Model reduction of port-Hamiltonian systems
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11. Conclusions

Port-based network modeling of physical systems leads directly to their representation as port-Hamiltonian systems. The port-Hamiltonian structure can be exploited for analysis and control. In particular, if the Hamiltonian function is non-negative then port-Hamiltonian systems are passive. At the same time network modeling of physical systems often leads to high-dimensional dynamical models. Large state-space dimensions are obtained as well if distributed parameter models are spatially discretized. Therefore an important issue concerns the structure preserving model reduction of these high-dimensional systems, both for analysis and control.

This thesis offers a series of structure preserving model reduction methods for linear port-Hamiltonian systems. The resulting reduced order models retain the port-Hamiltonian structure, and, therefore, if the Hamiltonian function is non-negative, the properties of passivity and stability. Preservation of the port-Hamiltonian structure implies as well the ability to interconnect the reduced order models with other subsystems in complex networks, e.g. electrical circuits.

11.1. Contributions

- Observability and controllability of port-Hamiltonian systems are in general not equivalent, even though there are important subclasses of port-Hamiltonian systems where this is the case. We have shown in Chapter 2 that non-minimal (uncontrollable/unobservable) port-Hamiltonian systems by the use of the Kalman decomposition can be reduced both in energy and co-energy coordinates to minimal (controllable/observable) systems which are again port-Hamiltonian. Minimality reduction procedures motivate two approximate reduction procedures – the effort- and the flow-constraint methods – for general port-Hamiltonian systems.

- A family of four structure preserving reduction methods for general port-Hamiltonian systems is obtained in Chapter 3 by reducing underlying full order Dirac structures. These structure preserving methods result in reduced order systems which are port-Hamiltonian by construction. The bond-graph modeling framework explains two of these
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structure preserving reduction methods – the effort-constraint method (seen already in Chapter 2) and the flow-constraint method. The effort-constraint method, apart from preserving the port-Hamiltonian structure, turns out to be related to projection-based methods, preserving the moments of the full order port-Hamiltonian system at specific points in the complex plane, provided that the full order system is in suitable coordinates. These coordinates as well as balanced coordinates are suggested for the effort- and flow-constraint methods, even thought the choice of the best coordinate system is a topic of future research.

• In Chapter 4 we show that positive real balancing can be used to reduce port-Hamiltonian systems with the preservation of the port-Hamiltonian structure. Positive real balancing results in families of reduced order port-Hamiltonian systems in energy and co-energy coordinates with at least two known reduced order models in each family.

• The Arnoldi and Lanczos methods are employed in Chapter 5 in order to preserve not only the port-Hamiltonian structure but also a specific number of the Markov parameters (moments at infinity) of the full order model. The Lanczos method preserves the port-Hamiltonian structure for port-Hamiltonian systems from a specific algebraically characterized subclass. For this subclass the Arnoldi and the Lanczos methods produce equivalent reduced order models in the sense of sharing the same transfer functions. As a result, the Arnoldi method preserves twice as many Markov parameters for the subclass of port-Hamiltonian systems as for a general linear system.

• In Chapter 6 we investigate how the rational Arnoldi and Lanczos methods preserve the port-Hamiltonian structure of the full order model as well as a specific number of moments in the complex plane. Both energy and co-energy coordinate representations are treated. The rational Lanczos method is structure preserving for a subclass of port-Hamiltonian systems (different from the subclass described in Chapter 5). For the same subclass the rational Arnoldi method preserves twice as many moments of the full order system as for a general linear system, resulting in a reduced order port-Hamiltonian model which shares the same transfer function with the reduced order model given by the rational Lanczos method.

• In Chapter 7 we have shown how to employ interpolatory model reduction for port-Hamiltonian systems, so that the reduced order system not only interpolates the original model at certain (distinct) points in the complex plane, but also preserves the port-Hamiltonian structure. We proposed an algorithm which chooses interpolation points that satisfy a subset of the (unstructured) $\mathcal{H}_2$-optimal necessary conditions.
The moment matching problem for a given input-generating system of [5],[6],[7] is specialized to port-Hamiltonian systems in Chapter 8. It is shown that if the full order system is port-Hamiltonian, then in family of reduced order systems under described conditions there are lossless and passive port-Hamiltonian models, which match a certain number of moments of the full order system in the complex plane.

In Chapter 9 we showed that the model reduction method for structured systems of [83], which requires the projection of the full order model on the dominant eigenspace of the corresponding reachability Gramian, is applicable to port-Hamiltonian systems. The method preserves the port-Hamiltonian structure for the reduced order models, allowing at the same time for an $\mathcal{H}_2$ error bound.

In Chapter 10 we conduct several numerical experiments in order to investigate the behavior of different structure preserving methods.

Multi-input multi-output (MIMO) port-Hamiltonian systems are treated in Chapters 1, 2, 3, 4 and 9, while single-input single-output (SISO) port-Hamiltonian systems are the subject of Chapters 5, 6, 7, 8, and 10.

If the Hamiltonian of the full order model is non-negative, and so the full order system is passive, then, as a direct consequence of the preservation of the port-Hamiltonian structure, all the reduced order port-Hamiltonian models obtained in this thesis are also passive. Furthermore, if the full order model has a Hamiltonian with a strict minimum at its equilibrium, and thus is stable, then so have the reduced order models, implying again stability.

11.2. Recommendations for future work

The effort- and flow-constraint methods of Chapters 2 and 3 motivate to investigate further important issues about a (numerically efficient) choice of suitable coordinates to perform the port-Hamiltonian structure preserving model reduction in such a way, that a certain error metric is minimized. Questions about general error bounds are closely related to the choice of the best coordinate system for model reduction.

The relation between the families of the reduced order models obtained using positive real balancing in Chapter 4, as well as the questions of systematic characterization of the families of the reduced order models, and the choice of the best reduced order model from the families to minimize a certain error measure, require further investigation.
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- The methods of Chapters 5 and 6 seem to allow for an extension to the MIMO case. The systematic characterization of the discovered subclasses of port-Hamiltonian systems from physical and/or system-theoretic point of view, as well as the numerical efficiency of the considered methods are the subjects for future work.

- $\mathcal{H}_2$-optimal model reduction of port-Hamiltonian systems of Chapter 7 gives a hope for an extension to the MIMO case. The algorithm proposed for $\mathcal{H}_2$-optimal structure preserving model reduction requires the study of the convergence behavior and different initialization strategies. First order optimality conditions for a constrained optimal $\mathcal{H}_2$ problem, when the space of $r$-dimensional reduced order models contains only port-Hamiltonian models, are another research question.

- The port-Hamiltonian moment matching problem of Chapter 8 can be further studied in order to understand if it is possible to find a reduced order port-Hamiltonian moment matching model with additional constraints (e.g. the reduced order model has prescribed eigenvalues, relative degree, etc.) The problem can be further generalized for the MIMO case.

- A modification of the method considered in Chapter 9, when the full order model is projected onto the dominant eigenspace of the product of the observability and reachability Gramians, with further relation to Lyapunov balancing, as well as the application of other methods preserving higher order structure to port-Hamiltonian systems, are recommended for future research.

- Chapter 10 motivates to investigate the numerical performance of the offered structure preserving methods for different physical systems.

- The structure preserving model reduction methods of Chapters 5, 6, 7 and 8 for the SISO port-Hamiltonian systems require an extension to the MIMO case.

- Physical realization of the reduced order port-Hamiltonian models for both the SISO and MIMO cases, e.g. as electrical circuits, is an open question.