( \mu = (r, r') \) with \( r = 0, 1, \ldots, p - \text{rank} \mathcal{F} \) and \( r' = p + 1 - \text{rank} \mathcal{F}, \ldots, p \):

\[
\mathcal{F} = \begin{pmatrix}
\mathcal{F}_{rr} & 0 \\
0 & \mathcal{F}_{rr'}
\end{pmatrix}.
\tag{2}
\]

We consider now the following decoupling limit (see, e.g., [2]). We take \( \epsilon \to 0 \) such that

\[
\eta_{rr'} \sim \epsilon \eta_{rr'}, \quad \alpha' \sim \epsilon^{1/2} \alpha'.
\tag{3}
\]

1 In (3) it is understood that the \( \eta_{rr'} \) and \( \alpha' \) occurring on the right-hand side are \( \epsilon \)-independent.
while keeping all other quantities fixed. The open string action scales as follows:

\[ S \sim \frac{1}{2e^{1/2}\alpha'} \int_{M^2} d^2\sigma \, \tilde{a} X^m \partial X^n \eta_{mn} + \frac{1}{2e^{1/2}\alpha'} \int_{M^2} d^2\sigma \, \tilde{a} X^r \partial X^s \eta_{rs} + \frac{e^{1/2}}{2\alpha'} \int_{\Sigma} d^2\sigma \, \tilde{a} X^r \partial X^s \eta_{rs} + \frac{1}{2} \int_{\partial M^2} d\tau \, F_{r's'} X^r X^s. \]

One may now argue (see, e.g., [2,3]) that the dynamics of the \( F_1/Dp \) system is dominated by the last so-called Wess–Zumino term, i.e.

\[ S \sim \frac{1}{2} \int_{\partial M^2} d\tau \, F_{r's'} X^r X^s. \]

Moreover, the open string metric is finite in this limit and is given by the maximal rank matrix

\[ G_{\mu\nu} = \begin{cases} \eta_{\mu\nu} & \text{for } \mu, \nu = r, s \\ -\alpha'^2 F_{\mu\rho} \eta^{\rho\sigma} F_{\sigma\nu} & \text{for } \mu, \nu = r', s'. \end{cases} \]

The equations of motion corresponding to the Wess–Zumino term read

\[ \dot{X}^r = 0, \]

i.e. there are additional Dirichlet conditions: the endpoint of the string is not allowed to move in the \( r' \) directions. The non-commutative nature of the \( D \)-brane arises from quantizing the Wess–Zumino term (5). Applying the standard canonical quantization procedure leads to the following non-zero Dirac brackets:

\[ \{ X^r(\sigma), X^{s'}(\sigma') \} = (F^{-1})^{r's'} \delta(\sigma - \sigma'). \]

We thus conclude that the string probing the \( Dp \)-brane sees a non-commutative geometry in the \( r' \) directions of the \( Dp \)-brane worldvolume.

2. The M2/M5 system

The M-theory origin of the \( F_1/Dp \) system is a M2/M5 system, i.e. an open membrane ending on a 5-brane in an 11-dimensional supergravity background. The membrane boundary is a string that is constrained to lie within the 5-brane. The action for the open bosonic membrane is as follows:

\[ S = S_k + \int_{M^3} f_2^* C + \int_{\partial M^3} f_1^* b. \]

where the kinetic term can be written in Polyakov form as

\[ S_k = \frac{1}{2(\ell_p)^2} \int_{M^3} d^3\sigma \sqrt{-\det\gamma} \left(-\gamma^{ab} \partial_a X^M \partial_b X^N \tilde{g}_{MN} + \ell_p^2 \right). \]

Here \( \ell_p \) is the \( D = 11 \) Planck constant, \( \tilde{g}_{MN} \) is the \( D = 11 \) spacetime metric and \( \gamma_{ab} \) is the auxiliary worldvolume metric. The maps \( f_2 \) and \( f_1 \) denote the embedding of the membrane and its boundary into the spacetime and the 5-brane, respectively. The worldvolume 3-form \( f_2^* C \) is the pull-back of the \( D = 11 \) 3-form potential \( C \) to the membrane worldvolume and, similarly, \( f_1^* b \) is the pull-back of the 5-brane 2-form potential \( b \) to the boundary of the membrane. In terms of components, we write

\[ (f_2^* C)_{ab} = \partial_a X^M \partial_b X^N \partial_p X^p C_{MNP}, \quad (f_1^* b)_{ij} = \partial_i X^a \partial_j X^b b_{\mu\nu}. \]
where $M = 0, 1, \ldots, 9, 11$ are spacetime indices, $\mu = 0, 1, \ldots, 5$ are 5-brane worldvolume indices, $\alpha = 0, 1, 2$ are membrane worldvolume indices and $i = 0, 1$ are indices on the boundary of the membrane.

The coupling of $b$ to the boundary of the membrane ensures that the open-membrane action is invariant under the spacetime gauge transformations $\delta C = d \Lambda$ provided that $\delta b = - f_5^* \Lambda$, where $f_5$ denotes the embedding of the 5-brane into spacetime. The 2-form $b$ satisfies the 5-brane field equations. These are equivalent to a nonlinear self-duality condition on the following gauge invariant 3-form field strength of $b$:

$$\mathcal{H} = db + f_5^* C. \quad (12)$$

Here the last term is the pull-back of the spacetime 3-form potential to the 5-brane:

$$(f_5^* C)_{\mu\nu\rho} = \partial_\mu x^M \partial_\nu x^N \partial_\rho x^P C_{MNP}, \quad (13)$$

where $x^M(X^\mu)$ are local embedding functions satisfying the 5-brane equations of motion.

We shall consider backgrounds where $H_{\mu\nu\rho}$ is constant. This is only consistent with (12) provided we require that the pull-back of the spacetime 4-form field strength $F = dC$ to the 5-brane vanishes, i.e. $f_5^* F = 0$. It is convenient to write $C = \tilde{C} + dC_2$ with $f_5^* \tilde{C} = 0$ and $f_5^* C_2 = c$. This enables us to rewrite the following bulk term as a boundary term:

$$\int_{M^3} f_5^* C = \int_{\partial M^3} f_5^* C_2 = \int_{\partial M^3} f_5^* f_5^* C_2 = \int_{\partial M^3} f_5^* c, \quad (14)$$

where we have applied Stoke’s theorem. Finally, since $f_5^* C = dc$ we have that $\mathcal{H} = d(b + c)$ or

$$(b + c)_{\mu\nu} = \mathcal{H}_{\mu\nu\rho} X^\rho. \quad (15)$$

This enables us to rewrite the Wess–Zumino term as

$$S_{WZ} = \frac{1}{3} \int_{\partial M^3} d^2 \sigma \mathcal{H}_{\mu\nu\rho} X^\mu X^\nu X^\rho. \quad (16)$$

A complicating feature of the M5-brane is that the 3-form curvature $\mathcal{H}$ satisfies a nonlinear self-duality condition [4]:

$$\sqrt{-\det g} \epsilon_{\mu\nu\rho\sigma\lambda\tau} \mathcal{H}^{\rho\sigma\lambda\tau} = \frac{1 + K}{2} (G^{-1})^{\lambda}_{\mu} \mathcal{H}_{\nu\rho\lambda}, \quad (17)$$

where $\epsilon^{012345} = 1$ and the scalar $K$ and the tensor $G_{\mu\nu}$ are given by

$$K = \sqrt{1 + \frac{\ell_6^8}{24} \mathcal{H}^2}, \quad (18)$$

$$G_{\mu\nu} = \frac{1 + K}{2K} \left( g_{\mu\nu} + \frac{\ell_6^8}{4} \mathcal{H}_{\mu\nu} \right). \quad (19)$$

In [3] it was argued that the tensor $G_{\mu\nu}$ is the metric on the 5-brane seen by an open membrane in the presence of a background 3-form field strength $\mathcal{H}$. It is understood that in the above three equations the indices are contracted with the induced 5-brane metric:

$$g_{\mu\nu} = \partial_\mu x^M \partial_\nu x^N \hat{g}_{MN}. \quad (20)$$
It will be useful to introduce a specific parametrization of the solutions of the self-duality condition (17) as follows:\(^2\):

\[ H_{\mu\nu\rho} = \frac{h}{\sqrt{1 + \ell_p^2 h^2}} \epsilon_{\alpha\beta\gamma} v^\alpha_{\mu} v^\beta_{\nu} v^\gamma_{\rho} + h \epsilon_{abu} u^a_{\mu} u^b_{\nu} u^c_{\rho}, \]  

\[ G_{\mu\nu} = \left( \frac{1}{1 + \ell_p^2 h^2} \right)^2 \left( \frac{1}{1 + \frac{1}{2} h^2 \ell_p^6} \epsilon_{\alpha\beta} v^\alpha_{\mu} v^\beta_{\nu} + \delta_{abu} u^a_{\mu} u^b_{\nu} \right). \]

Here \( h \) is a real field of dimension (mass)\(^3\) and \((v^\alpha_{\mu}, u^a_{\mu})\), \(\alpha = 0, 1, 2\), \(a = 3, 4, 5\), are sechtbein fields in the nine-dimensional coset \( SO(5,1)/SO(2,1) \times SO(3) \) satisfying

\[ g^{\mu\nu} v^\alpha_{\mu} v^\beta_{\nu} = \eta^{\alpha\beta}, \quad g^{\mu\nu} u^a_{\mu} u^b_{\nu} = 0, \quad g^{\mu\nu} u^a_{\mu} u^b_{\nu} = \delta^{ab}, \]  

\[ g_{\mu\nu} = \eta_{\alpha\beta} v^\alpha_{\mu} v^\beta_{\nu} + \delta_{abu} u^a_{\mu} u^b_{\nu}. \]

3. Limits on M5

We will now consider a limit of the open-membrane/5-brane system with the main property that the boundary string that lives in the 5-brane is governed solely by the Wess–Zumino term (16). Compared with the case of a string ending on a \( D_p \)-brane we are faced with two problems.

(1) The decoupling limit must be consistent with the nonlinear self-duality condition (17).

(2) Since both \( \tau_{M2} \sim 1 \) and \( \tau_{M5} \sim 1 \) we cannot use the membrane as a probe to study the worldvolume geometry of the M5-brane.

In this paper we will discuss a particular limit that avoids these two problems. Other limits were discussed at this conference by Per Sundell. Problem (1) is circumvented by using the explicit solution for \( H \) given by (21) and (24). To take care of problem (2) we consider, instead of a flat background, a \( D = 11 \) background consisting of a stack of \( N \) parallel 5-branes, given by the solution

\[ ds^2(\hat{g}) = H^{-1/3}(dx^\mu)^2 + H^{2/3}(dy^m)^2, \quad H = 1 + \frac{N_5 \ell_p^3}{\ell}, \quad F = N_5 \epsilon_4, \]  

where \( \mu = 0, 1, \ldots, 5; m = 6, 7, 8, 9, 11 \), \( N_5 \) is the number of stacked 5-branes and \( \epsilon_4 \) is the volume form on the transverse \( S^4 \). We let the open membrane end on one of these 5-branes removed from the stack and placed at radius \( r_0 \). If \( N_5 \gg 1 \) and \( r_0 \) is small, then the interactions between the stack and the separated 5-brane effectively stiffens the latter so that the membrane can probe it without deforming it. Under these conditions the induced metric on the 5-brane (20) is given by

\[ g_{\mu\nu} = H^{-1/3}(r_0) \eta_{\mu\nu}. \]

Moreover, from (25) it follows that the \( D = 11 \) background 4-form field strength satisfies \( f_2^4 F = 0 \). From the discussion in section 2 it follows that we may consider an open-membrane action given by

\[ S = \frac{1}{2(\ell_p)^2} \int_{M'} d^3\sigma \sqrt{-\det \gamma} \left( -H^{-1/3} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} + H^{2/3} \gamma^{\alpha\beta} \partial_\alpha Y^m \partial_\beta Y^n \delta_{mn} + \ell^2 \right) + N_5 \int_{M'} f_2^4 \hat{C} + \frac{1}{3} \int_{\partial M'} d^2\sigma \mathcal{H}_{\mu\nu\rho} X^\mu X^\nu X^\rho, \]  

\(^2\) This parametrization has been derived independently by [5].
where the \( D = 11 \) background 3-form potential \( \tilde{C} \) obeys \( d\tilde{C} = \epsilon_4 \) and \( f_2^* \tilde{C} = 0 \) and the background 3-form field strength \( \mathcal{H}_{\mu
u\rho} \) on the 5-brane is constant.

We now propose the following decoupling limit obtained by taking \( \epsilon \to 0 \) such that

\[
\ell_p \sim \epsilon \ell_p, \tag{28}
\]

\[
N_5 \sim \epsilon^{-\delta} N_5, \tag{29}
\]

\[
h \sim \epsilon^{-\lambda} h. \tag{30}
\]

For simplicity we shall assume that \( \delta > 1 \) such that we may drop the 1 from the harmonic function in the metric (25). It then follows from (26) that the induced 5-brane metric and the sechsbein fields in (24) scale as

\[
g_{\mu\nu} \sim \epsilon^{\delta - 1} g_{\mu\nu}, \quad u^a_{\mu} \sim \epsilon^{\frac{1}{2}(\delta - 1)} u^a_{\mu}, \quad v^a_{\mu} \sim \epsilon^{\frac{1}{2}(\delta - 1)} v^a_{\mu}. \tag{31}
\]

Furthermore, we assume that \( \lambda \leq 3 \). This implies that \( h\ell_p^3 \) remains finite which enables us to keep the 3-form field strength and the open-membrane metric (22) non-degenerate in the limit. Thus we find that the open-membrane action (27) scales as

\[
S \sim \epsilon^{-\Delta} \frac{1}{2\ell_p^2} \int_{M^4} d^3\sigma \sqrt{-\gamma} \left(-\epsilon^{\Delta + \delta - 3} H^{-1/3} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}ight.

\[
- \epsilon^{\Delta - 2\delta} H^{2/3} \gamma^{\alpha\beta} \partial_\alpha Y^m \partial_\beta Y^n \delta_{mn} + \delta \epsilon^2 \ell_p^2 + \frac{1}{2} \int_{\partial M^3} d^2\sigma \mathcal{H}_{\mu\nu\rho} X^\mu X^\nu X^\rho \right], \tag{32}
\]

where we have defined

\[
\Delta = \lambda - \frac{3}{2}(\delta - 1). \tag{33}
\]

We now impose the following requirements for our decoupling limit (for a more detailed discussion, see [3]).

(a) All interactions on the M5-brane worldvolume must vanish.

(b) The bulk modes must decouple.

(c) The open-membrane metric must remain non-degenerate after taking the decoupling limit.

Given these assumptions we find the following restrictions on our parameters [3]:

\[
\Delta + \frac{3}{2}(\delta - 1) \leq 3 < \Delta + \delta, \quad \Delta < 2\delta, \quad 1 < \delta < 3. \tag{34}
\]

These conditions are solved by \((\Delta, \delta)\) in a finite size region. For instance, \( \delta = \frac{5}{4}, \lambda = 3 \) and \( \Delta = 2 \) leads to a decoupled 5-brane theory in a background with a nonlinearly self-dual field strength, while \( \delta = \frac{3}{4}, \lambda = 2 \) and \( \Delta = 2 \) yields a linearly self-dual field strength.

A noteworthy feature is that (34) implies \( \Delta > 0 \), such that there is necessarily an overall scaling of the action in (32). Such a scaling was not required in the string case. A crucial difference between the string and the membrane is that only the string action (1) has a microscopic interpretation. On the other hand, the membrane action (10) should be seen as an effective action. One interpretation of the scaling (32) with \( \Delta > 0 \) is that actually we are taking a semiclassical limit.

Summarizing, in order to understand the geometry of the 5-brane worldvolume we are led to study the quantization of the Wess–Zumino action (16) with \( \mathcal{H} \) constant.
4. Canonical analysis

We now will canonically quantize the action (16) with constant field strength $H_{\mu\nu\rho}$. For convenience, we assume that the field strength can be diagonalized as follows [2]:

\[ H_{012} = - \frac{h}{\sqrt{1 + \ell_p^6 h^2}}, \quad H_{345} = h, \]  

where the dimensionless combination $h\ell_p^3$ is non-vanishing provided the decoupling limit (30) has been taken with $\lambda = 3$.

In the parametrization (35) the action (16) splits into two independent Lagrangians for the two sets of coordinates $X^{0,1,2}$ and $X^{3,4,5}$:

\[ S = \frac{h}{3\sqrt{1 + \ell_p^6 h^2}} \int_{\partial M^4} d^2\sigma \epsilon_{\alpha\beta\gamma} \dot{X}^\alpha X'^\beta X''^\gamma + \frac{h}{3} \int_{\partial M^4} d^2\sigma \epsilon_{abc} \dot{X}^a X'^b X''^c, \]  

where $\alpha = 0, 1, 2$ and $a = 3, 4, 5$. The action is invariant under worldsheet reparametrizations:

\[ \delta_\xi X^\alpha = \xi_i \partial_i X^\alpha, \quad \delta_\eta X^a = \eta_i \partial_i X^a, \quad i = 0, 1. \]

Note that, due to the absence of a worldsheet metric, there is no need to identify the vector fields $\xi$ and $\eta$.

The equations of motion are

\[ \epsilon_{\alpha\beta\gamma} \dot{X}^\alpha X'^\beta X''^\gamma = 0, \quad \epsilon_{abc} \dot{X}^a X'^b X''^c = 0. \]

Assume now that the string boundary inside the M5-brane has a non-compact extension in the time direction. In that case we can impose the gauge choice $X^0 = \tau$. Substituting this into the equations of motion we obtain

\[ X'^a = 0, \]  

which means that the spatial extension of the string must be in the $a$ direction. Assuming that $|\vec{X}'| \neq 0$ we obtain

\[ \vec{X}' = 0, \]  

which implies additional Dirichlet conditions in the $a$ directions.

Let us continue by analysing the phase space dynamics of the three coordinates $\vec{X} = (X^0, X^1, X^2)$. The canonical momenta are given by

\[ \Pi_a(\sigma) := \frac{\delta S}{\delta \dot{X}^a(\sigma)} = -\frac{1}{2} h \epsilon_{apq} \dot{X}^p X'^q, \]

indicating that there are three primary constraints $\phi_a(\sigma)$:

\[ \phi_a := \Pi_a + \frac{1}{2} h \epsilon_{abc} \dot{X}^b X'^c \approx 0. \]

The non-trivial canonical Poisson brackets are

\[ \{X^a(\sigma), \Pi_b(\sigma')\} = \delta^a_b \delta(\sigma - \sigma'), \]

and the non-zero Hamiltonian is given by

\[ H = \int d\sigma \lambda^a(\sigma) \phi_a(\sigma), \]
The M5-brane and non-commutative loop space

where \(\lambda^a(\sigma)\) are three Lagrange multipliers. To proceed with the canonical analysis we study the consistency conditions

\[
\dot{\phi}_a(\sigma) = \lambda^b(\sigma) M_{ba}(\sigma) \approx 0,
\]

(45)

where

\[
\{\phi_a(\sigma), \phi_b(\sigma')\} = M_{ab}(\sigma) \delta(\sigma - \sigma'), \quad M_{ab} = \hbar \epsilon_{abc} X'^c.
\]

(46)

Note that in the \(\alpha\) space we can impose \(X^0 = \tau\) and, via the equations of motion, \(X'^a = 0\). This implies that \(M_{ab} = 0\). In other words, the three primary constraints \(\phi_a(\sigma)\) are all first class.

In contrast, let us now consider the canonical analysis of the three Euclidean coordinates \(\vec{X} = (X^3, X^4, X^5)\). A similar analysis as above leads to the same result except that in this case we have assumed that \(|\vec{X}'| \neq 0\) and therefore

\[
M_{ab}X'^b = 0.
\]

(47)

The matrix \(M_{ab}\) is thus non-degenerate in the two-dimensional subspace orthogonal to \(\vec{X}'\). It is convenient to introduce a projection onto this subspace as follows \((I = 1, 2)\):

\[
P_I^a(\sigma) P_J^b(\sigma) \delta_{ab} = \delta_{IJ},
\]

(48)

\[
\delta^{IJ} P_I^a(\sigma) P_J^b(\sigma) = \delta^{ab} - \frac{X'^a X'^b}{|\vec{X}'|^2},
\]

(49)

\[
\epsilon^{IJ} P_I^a(\sigma) P_J^b(\sigma) = \frac{\epsilon^{abc} X'^c}{|\vec{X}'|}.
\]

(50)

The three constraints \(\phi_a\) now split into the two second-class constraints

\[
\chi_I := P_I^a \phi_a,
\]

(51)

with the now non-degenerate matrix

\[
\{\chi_I(\sigma), \chi_J(\sigma')\} := M_{IJ}(\sigma) \delta(\sigma - \sigma'), \quad M_{IJ} = P_I^a P_J^b M_{ab},
\]

(52)

and one first-class constraint

\[
\phi := X'^a \phi_a = X'^a \Pi_a,
\]

(53)

which acts as the generator of \(\sigma\) reparametrizations.

The presence of the two second-class constraints leads to a non-trivial Dirac bracket between the \(X^a\) coordinates given by

\[
[X^a(\sigma), X^b(\sigma')]^D = -\frac{1}{\hbar} \frac{\epsilon^{abc} X'^c(\sigma)}{|\vec{X}(\sigma)|^2} \delta(\sigma - \sigma').
\]

(54)

The conclusion is that the membrane probe sees a so-called non-commutative loop space geometry in the \(a\) directions of the M5-brane worldvolume.
5. Non-commutative loop space

The main conclusion of this paper is that, whereas $D$-branes lead to a non-commutative geometry of points, the M5-brane seems to lead to a non-commutative geometry of loops. To the best of our knowledge, such a non-commutative loop space geometry has not been considered before in the literature.

As a historical note, it is perhaps of interest to note that, whereas the idea of lightlike integrability applied to a superspace geometry naturally leads to the superspace constraints of Yang–Mills [6], the same idea when applied to a loop superspace geometry leads to the constraints of supergravity coupled to Yang–Mills [7]. In the latter work the definition of a loop space covariant derivative plays a central role. The gauge field part of this covariant derivative is given by the pull-back of the self-dual antisymmetric tensor, i.e.

$$D_{\mu}(\sigma) = \frac{\delta}{\delta X^\mu(\sigma)} + b_{\nu\mu}X^\nu. \quad (55)$$

Through this paper we are naturally led to consider a non-commutative version of loop superspace. One of the open questions is how to exactly construct a covariant derivative corresponding to such a non-commutative loop space. It suggests that this problem is related to the problem of how to construct a field theory for a set of $D = 6 (2, 0)$ non-Abelian tensor multiplets. The analogy is as follows. On the one hand, in the non-commutative case, one must replace the term $b_{\nu\mu}X^\nu$, present in the covariant derivative, by some non-commutative generalization with

$$\{X^\mu, X^\nu\} \neq 0. \quad (56)$$

On the other hand, in the non-Abelian case, one must replace this term by some non-Abelian generalization, i.e. $b_I^{\nu\mu}X^\nu T^I$ with

$$[T^I, T^J] \neq 0. \quad (57)$$

More generally, the suggestion is that, in order to describe a set of $D = 6 (2, 0)$ tensor multiplets, it will not suffice to work with a local field theory but instead, one should work with a non-local loop space where in the covariant derivative one makes the replacement:

$$b_{\nu\mu}(X)X^\nu \Rightarrow A_{\mu}(X(\sigma)). \quad (58)$$

The antisymmetric tensor is just one of the many components of the gauge field $A_{\mu}(X(\sigma))$. In this way one would also circumvent the no-go theorem of [8]. It would be of interest to investigate these issues in more detail.

Acknowledgments

The work described in this paper is based upon [3]. We are grateful to the organizers of the conference for providing such a stimulating environment. This work is supported by the European Commission TMR programme ERBFMRX-CT96-0045, in which EB, DB and JPvdS are associated with the University of Utrecht. The work of JPvdS and PS is part of the research programme of the ‘Stichting voor Fundamenteel Onderzoek der Materie’ (FOM).

References


(Douglas M R and Hull C 1997 Preprint hep-th/9711165)
The M5-brane and non-commutative loop space

   (Seiberg N and Witten E 1999 Preprint hep-th/9908142)

   Phys. B 590 173

   (Howe P S and Sezgin E 1996 Preprint hep-th/9607227)


   468 228
   (Bekaert X, Henneaux M and Sevrin A 1999 Preprint hep-th/9909094)