Chapter 2

Theory of many-particle systems. The Green function method.

Abstract

In this chapter we introduce the fundamental concepts behind the Green Function Theory. We start from a description of the second quantization method and then, after introducing the ensemble average of an operator, we define the main object of the theory, the Green function. Further on, after obtaining the equation of motion for the Green function, we introduce the self-energy operator and we discuss the Hartree-Fock approximation, the second-Born approximation and the $GW$ approximation to the self-energy.
2.1 Second quantization

Let us consider a system of \( N \) identical, non-relativistic particles. The physical state of each particle \( j \) is described by the label \( x_j \). The wave function for this system is \( \Psi(x_1 \ldots x_N) \). If the particles are identical, then the probability density \( |\Psi(x_1 \ldots x_N)|^2 \) must be unchanged under arbitrary exchanges of the labels we use to identify each particle. If the system is described by a scalar\(^1\), \( \Psi \) must transform according to a 1-D representation of the permutation group. Hence, for the system under consideration, we have two cases: a) \( \Psi \) is even under permutation \( (P \Psi = \Psi) \); and b) \( \Psi \) is odd under permutation \( (P \Psi = -\Psi) \). The latter case represents a system of bosons and the former a system of fermions. Since in this work we are only concerned with fermionic systems of bosons and the former a system of fermions.

While the usual representation of quantum mechanics can be used to represent the full Hilbert space of \( N \) particles, we will neglect the bosonic case hereafter. Since in this work we are only concerned with fermionic systems, we will neglect the bosonic case hereafter.

The method which removes all these shortcomings is the second quantization [3, 4]. Within this method the operator \( \hat{\psi}^\dagger(x) \) is called creation operator and the operator \( \hat{\psi}(x) \) is called annihilation operator. By applying a creation operator, a particle is added to the system and by applying an annihilation operator, the particle is removed [5]. These operators satisfy the commutation relations
\[
\hat{\psi}^\dagger(x) \hat{\psi}(x') + \hat{\psi}(x') \hat{\psi}^\dagger(x) = \delta(x - x'),
\]
\[
\hat{\psi}(x) \hat{\psi}(x') + \hat{\psi}(x') \hat{\psi}(x) = 0.
\]

Using the definitions of the field operators and their commutation relations 2.1 and 2.2 we can express the usual operators in terms of field operators. If we consider a general one-body operator
\[
\hat{O} = \int dx \hat{\psi}^\dagger(x) \sigma(x) \hat{\psi}(x),
\]
and the two-particle interaction
\[
\hat{W} = \frac{1}{2} \int dx_1 dx_2 \hat{\psi}^\dagger(x_1) \hat{\psi}^\dagger(x_2) w(r_1, r_2) \hat{\psi}(x_2) \hat{\psi}(x_1),
\]
\(1\)For higher order objects, such as tensors, higher order representations of the permutation group are possible.

one can easily write the general Hamiltonian in the second quantization form
\[
\hat{H} = \int dx \hat{\psi}^\dagger(x) \hat{h}(r, t) \hat{\psi}(x) + \frac{1}{2} \int dx_1 dx_2 \hat{\psi}^\dagger(x_1) \hat{\psi}^\dagger(x_2) w(r_1, r_2) \hat{\psi}(x_2) \hat{\psi}(x_1),
\]

where \( w(r_1, r_2) = \frac{1}{r_1 - r_2} \) and where the one-body part of the Hamiltonian is
\[
h(r, t) = -\frac{1}{2} \nabla^2 + v(r, t) - \mu.
\]

In Eq. 2.6 we introduced the chemical potential \( \mu \) and the external potential \( v(r, t) \) which will be switched on at \( t = t_0 \). Hence, for \( t < t_0 \) the Hamiltonian is time-independent.

Using the above considerations, we can now define our strategy and set up further the formalism. We want to study the time-evolution of a many-particle fermion system, after it has been perturbed from the ground state, or statistical equilibrium. We have seen that the formalism of the second quantization is appropriate since it allows for extending quantum mechanics to macroscopic number of particles and to describe the dynamical response and internal correlations of large systems. In the next section we will briefly go through the steps to determine the time-dependent expectation value for an operator in the grand canonical ensemble.

2.2 Evolution of ensembles

In the grand canonical ensemble [5, 2], the expectation value of an operator \( \hat{O} \), for a system at temperature \( T \) with the chemical potential \( \mu \), is
\[
\langle \hat{O} \rangle = \frac{\sum_i \langle \Psi_i | \hat{O} | \Psi_i \rangle e^{-\beta E_i}}{\sum_i e^{-\beta E_i}},
\]
where \( \{|\Psi_i\rangle\} \) is a complete set of states in the Fock space, \( \beta = 1/k_B T \), with \( k_B \) the Boltzmann constant and \( E_i \) are the eigenvalues of the Hamiltonian \( \hat{H} \). Note that the chemical potential is included in the one-body part. If the system evolves in time, the expectation value becomes [2]
\[
\langle \hat{O}(t) \rangle = \text{Tr} \{ \hat{P} \hat{O}(t) \},
\]
where the statistical operator \( \hat{P} \) acts as a weight function for the Heisenberg representation of the operator \( \hat{O} \), i.e.
\[
\hat{O}_H(t) = \hat{U}(t_0, t) \hat{O} \hat{U}(t, t_0),
\]
and where \( \hat{U} \) is an evolution operator.

Because the time-dependent quantities are switched on...
2.4. SELF-ENERGY APPROXIMATIONS

The one-particle Green function \( K_{\text{eldysh}} \) [7] provides us with the all ground state expectation values of one-particle operators, as well as the total energy. If we consider the Green function at time \( t_1 = t_0 - i\beta \) and use the cyclic property of the trace, we find that Green function defined in Eq. (2.13) obeys the boundary conditions

\[
G(x_1 t_0, 2) = -G(x_1 t_0 - i\beta, 2), \quad (2.14)
\]
\[
G(1, x_2 t_0) = -G(1, x_2 t_0 - i\beta). \quad (2.15)
\]

These boundary conditions are sometimes referred as the Kubo-Martin-Schwinger boundary conditions [9, 10, 11]. If we explicitly write the time-ordering operator for the Green function, we obtain

\[
G(1, 2) = \theta(t, t') G^>(1, 2) + \theta(t', t) G^<(1, 2). \quad (2.16)
\]

with \( \theta(t_1, t_2) \) being contour step functions generalized to time arguments on the contour i.e. \( \theta(t_1, t_2) = 1 \) if \( t_1 > t_2 \) and \( \theta(t_1, t_2) = 0 \) otherwise [12]. The greater and lesser components, \( G^> \) and \( G^< \) respectively, in Eq. 2.16, have the explicit form

\[
G^>(1, 2) = -i\langle \hat{\psi}_H(1) \hat{\psi}_H^\dagger(2) \rangle, \quad (2.17)
\]
\[
G^<(1, 2) = i\langle \hat{\psi}_H^\dagger(2) \hat{\psi}_H(1) \rangle. \quad (2.18)
\]

The greater component can be interpreted as a propagation of a particle in the system and the lesser component, as the propagation of a hole [13]. By taking the Fourier transform of the \( G^> \) component, we obtain a function that is peaked at the removal energies and by taking the Fourier transform of the \( G^< \) component we get a function peaked at addition energies (affinities) [14].

2.4 Self-energy approximations

By taking the commutators \([\hat{\psi}(x), \hat{O}]\) and \([\hat{\psi}(x), \hat{W}]\), we can immediately obtain the commutators of the

Green functions differ in the nature of the averaging process performed in order to obtain them, in the time arguments on which they depend and in their analytic properties.
field operators with the Hamiltonian, \([\hat{\psi}(x), \hat{H}]\) and \([\hat{\psi}^\dagger(x), \hat{H}]\). Further we have the equation of motion for the field operators

\[
i\partial_1 \hat{\psi}(1) = [\hat{\psi}(1), \hat{H}(1)] = h(1)\hat{\psi}(1) + \int d2w(1, 2)\hat{\psi}^\dagger(2)\hat{\psi}(2),
\]

\[
i\partial_1 \hat{\psi}^\dagger(1) = [\hat{\psi}^\dagger(1), \hat{H}(1)] = -h(1)\hat{\psi}^\dagger(1) - \int d2w(1, 2)\hat{\psi}^\dagger(1)\hat{\psi}(2),
\]

with \(w(1, 2) = \delta(t_1, t_2)/|r_1 - r_2|\) being the Coulomb interaction. If we take the derivative of the Green function 2.16, in respect to \(t_1\) and \(t_2\), and we use 2.19 and 2.20 respectively, we obtain the equation of motion for the Green function

\[
i\partial_1 G(1, 2) = \delta(1, 2) + h(1)G(1, 2)
\]

\[
- i \int d3w(1^+, 3)(T_C[\hat{\psi}(1)^\dagger(2)\hat{\psi}(2)\hat{\psi}(3)]),
\]

\[
i\partial_2 G(1, 2) = \delta(1, 2) + h(2)G(1, 2)
\]

\[
- i \int d3w(2^+, 3)(T_C[\hat{\psi}(1)^\dagger(2)\hat{\psi}(1)\hat{\psi}(3)^\dagger]),
\]

The product of field operators in the last term of 2.21 and 2.22 is the two particle Green function which describes the propagation of two particles, two holes or a particle and a hole through the system. We rewrite the Eqs. 2.21-2.22 as

\[
i\partial_1 G(1, 2) = \delta(1, 2)
\]

\[
+ h(1)G(1, 2) - i \int d3w(1^+, 3)G_2(1, 3, 3^+, 2),
\]

\[
i\partial_2 G(1, 2) = \delta(1, 2)
\]

\[
+ h(2)G(1, 2) - i \int d3w(2^+, 3)G_2(1, 3, 3^+, 2),
\]

where \(G_2\) is the two particle Green function was defined from the \(n\)-particle Green function

\[
G_n(1, \ldots, n, 1', \ldots, n') = (-i)^n(T_C[\hat{\psi}(1), \ldots, \hat{\psi}(n), \hat{\psi}^\dagger(1'), \ldots, \hat{\psi}^\dagger(n')].
\]

It can be proved by induction that the \(n\)-body Green function depends on the \(n + 1\)-body Green function [10, 3]. At the first level, the truncation of this hierarchy is performed by introducing the self-energy operator \(\Sigma\) such that \(-iG_{2V} = \Sigma G\) [14]. In other words, we define the self-energy \(\Sigma\) and its adjoint \(\Sigma^\dagger\) by

\[
\int d2\Sigma(1, 2)G(2, 1') = -i \int d2w(1^+, 2)G_2(1, 2, 2^+, 1'),
\]

\[
\int d2\Sigma(1, 2)\Sigma(2, 1') = -i \int d2w(1^+, 2)G_2(1, 2, 2^+, 1').
\]

One can show that for initial equilibrium conditions the self-energy operator \(\Sigma\) and its adjoint \(\Sigma^\dagger\) are the same. From here on we will assume that this is the case. With the considerations above, the equations of motion of the Green function will be written as [12]

\[
i\partial_1 G(1, 2) = \delta(1, 2) + h(1)G(1, 2)
\]

\[
+ \int d2\Sigma(1, 3)G(3, 2),
\]

\[
- i\partial_2 G(1, 2) = \delta(1, 2) + h(2)G(1, 2)
\]

\[
+ \int d2G(1, 3)\Sigma(3, 2).
\]

If we take the functional derivative of the Green function in respect to the external field, we find that the time ordered product of four field operators equals the functional derivative of the one-particle Green function plus the product of the expectation value of the density operator and the one-particle Green function

\[
-i(T_C[\hat{\psi}(1)^\dagger(2)\hat{\psi}^\dagger(3)]) = \frac{\delta G(1, 2)}{\delta v(3)}
\]

\[
+ \langle n_T(3)\rangle G(1, 2).
\]

This follows directly from the equation of motion for the evolution operator. Making use of the definition of the inverse Green function \(\int G(1, 2)G^{-1}(2, 1) = \delta(1, 2)\), we obtain an expression for the functional derivative of the one-particle Green function in respect to the external potential

\[
\frac{\delta G(1, 2)}{\delta v(3)} = - \int d4d5G(1, 4)\frac{\delta G^{-1}(4, 5)}{\delta v(3)}G(5, 2).
\]

If we define

\[
\frac{\delta G^{-1}(4, 5)}{\delta v(3)} = \Gamma(45; 3)
\]

to be the vertex function. Further, making use Eq. 2.29 and of

\[
G^{-1}(1, 2) = (i\partial_1 - h(1))\delta(1, 4) - \Sigma(1, 4),
\]

we write the vertex function \(\Gamma\) in the form

\[
\Gamma(14; 3) = \delta(1, 2)\delta(1, 4) + \frac{\delta \Sigma(1, 4)}{\delta v(3)}.
\]
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By inserting 2.29, with 2.32, into 2.28, we can rewrite the equation of motion for the one-particle Green function as

\[
[i\partial_{t_1} - h(1)G(1, 1')] = \delta(1, 1') \\
+ \int d\bar{t} \left[ i \int d^2dG(1, 3)w(1^+, 2)\Gamma(3; 2) \
+ \delta(1, 4)\delta(3, 2)w(4, 3)(\hat{n}(3)) \right] G(4, 1').
\] (2.33)

Using the expression for the density \( \langle \hat{n}(3) \rangle = -iG(3, 3^+) \) together with Eqs. 2.26 and 2.23, we can identify the self-energy as

\[
\Sigma(1, 4) = i \int d^2d\bar{t} \left[ G(1, 3)w(1, 2)\Gamma(3; 2) \
- \delta(1, 2)\delta(3, 2)w(1, 3)G(3, 3^+) \right].
\] (2.34)

We can now use the expression for the vertex \( \Gamma \) in Eq. 2.32 together with Eq. 2.34 to generate higher order approximations. If we insert 2.32 in 2.34 we have

\[
\Sigma(1, 4) = iG(1, 2)w(1^+, 2) \\
- i\delta(1, 2) \int d\bar{t}w(1, 3)G(3, 3^+) \\
+ i \int d^2dG(1, 3)w(1^+, 4)\delta\Sigma(3, 2) \] (2.35)

If we consider in the equation 2.35 only the first order terms in \( w \), we obtain the Hartree-Fock self-energy

\[
\Sigma(1, 2) = iG(1, 2)w(1^+, 2) - i\delta(1, 2) \int d\bar{t}w(1, 3)G(3, 3^+).
\] (2.36)

This approximation is the lowest order approximation for the many-body effects.

2.4.1 The second Born approximation

Inserting 2.36 back into 2.35 leads to the second Born approximation to the self-energy. By keeping the terms to second-order in \( w \) we have

\[
\Sigma(1, 2) = \Sigma^{HF}(1, 2) + \Sigma^{(2)}(1, 2),
\] (2.37)

where \( \Sigma^{HF} \) is the Hartree-Fock part of the self energy 2.36 and \( \Sigma^{(2)} = \Sigma^{(2a)} + \Sigma^{(2b)} \) is the sum of the two terms

\[
\Sigma^{(2a)}(1, 2) = -i^2G(1, 2) \int d^2d4w(1, 3) \\
\times G(3, 4)G(4, 3)w(4, 2),
\] (2.38)

\[
\Sigma^{(2b)}(1, 2) = i^2 \int d^2d4G(1, 3)w(1, 4)G(3, 4) \\
\times G(4, 2)w(3, 2).
\] (2.39)

These terms are usually referred to as the second-order direct and exchange terms. They can be represented diagrammatically as the two diagrams to second order in the two-particle interaction [15, 16]. Further iterations of Eq. 2.35 lead to higher order approximations.

2.4.2 The GW approximation

In the GW approximation the self-energy is expanded in terms of the Green function and the dynamically screened interaction \( W \) [17]. The screened interaction \( W \) describes the dynamical shielding in the Coulomb potential. Including the effective interaction of the electrons, the screened interaction is defined as

\[
W(1, 2) = \int d^2w(1, 3)\frac{\delta V(2)}{\delta v(3)}.
\] (2.40)

where

\[
V(2) = v(2) + \int d^2w(2, 4)(\hat{n}(4)),
\] (2.41)

and describes the effects of a test charge at point 2 on the potential at point 1. By substituting 2.41 in 2.40 and taking the functional derivative, we can use the identity

\[
\frac{\delta}{\delta v(1)} = \int d^2\frac{\delta V(2)}{\delta v(1)} \frac{\delta}{\delta V(2)},
\] (2.42)

to obtain

\[
W(1, 2) = \int d^2w(1, 3)
\] (2.43)

\[
\times \left[ \delta(2, 3) + \int d^2d5w(2, 4)\left(\delta(\hat{n}(4)), \delta V(5) \right) \right].
\]

In Eq.2.43 we identify the irreducible polarizability

\[
P(4, 5) = \frac{\delta(\hat{n}(4))}{\delta V(5)},
\] (2.44)

density response to the effective field. We take the functional derivative of the Green function with respect to \( V(2) \) and we obtain

\[
\frac{\delta G(1, 2)}{\delta V(3)} = -\int d^4d5G(1, 4)\frac{\delta G^{-1}(4, 5)}{\delta V(3)}G(1, 5),
\] (2.45)

where, using the definition of the inverse Green function, we define the vertex \( \Gamma \) as

\[
\Gamma(12; 3) = -\frac{\delta G^{-1}(1, 2)}{\delta V(3)} = \delta(1, 2)\delta(1, 3) + \frac{\delta \Sigma(1, 2)}{\delta V(3)}.
\] (2.46)

Note that this vertex is different from the vertex in Eq. 2.32. From the definition of the density \( \langle \hat{n}(4) =
\[ -iG(4, 4^+) \text{ we can rewrite the irreducible polarizability as} \]
\[ P(4,7) = i \int d5d6G(4,5) \frac{\delta G^{-1}(5,6)}{V(7)} G(6,4), \quad (2.47) \]
and we can use the vertex function 2.46 to write it in the form
\[ P(4,7) = i \int d5d6G(4,5)\Gamma(56,7)G(6,4). \quad (2.48) \]

From Eq. 2.21 and 2.28 we identify the self-energy as
\[ \Sigma(1,2) = -i\delta(1,2) \int d3w(1,3)G(3,3^+) \]
\[ -i \int d3dw(1,3)G(1,4) \frac{\delta G^{-1}(4,2)}{\delta v(3)} \]
where the first term on the right hand side is the Hartree term and the second term represents the exchange-correlation part. We can express the self-energy in terms of screened interaction and vertex function by using the identity 2.42. We obtain
\[ \Sigma(1,2) = i\delta(1,2) \int d3w(1,3)G(3,3^+) \]
\[ + i \int d3d4w(1,3)G(1,4)\Gamma(42;3). \quad (2.50) \]

Finally, we apply the chain rule in the last term of Eq. 2.46 and we obtain a self-consistent expression for the vertex function
\[ \Gamma(12;3) = -\frac{\delta G^{-1}(1,2)}{\delta v(3)} = \delta(1,2)\delta(1,3) \quad (2.51) \]
\[ + \int d4d5d6d7 \frac{\delta \Sigma(1,2)}{\delta G(4,5)} G(4,6)\Gamma(67;3)G(7,5), \]
which together with the self-energy 2.50, the irreducible polarizability 2.48 and the screened interaction \[ W(1,2) = w(1,2) + i \int d3d4d5dw(2,4)W(1,7)P(4,7) \quad (2.52) \]
constitute the Hedin equations [17]. We have achieved a systematic expansion of the self-energy in G and W. This is preferable for extended systems with long range interactions, where W is usually much smaller than the bare interaction w [18]. These equations are to be solved iteratively to obtain the self-energy Σ. However, such a calculation is, in principle, not possible even for very simple systems, mainly due to the presence of the vertex. We can take the simplest approximation to the vertex function 2.51, i.e., \[ \Gamma(12;3) = \delta(1,2)\delta(1,3) \]
and obtain the GW approximation [17]. With this approximation to the vertex Γ, the self-energy depends on the self-consistent Green function and the screened potential W, calculated from the full Green function and therefore the equations must be solved self-consistently.

2.5 Conserving approximations

One of the most important issues when making an approximation is the satisfaction of the conservation laws. In Green Function Theory it is possible to guarantee that the macroscopic conservation laws, such as those of particle, momentum and energy conservation, are obeyed. Baym showed [19] how the macroscopic conservation laws are related to the invariant properties of a functional \( \Phi \). If we know the Green function, we can calculate the density and the current density from
\[ \langle \hat{n}(1) \rangle = iG(1,1^+) \quad (2.55) \]
\[ \langle \hat{j}(1) \rangle = -i \left\{ \frac{\nabla_1}{2i} - \frac{\nabla_1^*}{2i} \right\} G(1,1^+) \quad (2.56) \]

These two quantities must satisfy the continuity equation
\[ \partial_t \langle \hat{n}(1) \rangle + \nabla \cdot \langle \hat{j}(1) \rangle = 0, \quad (2.57) \]
which relates the accumulation of charge in a spatial region to the current flow in that region. This can be readily proved by subtracting 2.26 and 2.27 (with \( 2 \to 1^+ \))
\[ [i\partial_t - i\partial_{1^+} - h(1) + h(1^+)]G(1,1^+) = \int d\vec{r}[\Sigma(1,1^+)G(1,1^+) - G(1,1^+)\Sigma(1,1^+)]. \]

If the system is perturbed by a potential corresponding to a gauge transformation, the one body part of the Hamiltonian becomes
\[ h_\rho(1) = \frac{1}{2} |\nabla - i\nabla A|^2 + i \frac{\partial A(1)}{\partial t} + v(1) - \mu. \quad (2.59) \]

With this form of the single-particle Hamiltonian, and the conservation of particles at vertices [16], we can
\[ \Phi[G] = \sum_{n,k} \frac{1}{2n} \text{Tr}[\Sigma_n^{(n)}G] \quad (2.53) \]
where \( n \) indicates the number of interaction lines and the index \( k \) labels topologically different self-energy diagrams. The trace implies a summation over all indices and a frequency integration. In other words they proved that for the exact self-energy there is a functional \( \Phi \) of \( G \) such that
\[ \Sigma(1,2) = \frac{\delta \Phi}{\delta G(2,1)}. \quad (2.54) \]
show that for the interacting Green function and the self-energy we have the transformations
\[ G(1, 2; \Lambda) = e^{-i\Lambda(1)} G(1, 2) e^{i\Lambda(2)}, \] (2.60)

\[ \Sigma(1, 2; \Lambda) = e^{-i\Lambda(1)} \Sigma(1, 2) e^{i\Lambda(2)}. \] (2.61)

This solutions obey the boundary condition if we assume \( \Lambda(t_0) = \Lambda(t_0 - i\beta) \). A first order change in \( G \) due to the gauge transformation is
\[ \delta G(1, 2) = -i(\delta \Lambda(1) - \delta \Lambda(2)) G(1, 2). \] (2.62)

The exponential factors cancel at the internal vertices of \( \Sigma \) because factors from the particles entering a vertex cancel the factors from outgoing particles. The only factors remaining are those at external vertices. The corresponding change in the functional \( \Phi \) is given by
\[ \delta \Phi = i \int d1d2 \Sigma(2, 1) G(1, 2) (\delta \Lambda(2) - \delta \Lambda(1)) = i \int d1d2 \Sigma(2, 1) G(1, 2) - \Sigma(1, 2) G(2, 1) \delta \Lambda(2). \]

On the other hand, from the fact that \( \Phi[G(\Lambda)]=\Phi[G] \) we have \( \delta \Phi/\delta \Lambda = 0 \) and therefore from Eq. 2.63 it follows that
\[ \int d2 \{ \Sigma(2, 1) G(1, 2) - G(2, 1) \Sigma(1, 2) \} = 0. \] (2.63)

So the integral part of the right hand term of Eq. 2.58 is zero. From Eq. 2.58 and Eq. 2.55-2.56 it follows that the continuity equation is satisfied
\[ \partial_t \langle n(1) \rangle = -\nabla_v \langle j(1) \rangle. \] (2.64)

The conservation law for momentum follows from considering a translation of the coordinate system by a vector \( \mathbf{R}(t) \). This implies an observer whose origin of coordinates is at the time varying point \( \mathbf{R}(t) \) and who will describe the system with an extra term of the first order in \( \mathbf{R} \) added to the Hamiltonian. In analogy with the above considerations for particle conservation, we obtain the equation
\[ \frac{d}{dt} \langle P \rangle = -\int \delta x_1 \langle n(1) \rangle \nabla_v w(1). \] (2.65)

The proof for energy conservation involves the description for the \( \Phi \)-invariance when the system is described by an observer with a "rubbery clock" [19], since the energy conservation requires time-translational invariance. Following the same lines as for momentum conservation proof, we obtain
\[ \frac{d}{dt} \langle H_0 \rangle = -\int \delta x_1 \langle j(1) \rangle \cdot (-\nabla_v v(1)). \] (2.66)

where \( H_0 \) represents the system without the added field \( \langle H_0 \rangle = \langle H - V \rangle \). In Ref. [19], Baym has shown that the condition for a self-energy approximation to be conserving is that is has to be \( \Phi \)-derivable. This means that for the approximate self-energy there is a functional \( \Phi \) such that the equation (2.54) is obeyed. Such approximations to the self-energy are called conserving or \( \Phi \)-derivable approximations. Well-known conserving approximations are the Hartree approximation — where \( \Phi = 0 \) —, the Hartree-Fock approximation [16], the second Born [15] approximation, the \( GW \) approximation [17] and the \( T \)-matrix [15] approximation.

### References


