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THE STRUCTURE OF THE SUPER- $W_\infty(\lambda)$ ALGEBRA

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We give a comprehensive treatment of the super- $W_\infty(\lambda)$ algebra, an extension of the super-Virasoro algebra that contains generators of spin $s \geq \frac{1}{2}$. The parameter λ defines the embedding of the Virasoro subalgebra. We describe how to obtain the super- $W_\infty(\lambda)$ algebra from the associative algebra of superspace differential operators. We discuss the structure of this associative algebra and its relation with the so-called wedge algebra, in which the generators for given spin are restricted to finite-dimensional representations of $\mathfrak{sl}(2)$. From the super- $W_\infty(\lambda)$ algebra one can obtain a variety of W_∞ algebras by consistent truncations for specific values of λ . Without truncation the algebras are formally isomorphic for different values of λ . We present a realization in terms of the currents of a supersymmetric bc system.

1. Introduction

Extensions of the Virasoro algebra play an important role in the study of conformal field theories with central charge $c \geq 1$. The so-called W_N algebras are extensions of the Virasoro algebra that contain generators of all conformal spins from $s = 2$ up to $s = N$. The first such algebra is the W_3 algebra proposed by Zamolodchikov [1], which contains a spin-3 field in addition to the spin-2 energy-momentum tensor associated with the Virasoro algebra. A characteristic feature of these algebras is that they are non-linear: the closure of the operator algebra is achieved because operators of spin higher than N that arise naturally in the operator product expansions, take the form of normal-ordered products of the lower-spin fields. These W_N algebras, both the bosonic and the supersymmetric versions, have been studied from a variety of viewpoints (see for instance refs. [2, 3]).

It may be possible to interpret the W_N algebras in the context of a W_∞ -type algebra. Such an algebra is based on an infinite number of higher-spin operators with spin $s \geq 1$, or 2 and multiplicity 1. In contradistinction with the W_N algebras, these algebras are linear. It is then suggestive to view the W_N algebras as a W_∞ -type algebra subject to certain non-linear constraints. The W_∞ -type algebras may also arise in an appropriate large- N limit of W_N [4, 5]. This was in fact one of the original motivations for studying the W_∞ -type algebras and in ref. [6] two such algebras were constructed, denoted by W_∞ and $W_{1+\infty}$.

The W algebras are not only relevant for conformal field theory. For instance, in recent studies of 2-dimensional gravity by means of matrix models, it was shown that the square root of the partition function is subject to the vacuum conditions of a W algebra. The simplest case concerns the 1-matrix model where the relevant algebra is the Virasoro algebra [7]. More generally, for the $(N-1)$ -matrix model we have W_N , but in fact one can show that there are more conditions [8, 9]. According to ref. [9] the square root of the partition function is subject to the vacuum condition of the $W_{1+\infty}$ algebra. In fact there is an intricate relationship between W algebras and systems of non-linear integrable differential equations [10]. Recently it was shown for the KP hierarchy that the Poisson-bracket algebra corresponding to the first hamiltonian structure generates the $W_{1+\infty}$ algebra [11].

Just as the Virasoro algebra arises as a residual symmetry upon gauge-fixing the two-dimensional diffeomorphism invariance, one may wonder whether a corresponding situation exists for the W algebras. This leads to the notion of W gravity. Field-theoretic realizations of W algebras in which all W symmetries are gauged by introducing corresponding gauge fields were discussed in ref. [12].

Certain versions or subalgebras of W_∞ correspond to area-preserving (or symplectic) diffeomorphisms. For instance, the so-called w_∞ and $w_{1+\infty}$ algebras correspond to the area-preserving diffeomorphisms of a cylinder [13, 6]. Then there are the so-called wedge subalgebras, which follow from restricting the spin- s generators to $(2s-1)$ -dimensional representations of $\mathfrak{sl}(2)$. These algebras play a role in the residual invariance group of the light-cone formulation of relativistic (super)membranes [14]. Algebras of this type were also introduced [15–17] to describe interacting field theories of higher-spin gauge fields in $3+1$ dimensions [18]. Analogous constructions were used in $2+1$ dimensions [19]. Furthermore, the area-preserving diffeomorphisms are related to self-dual four-dimensional gravity (see for instance ref. [20] and references quoted therein).

In this paper we give a self-contained treatment of the supersymmetric W_∞ -type algebras based on the so-called super- $W_\infty(\lambda)$ algebra [21]. The generic algebra has $N=2$ supersymmetry and has generators of integer and half-integer spin $s \geq 1$ with multiplicity 2, and a single spin- $\frac{1}{2}$ generator. It contains an infinite variety of Virasoro subalgebras. The parameter λ specifies a particular Virasoro subalgebra, and tied to it, a so-called wedge subalgebra. Nevertheless, $W_\infty(\lambda)$ may be regarded as a single algebra written in a one-parameter family of bases, so that algebras with

different values of λ are isomorphic, at least in the formal sense. In truncations of the algebra this ceases to be true. These truncations require specific values for λ , so that the choice of the Virasoro subalgebra is fixed. The spin content of all higher-spin generators as (quasi-)conformal operators is then characterized with respect to this subalgebra. The wedge subalgebras depend on λ and belong to a one-parameter family of inequivalent algebras [22, 23]. When including the central charge, also the Virasoro subalgebras are inequivalent for different values of λ .

In many respects the situation is the same for extensions of the Virasoro algebra with generators of spin $s \leq 2$, such as the super-Virasoro algebra. For instance, if one defines the super-Virasoro algebra for a supersymmetric bc system [24] of conformal weight λ , then the algebras for different values of λ are isomorphic. Unlike the case of the $W_\infty(\lambda)$ algebra, the structure constants of the super-Virasoro algebra are independent of λ , and furthermore, one can directly rewrite the super-Virasoro currents defined for one value of λ in terms of those for another value of λ .

As mentioned above truncations of the algebra can be performed for particular values of the parameter λ . For instance, in the bosonic case, $\lambda = \frac{1}{2}$ corresponds to the $W_{1+\infty}$ algebra while $\lambda = 0$ yields the W_∞ algebra upon truncation to generators of spin $s \geq 2$. This particular truncation is only possible for this value of λ . The algebras w_∞ and $w_{1+\infty}$ are found by a contraction [6]. For the supersymmetric case we have the same situation. For $\lambda = 0$ one can truncate to the superalgebra found in ref. [25], which can be truncated further to an algebra with $N = 1$ supersymmetry, while for $\lambda = \frac{1}{4}$ we find the superalgebra exhibited in ref. [21] and another one without supersymmetry. The generic super- $W_\infty(\lambda)$ algebra contains $W_\infty(\lambda) \oplus W_\infty(\lambda + \frac{1}{2})$ as its bosonic subalgebra. Within the framework of the super- $W_\infty(\lambda)$ algebra one can conveniently clarify the nature of the various truncations of the algebra and elucidate many of the results found previously from a different perspective. Just as in ref. [21] we define the algebra both in the context of the associative algebra of superspace differential operators and the operator algebra of quasi-primary currents of a (supersymmetric) bc system. We will explicitly evaluate the multiplication properties of the former. Furthermore we discuss the relation with the so-called wedge algebras and make contact with previous work on these algebras [6, 22, 23, 26].

This paper is organized as follows. In sect. 2 we introduce the super- $W_\infty(\lambda)$ algebra expressed in terms of superspace differential operators. We discuss the main features of this algebra and establish the existence of an anti-automorphism which allows us to derive a number of important properties of the structure constants. In sect. 3 we describe the structure of the bosonic associative algebra, paying particular attention to the $\mathfrak{sl}(2)$ subalgebra. We derive an explicit expression for the multiplication constants of the associative algebra for arbitrary value of λ . We analyze the so-called wedge subalgebra, which is isomorphic to the enveloping algebra of $\mathfrak{sl}(2)$ factored by an appropriate ideal. In sect. 4 we extend this

treatment to the supersymmetric case. Here the wedge subalgebra is isomorphic to the enveloping algebra of $\text{osp}(1, 2)$ or $\text{osp}(2, 2)$, factored by appropriate ideals. In sect. 5 we construct a representation in terms of a supersymmetric bc system. This leads to central charges in the algebra. In the generic case these charges have a more complicated structure than in the specific examples exhibited so far. Some convenient formulae have been collected in appendix A.

2. Super- $W_\infty(\lambda)$

In ref. [21] we presented the super- $W_\infty(\lambda)$ algebra in terms of superspace differential operators, by generalizing the corresponding expressions for the generators of the super-Virasoro algebra,

$$\begin{aligned} L_n &= z^{-n+1} \partial - \lambda(n-1) z^{-n} - \frac{1}{2}(n-1) z^{-n} \theta \frac{\partial}{\partial \theta}, \\ G_r &= [z^{-r+1/2} \partial - \lambda(2r-1) z^{-r-1/2}] \theta + z^{-r+1/2} \frac{\partial}{\partial \theta}. \end{aligned} \quad (2.1)$$

Here, superspace coordinates are denoted by z and θ , $\partial \equiv d/dz$, $\partial/\partial\theta$ is the left-derivative, and λ is the conformal weight. Motivated by these expressions we constructed a complete set of operators in terms of positive powers of derivatives ∂ , arbitrary powers of z and the fermionic coordinate and its derivative. A suitable basis is found by classifying these generators according to their maximal power $s-1$ of ∂ , and requiring that they transform according to appropriate nondecomposable representations of $\text{sl}(2)$. Hence s takes positive integer values ($s \geq 1$), so that bosonic operators carry spin s and the fermionic operators spin $s - \frac{1}{2}$. The explicit expressions are

$$\begin{aligned} L_n^{(s)+} &= \sum_{i=0}^{s-1} (n-s+1)_{s-1-i} a^i(s, \lambda) z^{-n+i} \partial^i \\ &\quad + \theta \frac{\partial}{\partial \theta} \sum_{i=0}^{s-1} (n-s+1)_{s-1-i} \left[a^i(s, \lambda + \tfrac{1}{2}) - a^i(s, \lambda) \right] z^{-n+i} \partial^i, \\ L_n^{(s)-} &= -\frac{s-1+2\lambda}{2s-1} \sum_{i=0}^{s-1} (n-s+1)_{s-1-i} a^i(s, \lambda) z^{-n+i} \partial^i \\ &\quad + \theta \frac{\partial}{\partial \theta} \sum_{i=0}^{s-1} (n-s+1)_{s-1-i} \\ &\quad \times \left[\frac{s-2\lambda}{2s-1} a^i(s, \lambda + \tfrac{1}{2}) + \frac{s-1+2\lambda}{2s-1} a^i(s, \lambda) \right] z^{-n+i} \partial^i, \\ G_r^{(s)\pm} &= \theta \sum_{i=0}^{s-1} (r-s+\tfrac{3}{2})_{s-1-i} \alpha^i(s, \lambda) z^{-r+i-1/2} \partial^i \\ &\quad \pm \frac{\partial}{\partial \theta} \sum_{i=0}^{s-2} (r-s+\tfrac{3}{2})_{s-2-i} \beta^i(s, \lambda) z^{-r+i+1/2} \partial^i, \end{aligned} \quad (2.2)$$

where we made use of the definition

$$(a)_n \equiv \frac{\Gamma(a+n)}{\Gamma(a)} = a(a+1)(a+2)\dots(a+n-1), \quad \text{with } (a)_0 = 1. \quad (2.3)$$

Furthermore, $a^i(s, \lambda)$, $\alpha^i(s, \lambda)$ and $\beta^i(s, \lambda)$ are defined by

$$\begin{aligned} a^i(s, \lambda) &= \binom{s-1}{i} \frac{(-2\lambda - s + 2)_{s-1-i}}{(s+i)_{s-1-i}} & (0 \leq i \leq s-1), \\ \alpha^i(s, \lambda) &= \binom{s-1}{i} \frac{(-2\lambda - s + 2)_{s-1-i}}{(s+i-1)_{s-1-i}} & (0 \leq i \leq s-1), \\ \beta^i(s, \lambda) &= \binom{s-2}{i} \frac{(-2\lambda - s + 2)_{s-2-i}}{(s+i)_{s-2-i}} & (0 \leq i \leq s-2). \end{aligned} \quad (2.4)$$

Some properties of these coefficients are discussed in appendix A.

One easily verifies that $L_n^{(2)+}$ and $G_r^{(2)+}$ coincide with the super-Virasoro generators defined above (cf. (2.1)). Furthermore, apart from spin $\frac{1}{2}$ where $G_r^{(1)+} = G_r^{(1)-}$, there is a two-fold degeneracy for each spin. In the Neveu-Schwarz sector, where the indices r take half-integer values, $L_{\pm 1}^{(2)+}$, $L_0^{(2)+}$ and $G_{\pm 1/2}^{(2)+}$ generate the $\text{osp}(1, 2)$ algebra. The operators (2.2) then transform according to non-decomposable representations of $\text{osp}(1, 2)$, such that $(L_m^{(s)+}, G_r^{(s)+})$ and $(L_m^{(s)-}, G_r^{(s+1)-})$ constitute spin $(s, s - \frac{1}{2})$ and $(s, s + \frac{1}{2})$ supermultiplets, respectively.

The differential operators given in (2.2) form an associative algebra, which is isomorphic to a superextension of a version of the $\text{Gl}(\infty)$ algebra. On the basis of their (anti)commutator they define an explicit representation of the super- $W_\infty(\lambda)$ algebra. Special cases of this infinite-dimensional Lie superalgebra have been considered in the literature. For instance, for $\lambda = 0$, one can verify from (2.2) and (2.4) that all generators are proportional to either one of the derivatives ∂ or $\partial/\partial\theta$, with the exception of $L_n^{(1)+}$ and $G_r^{(1)}$. Therefore the algebra may be consistently restricted* to $L_n^{(1)-}$ and the higher-spin generators $L_n^{(s)\pm}$ and $G_r^{(s)\pm}$ with $s \geq 2$. One is then left with the super- W_∞ algebra constructed in ref. [25]. Of course the same argument applies to the bosonic case. The bosonic Lie algebra $W_\infty(\lambda)$ can be realized by the part of the generators $L_n^{(s)+}$ independent of $\theta\partial/\partial\theta$. When $\lambda = 0$ one can consistently reduce the $W_\infty(\lambda)$ algebra to the W_∞ algebra of ref. [6] after suppressing the spin-1 operator. For general λ such a decoupling of the spin-1 operator does not take place. For instance, the bosonic algebra $W_\infty(\frac{1}{2})$ coincides

*Observe that similar truncations are possible whenever λ takes negative integer or half-integer values. However, the resulting algebras no longer contain the Virasoro algebra.

with the $W_{1+\infty}$ algebra proposed in ref. [6]. Finally the w_∞ and $w_{1+\infty}$ algebras may be obtained by a contraction.

It is often convenient to sum over all the modes and to define corresponding differential operators in terms of commuting functions $\Lambda^{(s)\pm}(z)$ and anticommuting functions $\Theta^{(s)\pm}(z)$,

$$\begin{aligned}
 L_\lambda^{(s)+} &= \sum_{i=0}^{s-1} a^i(s, \lambda) ((-\partial)^{s-i-1} \Lambda^{(s)+}(z)) \partial^i \\
 &\quad + \theta \frac{\partial}{\partial \theta} \sum_{i=0}^{s-1} \left[a^i(s, \lambda + \tfrac{1}{2}) - a^i(s, \lambda) \right] ((-\partial)^{s-i-1} \Lambda^{(s)+}(z)) \partial^i, \\
 L_\lambda^{(s)-} &= -\frac{s-1+2\lambda}{2s-1} \sum_{i=0}^{s-1} a^i(s, \lambda) ((-\partial)^{s-i-1} \Lambda^{(s)-}(z)) \partial^i \\
 &\quad + \theta \frac{\partial}{\partial \theta} \sum_{i=0}^{s-1} \left[\frac{s-2\lambda}{2s-1} a^i(s, \lambda + \tfrac{1}{2}) + \frac{s-1+2\lambda}{2s-1} a^i(s, \lambda) \right] ((-\partial)^{s-i-1} \Lambda^{(s)-}(z)) \partial^i, \\
 G_\lambda^{(s)\pm} &= \theta \sum_{i=0}^{s-1} \alpha^i(s, \lambda) ((-\partial)^{s-i-1} \Theta^{(s)\pm}(z)) \partial^i \\
 &\quad \pm \frac{\partial}{\partial \theta} \sum_{i=0}^{s-2} \beta^i(s, \lambda) ((-\partial)^{s-i-2} \Theta^{(s)\pm}(z)) \partial^i, \tag{2.5}
 \end{aligned}$$

where $\Lambda^{(s)\pm}(z) = \sum_n \Lambda_n^{(s)\pm} z^{n+s-1}$ and $\Theta^{(s)\pm}(z) = \sum_r \Theta_r^{(s)\pm} z^{r+s-3/2}$.

Important information regarding the λ -dependence of $W_\infty(\lambda)$ follows from the existence of an anti-automorphism which is closely related to hermitian conjugation for superspace differential operators. The anti-automorphism is implemented by changing the order of the various operators (including the parameters contained in (2.5)) and by replacing

$$z \rightarrow z, \quad \partial \rightarrow -\partial, \quad \theta \rightarrow -i\theta, \quad \frac{\partial}{\partial \theta} \rightarrow i \frac{\partial}{\partial \theta}, \tag{2.6}$$

such that the basic (anti)commutators between z and ∂ and θ and $\partial/\partial\theta$ remain unchanged. Under this anti-automorphism operators change according to

$$L_\lambda^{(s)\pm} \rightarrow \pm (-)^{s-1} L_{1/2-\lambda}^{(s)\pm}, \quad G_\lambda^{(s)\pm} \rightarrow i (-)^{s-1} G_{1/2-\lambda}^{(s)\pm}. \tag{2.7}$$

This result follows from the identities (A.8)–(A.10) given in appendix A. Alternatively, one can first verify this result for the Virasoro generators, which shows that the (quasi-conformal) spin does not change under the anti-automorphism. There-

fore, to verify the validity of (2.7), it suffices to check the leading terms (i.e. the terms with the maximal number of derivatives).

From (2.7) it follows that the $W_\infty(\lambda)$ structure constants on the basis specified in (2.2) or (2.5) are either symmetric or antisymmetric under the replacement $\lambda \rightarrow \frac{1}{2} - \lambda$, depending on the values of the spins involved. To be precise, the structure constants involving three $L^{(s)+}$ operators, or one $L^{(s)+}$ and two $L^{(s)-}$ operators, are symmetric (antisymmetric) under the change $\lambda \rightarrow \frac{1}{2} - \lambda$ whenever the sum of their spins is even (odd), while the structure constants that involve two $L^{(s)+}$ and one $L^{(s)-}$ operator, or three $L^{(s)-}$ operators, are antisymmetric (symmetric) whenever the sum of the three spins is even (odd). For the structure constants that involve two fermionic operators, we have a similar result. If the third (bosonic) operator is $L^{(s)+}$, the structure constants are symmetric (antisymmetric) whenever the sum of the spins is odd (even), while, if the third operator is $L^{(s)-}$, the structure constants are symmetric (antisymmetric) whenever the sum of the spins is even (odd).

Furthermore we can deduce from (2.7) that $\pm(-)^s L_{1/2-\lambda}^{(s)\pm}$ and $(-)^s G_{1/2-\lambda}^{(s)\pm}$ exhibit the same commutator algebra as the original operators $L_\lambda^{(s)\pm}$ and $G_\lambda^{(s)\pm}$. Therefore, we conclude that the superalgebras $W_\infty(\lambda)$ and $W_\infty(\frac{1}{2} - \lambda)$ are isomorphic.

Clearly the value $\lambda = \frac{1}{4}$ is special in this respect. On the basis of the anti-automorphism one can show that many of the structure constants will vanish in that case. This enables a consistent reduction of the algebra where one retains only the generators $L^{(s)+}$ and $G^{(s)\pm}$ with even s , and $L^{(s)-}$ with odd s . This algebra has $N=2$ supersymmetry. Another consistent truncation is to retain the generators $L^{(s)+}$ with even s and the generators $L^{(s)-}$ and $G^{(s)\pm}$ with odd s . Although this is still a superalgebra we lost supersymmetry in this truncation. As we shall discuss in sect. 4, the algebra with $\lambda = \frac{1}{4}$ is also related to the symplecton higher-spin superalgebra [15, 27]. The above analysis also applies to the purely bosonic case. Then the anti-automorphism relates the parameter values λ and $1 - \lambda$ so that the value $\lambda = \frac{1}{2}$ is special.

The above results can be conveniently expressed in superfield notation. Superspace operators are then defined in terms of superfield transformation parameters $\Omega^{(s)\pm}(z, \theta)$. The operators $L^{(s)+}$ and $G^{(s)+}$ are associated with commuting parameters $\Omega^{(s)+}$, and $L^{(s)-}$ and $G^{(s+1)-}$ with anticommuting parameters $\Omega^{(s)-}$. The superspace operators read

$$\begin{aligned} L_\lambda^{(s)+} &= \sum_{k=0}^{2s-2} A_+^k(s, \lambda) (D^{2s-2-k} \Omega^{(s)+}) D^k, \\ L_\lambda^{(s)-} &= \sum_{k=0}^{2s-1} A_-^k(s, \lambda) (D^{2s-1-k} \Omega^{(s)-}) D^k, \end{aligned} \quad (2.8)$$

where we made use of the superspace derivative

$$D = \frac{\partial}{\partial \theta} - \theta \partial, \quad \text{with} \quad D^2 = -\partial. \quad (2.9)$$

The coefficients $A_{\pm}^k(s, \lambda)$ are defined by

$$\begin{aligned} A_+^{2i}(s, \lambda) &= (-)^i a^i(s, \lambda), & A_-^{2i}(s, \lambda) &= (-)^{i+1} \frac{s-1+2\lambda}{2s-1} a^i(s, \lambda), \\ A_+^{2i+1}(s, \lambda) &= -\frac{1}{2}(-)^i \beta^i(s, \lambda), & A_-^{2i+1}(s, \lambda) &= (-)^i \beta^i(s+1, \lambda), \end{aligned} \quad (2.10)$$

and the superfield parameters $\Omega^{(s)\pm}$ are related to the parameters in (2.5) according to

$$\begin{aligned} \Omega^{(s)+}(z, \theta) &= \Lambda^{(s)+}(z) + 2\theta \Theta^{(s)+}(z), \\ \Omega^{(s)-}(z, \theta) &= \Theta^{(s+1)-}(z) + \theta \Lambda^{(s)-}(z). \end{aligned} \quad (2.11)$$

For future use we describe some properties of the super-Virasoro subalgebra. For clarity let us first discuss the bosonic case. The Virasoro algebra is then expressed in terms of a function $\Lambda^{(2)}(z)$,

$$L_{\lambda}^{(2)} = \Lambda^{(2)} \partial + \lambda (\partial \Lambda^{(2)}). \quad (2.12)$$

For functions satisfying $\partial^3 \Lambda^{(2)} = 0$, the generators (2.12) define the $\mathfrak{sl}(2)$ subalgebra. To elucidate some of the characteristic features of the commutator algebra, let us evaluate the first few terms in the commutator of $L_{\lambda}^{(2)}$ and $L_{\lambda}^{(s)}$, characterized by functions $\Lambda^{(2)}(z)$ and $\Lambda^{(s)}(z)$,

$$[L_{\lambda}^{(2)}, L_{\lambda}^{(s)}] = L_{\lambda}^{(s)} + L_{\lambda}^{(s-1)} + L_{\lambda}^{(s-2)} + L_{\lambda}^{(s-3)} +, \quad (2.13)$$

where the generators on the right-hand side are expressed in terms of the functions

$$\xi^{(s)}(z) = \Lambda^{(2)}(\partial \Lambda^{(s)}) - (s-1)(\partial \Lambda^{(2)}) \Lambda^{(s)},$$

$$\xi^{(s-1)}(z) = 0,$$

$$\xi^{(s-2)}(z) = \frac{(s-1)(s-2)}{2s-3} [\lambda(\lambda-1) - \frac{1}{12}s(s-3)] \Lambda^{(s)}(\partial^3 \Lambda^{(2)}),$$

$$\xi^{(s-3)}(z) = -\frac{1}{6}(2\lambda-1)\lambda(\lambda-1) [2(\partial \Lambda^{(s)})(\partial^3 \Lambda^{(2)}) + (s-1)\Lambda^{(s)}(\partial^4 \Lambda^{(2)})]. \quad (2.14)$$

Observe that the commutator of two even-spin generators gives, in general, rise to generators of both even and odd spin. Furthermore the structure constants are either symmetric or antisymmetric under $\lambda \rightarrow 1 - \lambda$, in accordance with the results quoted below (2.7). The function $\xi^{(s)}$ in (2.14) specifies that $L^{(s)}$ has spin s . However, the presence of the lower-spin generators in (2.13) implies that $L^{(s)}$ is, in general, not a primary operator. Because all generators transform covariantly under $\mathfrak{sl}(2)$, all lower-spin contributions should depend on third- and higher-order derivatives of $\Lambda^{(2)}$, as is confirmed by the explicit results quoted in (2.14). In sect. 3 we shall see how $\mathfrak{sl}(2)$ puts further restrictions on the derivative terms in the commutators.

The situation is analogous for the superalgebras. The super-Virasoro generators are then expressed in terms of a superfield $\Omega^{(2)+}(z, \theta)$ and take the form

$$L_\lambda^{(2)+} = -\Omega^{(2)+} D^2 - \frac{1}{2} (D\Omega^{(2)+}) D - \lambda (D^2 \Omega^{(2)+}). \quad (2.15)$$

For functions $\Omega^{(2)+}$ satisfying the condition $D^5 \Omega^{(2)+} = 0$, the super-Virasoro algebra is restricted to the $\mathfrak{osp}(1, 2)$ subalgebra (this requires that the generators be defined in the Neveu–Schwarz sector).

Again we may evaluate the leading terms in the commutator of $L_\lambda^{(2)+}$ and $L_\lambda^{(s)\pm}$, characterized by functions $\Omega^{(2)+}(z, \theta)$ and $\Omega^{(s)\pm}(z, \theta)$, respectively,

$$[L_\lambda^{(2)+}, L_\lambda^{(s)\pm}] = L_\lambda^{(s)\pm} + \dots \quad (2.16)$$

The leading-spin operator $L_\lambda^{(s)\pm}$ is then characterized by the functions

$$\begin{aligned} \xi^{(s)+}(z, \theta) &= -\Omega^{(2)+} D^2 \Omega^{(s)+} - \frac{1}{2} (D\Omega^{(2)+}) (D\Omega^{(s)+}) + (s-1) (D^2 \Omega^{(2)+}) \Omega^{(s)+}, \\ \xi^{(s)-}(z, \theta) &= -\Omega^{(2)+} D^2 \Omega^{(s)-} - \frac{1}{2} (D\Omega^{(2)+}) (D\Omega^{(s)-}) + (s - \frac{1}{2}) (D^2 \Omega^{(2)+}) \Omega^{(s)-}. \end{aligned} \quad (2.17)$$

Because all operators transform covariantly with respect to $\mathfrak{osp}(1, 2)$, the generators of non-leading spin on the right-hand side of (2.16) will involve only fifth- or higher-order fermionic derivatives of $\Omega^{(2)+}$.

It is convenient to use a more uniform notation by introducing also half-integer values for s . The generators $L_\lambda^{(s)\pm}$ and corresponding parameters $\Omega^{(s)\pm}$ are then written as

$$\begin{aligned} L_\lambda^{(s)} &= L_\lambda^{(s)+}, & \Omega^{(s)} &= \Omega^{(s)+}, \\ L_\lambda^{(s+1/2)} &= L_\lambda^{(s)-}, & \Omega^{(s+1/2)} &= \Omega^{(s)-}. \end{aligned} \quad (2.18)$$

The parameter s ($s \geq 1$) thus coincides with the highest-spin value of the bosonic

or fermionic generator contained in $L_\lambda^{(s)}$. For integer s the highest-spin generator is a bosonic operator while for half-integer s the highest-spin generator is a fermionic operator. For general integer or half-integer s , the generators $L_\lambda^{(s)}$ are now defined by

$$L_\lambda^{(s)}(\Omega^{(s)}) = \sum_{i=0}^{2s-2} A^i(s, \lambda) (D^{2s-2-i} \Omega^{(s)}) D^i, \quad (2.19)$$

where the coefficients $A^i(s, \lambda)$ coincide with the coefficients $A_\pm^i(s, \lambda)$ defined in (2.10)*,

$$\begin{aligned} A^i(s, \lambda) &= A_+^i([s], \lambda) & (s \text{ integer}) \\ A^i(s, \lambda) &= A_-^i([s], \lambda) & (s \text{ half-integer}). \end{aligned} \quad (2.20)$$

Here and henceforth $[a]$ denotes the integer part of a . For half-integer a we thus have $[a] = a - \frac{1}{2}$. Observe that (2.19) is always a bosonic operator, because $\Omega^{(s)}$ is commuting (anticommuting) for integer (half-integer) value of s .

For convenience we list the generators with $s \leq 2$,

$$\begin{aligned} L_\lambda^{(1)}(\Omega^{(1)}) &= \Omega^{(1)}, \\ L_\lambda^{(3/2)}(\Omega^{(3/2)}) &= \Omega^{(3/2)} D - 2\lambda (D \Omega^{(3/2)}), \\ L_\lambda^{(2)}(\Omega^{(2)}) &= -\Omega^{(2)} D^2 - \frac{1}{2} (D \Omega^{(2)}) D - \lambda (D^2 \Omega^{(2)}). \end{aligned} \quad (2.21)$$

The commutation relation of the super-Virasoro generator $L_\lambda^{(2)}$ with $L_\lambda^{(s)}$ leads again to $L_\lambda^{(s)}$ plus generators of lower spin (cf. (2.16)). The leading-spin generator is then characterized by the function (cf. (2.17))

$$\xi^{(s)}(z, \theta) = -\Omega^{(2)} D^2 \Omega^{(s)} - \frac{1}{2} (D \Omega^{(2)}) (D \Omega^{(s)}) + (s-1) (D^2 \Omega^{(2)}) \Omega^{(s)}, \quad (2.22)$$

for both integer and half-integer values of s .

Finally we discuss the anti-automorphism (2.6) in this notation. Under the automorphism one replaces the order of all quantities and replaces the superspace derivative and the anticommuting coordinate by

$$D \rightarrow iD, \quad \theta \rightarrow -i\theta. \quad (2.23)$$

* A single formula for the coefficients $A^i(s, \lambda)$ is

$$A^i(s, \lambda) = (-)^{[s]+1+2s(i+1)} \begin{bmatrix} s-1 \\ i/2 \end{bmatrix} \frac{([(i+1)/2] + 2\lambda)_{[s-1/2]-[(i+1)/2]}}{([s+i/2])_{2s-1-[s+i/2]}}.$$

The modified binomial coefficient is defined in eq. (4.5).

Functions $\Omega^{(s)}$ thus change into functions $\tilde{\Omega}^{(s)}$ obtained by changing the order of the anticommuting parameters and replacing θ by $-\theta$. One can then derive that the generators change according to

$$L_\lambda^{(s)}(\Omega^{(s)}) \rightarrow i^{6s-2} L_{1/2-\lambda}^{(s)}(\tilde{\Omega}^{(s)}). \quad (2.24)$$

In order to prove this one may employ the following relation for functions that can be written as multiple superspace derivatives of other functions

$$\widetilde{D^n \Omega} = i^{|n|_2} (-)^{n(2s+1)} D^n \tilde{\Omega}. \quad (2.25)$$

Here $|n|_2 \equiv n - 2[n/2]$, which is equal to 0 or 1 depending on whether n is even or odd, respectively. With the aid of eq. (2.25) it is easy to verify the correctness of (2.24) for the low-spin generators listed in (2.21).

3. The bosonic algebra

In this section we shall discuss the structure of the bosonic associative algebra. The results lead directly to explicit expressions for the structure constants of the $W_\infty(\lambda)$ algebra. In sect. 4 we will generalize these results to the corresponding superalgebra. We start by deriving the consequences of the $\mathfrak{sl}(2)$ covariance for the associative algebra. Consider the product of $L_\lambda^{(s)}$ and $L_\lambda^{(t)}$ expressed in terms of corresponding functions $\Lambda^{(s)}$ and $\Lambda^{(t)}$,

$$L_\lambda^{(s)}(\Lambda^{(s)}) L_\lambda^{(t)}(\Lambda^{(t)}) = \sum_{u=1}^{s+t-1} L_\lambda^{(s+t-u)}(\xi_{(s)(t)}^{(s+t-u)}), \quad (3.1)$$

where the operators on the right-hand side depend on corresponding functions $\xi_{(s)(t)}^{(s+t-u)}$ which can be expanded according to

$$\xi_{(s)(t)}^{(s+t-u)} = \sum_{i=0}^{u-1} F_{st}^u(i, \lambda) (\partial^i \Lambda^{(s)}) (\partial^{u-1-i} \Lambda^{(t)}). \quad (3.2)$$

The $\mathfrak{sl}(2)$ covariance of the operators $L^{(s)}$ fixes the i -dependence of the coefficients $F_{st}^u(i, \lambda)$. To see this one takes the commutator of both sides of eq. (3.1) with $L^{(2)}(\Lambda^{(2)})$, where the corresponding function $\Lambda^{(2)}$ satisfies the $\mathfrak{sl}(2)$ condition $\partial^3 \Lambda^{(2)} = 0$. This exercise leads to three equations proportional to $\Lambda^{(2)}$, $\partial \Lambda^{(2)}$ and $\partial^2 \Lambda^{(2)}$, respectively. While the first two equations just ensure that the coefficients in (3.2) are z -independent and fix the total number of derivatives, the third equation yields a recurrence relation for the coefficients $F_{st}^u(i, \lambda)$, which determines their i -dependence and shows that the range of i -values in the sum (3.2) is restricted in a way which depends on the values of s , t and u . One may distinguish three possible

situations:

$$\begin{aligned} \mathbf{I}_{++}: \quad & 0 \leq i \leq u-1, & u \leq 2s-1, & u \leq 2t-1, \\ \mathbf{I}_{+-}: \quad & 0 \leq i \leq u-2t, & u \leq 2s-1, & u \geq 2t, \\ \mathbf{I}_{-+}: \quad & 2s-1 \leq i \leq u-1, & u \geq 2s, & u \leq 2t-1, \end{aligned} \quad (3.3)$$

where we must take into account that $1 \leq u \leq s+t-1$. A solution of the recurrence relation which automatically satisfies the above restrictions on the range of i values, is given by

$$\xi_{(s)(t)}^{(s+t-u)} = F_{st}^u(\lambda) \sum_{i=0}^{u-1} (-)^i \binom{u-1}{i} (2s-u)_{u-1-i} (2t-u)_i (\partial^i \Lambda^{(s)}) (\partial^{u-1-i} \Lambda^{(t)}). \quad (3.4)$$

The region $\mathbf{I}_{++}$ is characterized by the fact that the right-hand side of (3.2) contains no multiple derivatives of higher order than $\partial^{2s-2} \Lambda^{(s)}$ and $\partial^{2t-2} \Lambda^{(t)}$. This in contradistinction with $\mathbf{I}_{+-}$ ($\mathbf{I}_{-+}$), where no multiple derivatives appear of higher order than $\partial^{2s-2} \Lambda^{(s)} (\partial^{2t-2} \Lambda^{(t)})$ and of lower order than $\partial^{2t-1} \Lambda^{(t)} (\partial^{2s-1} \Lambda^{(s)})$.

It turns out that the functions satisfying (for all s)

$$\partial^{2s-1} \Lambda^{(s)} = 0 \quad (3.5)$$

define an infinite-dimensional associative algebra, whose corresponding Lie algebra is a subalgebra of $W_\infty(\lambda)$. This algebra is thus based on the generators $L_n^{(s)}$ with $|n| \leq s-1$, corresponding to a wedge in the parameter space spanned by the value of the spin s and the L_0 eigenvalue n . In this way the generators of given spin are restricted to a finite-dimensional irreducible representation of $\mathfrak{sl}(2)$. Observe also that the corresponding differential operators defined in (2.2) are regular near $z=0$. The fact that we are restricting ourselves to finite $\mathfrak{sl}(2)$ representations is already an indication that the wedge generators must constitute an associative subalgebra. Nevertheless it is not so easy to give a direct proof of this. Such a proof requires to show that $\partial^{2s+2t-2u-1} \xi_{(s)(t)}^{(s+t-u)} = 0$, for a function $\xi_{(s)(t)}^{(s+t-u)}$ defined according to eq. (3.2) in terms of functions $\Lambda^{(s)}$ and $\Lambda^{(t)}$ that satisfy eq. (3.5). However, the existence of the wedge subalgebra can also be established on the basis of the following arguments. Consider polynomials of the $\mathfrak{sl}(2)$ generators, L_0 and $L_\pm$, defined for a given value of λ . Because of the $\mathfrak{sl}(2)$ commutation relations, one may restrict oneself to symmetrized products, which constitute irreducible representations of $\mathfrak{sl}(2)$ provided one takes out the trace associated with the bilinear $\mathfrak{sl}(2)$ invariant. However, this trace is proportional to the identity, as the $\mathfrak{sl}(2)$ Casimir operator in the representation that we are dealing with, is

equal to

$$C_2(\mathfrak{sl}(2)) \equiv L_0^2 - \frac{1}{2}\{L_1, L_{-1}\} = \lambda(\lambda - 1). \quad (3.6)$$

From the structure of the generators $L_n^{(s)}$ and the $\mathfrak{sl}(2)$ assignments of the symmetrized traceless polynomials^{*}, one concludes that the wedge generators coincide with the generators of the enveloping algebra of $\mathfrak{sl}(2)$, factored over the ideal generated by $C_2(\mathfrak{sl}(2)) - \lambda(\lambda - 1)$. This confirms the required result that the wedge generators constitute an associative algebra. The multiplication properties of this algebra will only depend on λ through the Casimir operator value $\lambda(\lambda - 1)$.

We denote the Lie algebra based on the wedge generators by $\mathcal{B}_\lambda$. As explained in the previous section, $\mathcal{B}_\lambda$ and $\mathcal{B}_{1-\lambda}$ are isomorphic. In this particular case, this is obvious in view of the fact that the multiplication properties of the associative algebra depend only on $\lambda(\lambda - 1)$. Observe that the generator $L_0^{(1)}$ is just proportional to the identity, so that it may be dropped from the Lie algebra $\mathcal{B}_\lambda$. (The corresponding central extension for $\mathcal{B}_\lambda$ is trivial.) However, it does belong to the associative algebra based on products of the wedge generators. The Lie algebras $\mathcal{B}_\lambda$ have been encountered previously in a different context. Sometime ago [22, 23] it was found that both the $(2+1)$ -dimensional higher-spin algebra $\mathfrak{hs}(1, 1)$ that is used for the construction of interacting higher-spin gauge theories, and the algebra of area-preserving diffeomorphisms of the two-dimensional hyperboloid $\mathfrak{sdiff}(S^{1,1})$, belong to the one-parameter family $\mathcal{B}_\lambda$ of infinite-dimensional algebras. The precise correspondence is given by $\mathfrak{hs}(1, 1) = \mathcal{B}_{1/4}$ and $\mathfrak{sdiff}(S^{1,1}) = \mathcal{B}_\infty$. Furthermore, it was shown that different values of λ with $\lambda \leq \frac{1}{2}$ always lead to inequivalent algebras [22, 23]. This is consistent with the fact that in our representation, the wedge generators for given value of λ cannot, in general, be expressed in terms of the wedge generators corresponding to some other value of λ .

It is possible to view the $W_\infty(\lambda)$ as an extension of the $\mathcal{B}_\lambda$ algebra by including the generators outside the wedge. The same point of view can be taken with respect to the associative algebra. According to (3.3) the constants $F_{st}^u(\lambda)$ associated with the region I_{++} are completely determined by the wedge algebra, whereas the regions I_{+-} and I_{-+} are only relevant for the extension outside the wedge. The possibility for extending the $\mathcal{B}_\lambda$ algebras for general value of λ was already discussed before [6, 26] (note that the relevance of algebras of differential operators to the solution of this problem was emphasized in ref. [26]). The question is whether such extensions are possible without introducing generators of negative spin. An additional restriction imposed in the work of ref. [6] is, that the Lie algebra exhibits a certain grading according to which the commutator of two even-spin generators can only give rise to even-spin generators. As it turns out [21],

^{*} Alternatively, one may start from the simple identity $L_{s-1}^{(s)} = (L_1^{(2)})^{s-1} = \partial^{s-1}$, and generate similar identities by applying infinitesimal $\mathfrak{sl}(2)$ transformations. These identities suffice to reach the same conclusion.

the $W_\infty(\lambda)$ algebra only satisfies this requirement for $\lambda = \frac{1}{2}$ and, provided that one suppresses the spin-1 generator, for $\lambda = 0$, or 1. We will return to this point at the end of this section.

Now we turn to a more explicit evaluation of the constants $F_{st}^u(\lambda)$. From the product rule (3.1) one can establish the following set of relations,

$$\begin{aligned} & \sum_{j=0}^{t-1} \theta(i) \theta(s-1-i) \theta(k-j) \theta(i+j-k) \binom{i}{k-j} (-)^{i+j} a^i(s, \lambda) a^j(t, \lambda) \\ &= \sum_{u=1}^{s+t-1} \sum_{l=0}^{s+t-u-1-k} \theta(k) \theta(s+t-u-1-k) F_{st}^u(\lambda) a^k(s+t-u, \lambda) \\ & \quad \times \theta(s-1-i-l) \theta(u-s+i+l) (-)^{s-u-k-i-l} \\ & \quad \times \binom{s+t-u-1-k}{l} \binom{u-1}{s-1-i-l} (2s-u)_{u-s+i+l} (2t-u)_{s-1-i-l}, \end{aligned} \quad (3.7)$$

where i and k are arbitrary integers and $\theta(n) = 1$ for $n \geq 0$ and zero otherwise. The equations (3.7) are not all independent; some of them are related by $\mathfrak{sl}(2)$.

In order to solve (3.7) and find an expression for $F_{st}^u(\lambda)$, one sums over i including a sign factor $(-)^i$. Then, after shifting the summation parameter l to $l-i$, one uses the equation

$$\sum_{i=0}^n (-)^i \binom{n}{i} = \delta(n), \quad (3.8)$$

for integer $n > 0$. This equation expresses the fact that $(1-1)^n = 0$ if $n \geq 0$. After application of the Gauss summation theorem (cf. eq. (A.7)) on both sides of the resulting equation, one finds

$$\begin{aligned} F_{st}^u(\lambda) &= (-)^{u+1} \frac{(2s+2t-2u-1)!}{(2s+2t-u-2)!} \\ & \quad \times \sum_{i=0}^{s-1} \sum_{j=0}^{t-1} \delta(i+j-s-t+u+1) a^i(s, 1-\lambda) a^j(t, \lambda). \end{aligned} \quad (3.9)$$

This expression for $F_{st}^u(\lambda)$ satisfies the relation

$$F_{st}^u(\lambda) = F_{ts}^u(1-\lambda), \quad (3.10)$$

in accord with the bosonic version of the anti-automorphism (2.6).

Performing one of the summations in (3.9) one can remove the Kronecker delta and obtain the result

$$F_{st}^u(\lambda) = (-)^{u+1} \frac{(2s+2t-2u-1)!}{(2s+2t-u-2)!} \\ \times \sum_i \binom{s-1}{i} \binom{t-1}{u-1-i} \frac{(2\lambda-s)_i}{(2s-1-i)_i} \frac{(-2\lambda-t+2)_{u-1-i}}{(2t-u+i)_{u-1-i}}, \quad (3.11)$$

where the boundaries of the summation domain are implied by the zeroes of the two binomial coefficients. Depending on the values for s , t and u , the sum runs over s , t , u or $s+t-u$ terms and can be expressed in the form of a generalized hypergeometric series.

It is instructive to evaluate some of the explicit expressions for the constants $F_{st}^u(\lambda)$. A straightforward calculation gives for $u = 1, 2, 3$,

$$F_{st}^1(\lambda) = 1, \\ F_{st}^2(\lambda) = \frac{1}{4}\theta(s-2)\theta(t-2) + \frac{1}{4}\theta(s-2)\delta(t-1)\frac{s-2\lambda}{s-1} \\ + \frac{1}{4}\theta(t-2)\delta(s-1)\frac{t-2+2\lambda}{t-1}, \\ F_{st}^3(\lambda) = \frac{1}{32}\theta(s-2)\theta(t-2)\left\{1 - \frac{16\lambda(\lambda-1)+3}{(2s-3)(2t-3)(2s+2t-5)}\right\} \\ + \frac{1}{8}\theta(s-3)\delta(t-1)\left\{\frac{4\lambda(\lambda-1)}{(2s-3)^2} + \frac{s(s-1)}{(2s-3)^2} - \frac{2\lambda}{2s-3}\right\} \\ + \frac{1}{8}\theta(t-3)\delta(s-1)\left\{\frac{4\lambda(\lambda-1)}{(2t-3)^2} + \frac{t(t-1)}{(2t-3)^2} - \frac{2(1-\lambda)}{2t-3}\right\}. \quad (3.12)$$

Clearly, the evaluation of these expressions becomes very cumbersome for large values of u . The terms with the Kronecker deltas correspond to the regions I_{+-} and I_{-+} indicated in (3.3). The other terms belong to the region I_{++} and coincide with the multiplication constants of the wedge subalgebra. As mentioned previously, these coefficients can only depend on λ through the combination $\lambda(\lambda-1)$. By virtue of eq. (3.10), these coefficients must therefore be symmetric under the exchange of s and t . This exchange property, which is manifest in the results listed in (3.12), can also be derived directly for the wedge subalgebra. Namely, one

considers an anti-automorphism induced by the substitution $L^{(2)} \rightarrow -L^{(2)}$ accompanied by a reversal in the order of the operators in products of the $L^{(2)}$. This operation leaves both the $\mathfrak{sl}(2)$ commutation relations and the Casimir operator unchanged. As the spin- s generators inside the wedge are built from symmetrized products of the $\mathfrak{sl}(2)$ generators, they change under the anti-automorphism by a factor $(-)^{s-1}$. This suffices to show that the constants $F_{st}^u(\lambda)$ are symmetric in s and t inside the wedge.

The structure constants for the $W_\infty(\lambda)$ algebra can be expressed as in (3.4) but now with coefficients $f_{st}^u(\lambda)$ equal to

$$f_{st}^u(\lambda) = F_{st}^u(\lambda) + (-)^u F_{ts}^u(\lambda) = F_{st}^u(\lambda) + (-)^u F_{st}^u(1-\lambda). \quad (3.13)$$

In the last equation we made use of eq. (3.10). Inside the wedge one thus finds that the structure constants vanish for odd values of u , irrespective of the value for λ . For the full $W_\infty(\lambda)$ algebra this phenomenon also takes place for $\lambda = \frac{1}{2}$, as is manifest in eq. (3.13). Actually, there is one more value for λ where this is also the case, namely for $\lambda = 0$ (or, equivalently, for $\lambda = 1$), provided that we exclude the spin-1 generator. In that case all the generators $L^{(s)}$ are proportional to a derivative, and we may write

$$L^{(s)} = \hat{L}^{(s)} \partial \quad (s \geq 2). \quad (3.14)$$

It turns out that the differential operator $\hat{L}^{(s)}$ changes under the anti-automorphism (2.6) in the following way*

$$\hat{L}^{(s)} \rightarrow (-)^s \hat{L}^{(s)}. \quad (3.15)$$

This allows one to show that

$$F_{st}^u(0) = F_{ts}^u(0) \quad (s, t \geq 2). \quad (3.16)$$

From the expression for $\xi^{(s-3)}$ in (2.14) we can verify that $\lambda = 0, \frac{1}{2}, 1$ represent the only cases where the exchange property holds also outside the wedge. Hence, for these values of λ one can truncate the Lie algebra to the even-spin generators. One then obtains the algebras denoted by $W_{(1+\infty)/2}$ and $W_{\infty/2}$ [6]. For the supersymmetric algebra a similar phenomenon takes place, as we shall discuss later.

4. The superalgebra

We now apply the analysis of the previous section to the associative superalgebra based on the generators $L_\lambda^{(s)}$, where s takes integer and half-integer values

* This implies that $\hat{L}^{(s)}$ can be written as a sum of $L^{(s-1)}$, $L^{(s-3)}$, etc., defined for $\lambda = \frac{1}{2}$.

with $s \geq \frac{1}{2}$. We begin by requiring the covariance of the algebra with respect to $\text{osp}(1, 2)$. The associative algebra is defined by specifying how the product of two generators, say $L_\lambda^{(s)}$ and $L_\lambda^{(t)}$, decomposes in terms of the generators. From dimensional arguments, using the parametrization (2.19), we may again write the product as

$$L_\lambda^{(s)}(\Omega^{(s)})L_\lambda^{(t)}(\Omega^{(t)}) = \sum_{u=1}^{s+t-1} L_\lambda^{(s+t-u)}(\xi_{(s)(t)}^{(s+t-u)}), \quad (4.1)$$

where, in contrast with eq. (3.1), s , t and u may take both integer and half-integer values. The generators on the left-hand side depend on corresponding functions $\Omega^{(s)}(z, \theta)$ and $\Omega^{(t)}(z, \theta)$. The generators on the right-hand side depend on functions $\xi_{(s)(t)}^{(s+t-u)}(z, \theta)$, which can be written in the form

$$\xi_{(s)(t)}^{(s+t-u)} = \sum_{i=0}^{2u-2} F_{st}^u(i, \lambda)(D^i \Omega^{(s)})(D^{2u-2-i} \Omega^{(t)}). \quad (4.2)$$

We stress that the summation index i takes only integer values.

The $\text{osp}(1, 2)$ covariance is implemented by taking the commutator of both sides of eq. (4.1) with the super-Virasoro operator $L^{(2)}(\Omega^{(2)})$, where the corresponding function $\Omega^{(2)}$ satisfies the $\text{osp}(1, 2)$ condition $D^5 \Omega^{(2)} = 0$. We thus find five equations. Three are identically satisfied (those are just the conditions that require $\xi_{(s)(t)}^{(s+t-u)}$ to be of the form quoted in (4.2)), while the equation proportional $D^4 \Lambda^{(2)}$ follows from the one proportional to $D^3 \Lambda^{(2)}$. The latter yields a recurrence relation for the coefficients $F_{st}^u(i, \lambda)$, which determines their i -dependence and shows, just as in the bosonic case, that the range of i -values in the sum (4.2) is restricted in a certain way. Again three possible situations can be distinguished:

$$\begin{aligned} \text{I}_{++}: \quad & 0 \leq i \leq 2u - 2, & 2u \leq 4s - 2, & \quad 2u \leq 4t - 2, \\ \text{I}_{+-}: \quad & 0 \leq i \leq 2u - 4t + 1, & 2u \leq 4s - 2, & \quad 2u \geq 4t - 1, \\ \text{I}_{-+}: \quad & 4s - 3 \leq i \leq 2u - 2, & 2u \geq 4s - 1, & \quad 2u \leq 4t - 2. \end{aligned} \quad (4.3)$$

A solution of the recurrence relation which incorporates these restrictions on the range of i values, is given by

$$\begin{aligned} \xi_{(s)(t)}^{(s+t-u)} = F_{st}^u(\lambda) \sum_{i=0}^{2u-2} (-)^{[i/2]+2i(s+u)} \begin{bmatrix} u-1 \\ i/2 \end{bmatrix} ([2s-u])_{[u-1-i/2]+[2u]_2[2u-2-i]_2} \\ \times ([2t-u])_{[i/2]+[2u]_2[i]_2} (D^i \Omega^{(s)})(D^{2u-2-i} \Omega^{(t)}), \end{aligned} \quad (4.4)$$

where the notation $|n|_2$ was introduced in sect. 2. We also introduced a slightly

modified version of the binomial coefficient,

$$\begin{bmatrix} a \\ b \end{bmatrix} \equiv \frac{[a]!}{[b]![a-b]!}. \quad (4.5)$$

It is easy to verify that for integer values of s , t and u (4.4) reduces to (3.4) if we restrict the sum in (4.4) to even values of i .

The region I_{++} is characterized by the fact that the right-hand side of (4.4) contains no multiple derivatives of higher order than $D^{4s-4}\Omega^{(s)}$ and $D^{4t-4}\Omega^{(t)}$. This in contradistinction with I_{+-} (I_{-+}), where no multiple derivatives appear of higher order than $D^{4s-4}\Omega^{(s)}(D^{4t-4}\Omega^{(t)})$ and of lower order than $D^{4t-3}\Omega^{(t)}(D^{4s-3}\Omega^{(s)})$. The generators corresponding to functions $\Omega^{(s)}$ satisfying (for all s)

$$D^{4s-3}\Omega^{(s)} = 0, \quad (4.6)$$

form an infinite-dimensional subalgebra. Its multiplication rules are governed by the constants $F_{st}^u(\lambda)$ belonging to the region I_{++} . This algebra will be called the superwedge algebra, because, just as before in the bosonic case, these generators belong to a wedge in the parameter plane spanned by the value of the spin and the L_0 eigenvalue. In terms of the bosonic and fermionic generators defined in (2.2), the superwedge generators consist of the bosonic generators $L_n^{(s)\pm}$ with $|n| \leq s-1$ and the fermionic generators $G_r^{(s)\pm}$ with $|r| \leq s - \frac{3}{2}$. The generators are thus restricted to finite-dimensional irreducible representations of $\text{osp}(1, 2)^*$.

In order to prove that the superwedge generators constitute an infinite-dimensional associative algebra, we make use of the fact that these generators span the enveloping algebra of $\text{osp}(1, 2)$ factored by certain ideals. This enveloping algebra is based on functions of the $\text{osp}(1, 2)$ generators, $L_{\pm 1}$, L_0 and $G_{\pm 1/2} \equiv G_{\pm 1/2}^{(2)+}$. Since the $\text{sl}(2)$ generators $L_{\pm 1}$ and L_0 can be expressed in terms of anticommutators of the supercharges G_α we may restrict ourselves to functions of the latter. Here we employ a two-component notation where G_α with $\alpha = 1, 2$ denotes $(G_{1/2}, G_{-1/2})$. The supercharges do not commute,

$$[G_\alpha, G_\beta] = (Q + \frac{1}{2})\varepsilon_{\alpha\beta}, \quad (4.7)$$

where Q is some bosonic operator, which belongs to the enveloping algebra. From the $\text{osp}(1, 2)$ commutation relations one finds that

$$\{Q, G_\alpha\} = 0. \quad (4.8)$$

* We remind the reader that for the generators (2.2) s was defined to be an integer. Furthermore, we note that the superwedge algebra can only be defined for the Neveu-Schwarz sector. In terms of the functions $\Lambda^{(s)\pm}(z)$ and $\Theta^{(s)\pm}(z)$, (4.6) reads $\partial^{2s-1}\Lambda^{(s)\pm} = \partial^{2s-2}\Theta^{(s)\pm} = 0$, so that $\Lambda^{(s)\pm}$ and $\Theta^{(s)\pm}$ are polynomials in z of degree $2s-2$ and $2s-3$, respectively.

As none of the $\mathfrak{sl}(2)$ commutation relations imposes further conditions on G_α and Q , the enveloping algebra of $\mathfrak{osp}(1,2)$ is isomorphic to the associative algebra generated by G_α and Q . It turns out the Q is directly related to the $\mathfrak{osp}(1,2)$ Casimir operator,

$$C_2(\mathfrak{osp}(1,2)) = L_0^2 - \frac{1}{2}\{L_1, L_{-1}\} + \frac{1}{4}[G_{1/2}, G_{-1/2}] = \frac{1}{4}Q^2 - \frac{1}{16}. \quad (4.9)$$

In the representation at hand the Casimir operator is equal to

$$C_2(\mathfrak{osp}(1,2)) = \lambda(\lambda - \frac{1}{2}). \quad (4.10)$$

In this way we establish the existence of the Klein operator K , which in this representation is equal to $K = 1 - 2\theta \partial/\partial\theta$. Indeed K satisfies $K^2 = \mathbb{1}$ and anti-commutes with θ and $\partial/\partial\theta$. (An analogous realization of K was proposed in ref. [17] in terms of Clifford algebras.) It is related to the operator Q by

$$Q = 2(\lambda - \frac{1}{4})K. \quad (4.11)$$

The enveloping algebra of $\mathfrak{osp}(1,2)$ factored over the ideal generated by $C_2(\mathfrak{osp}(1,2)) - \lambda(\lambda - \frac{1}{2})$ is thus isomorphic to the algebra generated by the G_α and K with $K^2 = \mathbb{1}$. Its elements can be written as [23]

$$F(G_\alpha, K) = \sum_{k=0}^{\infty} \{F_0^{\alpha_1 \dots \alpha_k} + K F_1^{\alpha_1 \dots \alpha_k}\} G_{\alpha_1} \dots G_{\alpha_k}, \quad (4.12)$$

where the coefficients are symmetric in $\mathfrak{sl}(2)$ indices $\alpha_1 \dots \alpha_k$. From the $\mathfrak{sl}(2)$ assignments we may conclude that these functions coincide with (linear combinations of) the generators in the wedge. Therefore these generators constitute an associative algebra which is isomorphic to the enveloping algebra of $\mathfrak{osp}(1,2)$ factored by an ideal.

In general the superwedge algebra admits $N = 2$ supersymmetry, as was already observed in ref. [25] for the special case of $\lambda = 0$ and in ref. [16] for the case $\lambda = \frac{1}{4}$. The reason is that the algebra contains two kinds of supersymmetry charges, $G_\alpha^{(2)\pm}$. Together with $L_{\pm 1}^{(2)+}$, $L_0^{(2)+}$ and $L_0^{(1)-}$, they generate the $\mathfrak{osp}(2,2)$ algebra. The two supercharges are related by the Klein operator according to

$$G_\alpha^{(2)-} = G_\alpha K = -K G_\alpha. \quad (4.13)$$

Therefore the superwedge algebra is also contained in the enveloping algebra of $\mathfrak{osp}(2,2)$, factored by certain ideals related to conditions such as (4.10), (4.13) and $K^2 = \mathbb{1}$.

For $\lambda \neq \frac{1}{4}$ we may choose a basis for the superwedge algebra consisting of the symmetrized polynomials $G_{\alpha_1} \dots G_{\alpha_k}$ and $Q G_{\alpha_k}$. It is then clear that the product

structure of the algebra depends on λ through the value of the Casimir operator (4.10), just as in the bosonic case. The anti-automorphism (2.6)–(2.7) is induced here by the substitutions $G_\alpha \rightarrow iG_\alpha$, $Q \rightarrow Q$, where we simultaneously reverse the order of all operators G_α and Q in (4.12). Under this operation the $\text{osp}(1,2)$ commutation relations, the Casimir operator (4.10) and the commutator (4.7) remain unchanged.

As mentioned previously the value $\lambda = \frac{1}{4}$ is special. In that case it follows from eqs. (4.9)–(4.10) that $Q^2 = 0$. As a result the elements proportional to Q form an ideal of the enveloping algebra of $\text{osp}(1,2)$ which can be factored out. Therefore one has the option of restricting the superwedge algebra to the K -independent part of (4.12), so that one is dealing with $N=1$ supersymmetry only. The corresponding $N=1$ Lie algebra is isomorphic to the so-called symplectic higher-spin superalgebra [15]. Two other truncations, based on the anti-automorphism (2.6)–(2.7), amount to retaining the generators $L_n^{(s)+}$ with s even and $L_n^{(s)-}$ with s odd and the generators $G_n^{(s)\pm}$ with s either even or odd. In the case $\lambda = \frac{1}{4}$ the multiplication rules within the wedge algebra often take a simpler form. For instance, one can easily deduce that $F_{st}^u(\frac{1}{4})$ vanishes inside the wedge for half-integer values of u .

Before continuing the discussion of the full algebra, we note that the existence of the anti-automorphism implies the following relation for the constants $F_{st}^u(\lambda)$,

$$F_{st}^u(\lambda) = (-)^{2u} F_{ts}^u(\frac{1}{2} - \lambda). \quad (4.14)$$

To determine the constants $F_{st}^u(\lambda)$ we proceed as in sect. 3. First we substitute (4.4) into (4.1) and evaluate the term proportional to $(D^{2s-2-i}\Omega^{(s)})(D^{2t-2+i-k}\Omega^{(t)})D^k$. This leads to a generalization of (3.7), which we sum over i after including a factor $\hat{i}$. To distinguish the imaginary unit from the index i , we denote it by $\hat{i}$. After using eq. (3.8) the result reads

$$\begin{aligned} & \sum_{i=0}^{2s-2} \sum_{j=0}^{2t-2} \theta(k-j) \theta([i/2] - [(k-j)/2]) \binom{[i/2]}{[(k-j)/2]} A^i(s, \lambda) A^j(t, \lambda) \\ & \quad \times \hat{i}^j \{ |k-j-1|_2 + (-)^j |i|_2 |k-j|_2 \} \\ & = \sum_{u=1}^{s+t-1} \theta(k) \theta(2s+2t-2u-2-k) F_{st}^u(\lambda) A^k(s+t-u, \lambda) \\ & \quad \times \sum_{i=0}^{2u-2} (-)^{[i/2]+2i(s+u)} \hat{i}^{2s-2-i} \left[\frac{u-1}{i/2} \right] \\ & \quad \times ([2s-u])_{[u-1-i/2]+|2u|_2|2u-2-i|_2} ([2t-u])_{[i/2]+|2u|_2|i|_2} \\ & \quad \times \left\{ \delta(s+t-u-1-\frac{1}{2}k) + ((-)^{2s-2-i} - \hat{i}) \delta(s+t-u-\frac{3}{2}-\frac{1}{2}k) \right\}, \end{aligned} \quad (4.15)$$

where i, j, k always take integer values, while the parameters s, t, u can be integer or half-integer.

After shifting the summation index i on the left-hand side and using

$$A^{2(i+k)}(s, \lambda) = (-)^k \frac{(i+1-[s])_k ([s]+i)_k}{(2\lambda+i)_k (i+1)_k} A^{2i}(s, \lambda),$$

$$A^{2(i+k)+1}(s, \lambda) = (-)^k \frac{(i+2-[s+\frac{1}{2}])_k ([s+\frac{1}{2}]+i)_k}{(2\lambda+i+1)_k (i+1)_k} A^{2i+1}(s, \lambda), \quad (4.16)$$

we can perform the sum over i with the aid of the Gauss summation theorem (cf. eq. (A.7)). Also on the right-hand side this theorem can be used to perform the i -summation. The result takes the following form

$$\begin{aligned} & \sum_{i=0}^{2s-2} \sum_{j=0}^{2t-2} \delta([i/2] - [(k-j)/2]) A^i(s, \lambda) A^j(t, \lambda) (-)^{[(k-j)/2]} \\ & \times \left\{ \frac{(2\lambda - [s])_{[s-1-i/2]}}{(2\lambda + [i/2])_{[s-1-i/2]}} |i+1|_2 |k-j-1|_2 \right. \\ & \quad \left. + \hat{i} \frac{(2\lambda - [s-\frac{1}{2}])_{[s-1-i/2]}}{(2\lambda + [i/2] + 1)_{[s-1-i/2]}} |i|_2 \left\{ |k-j-1|_2 + (-)^j |k-j|_2 \right\} \right\} \\ & = \sum_{u=1}^{s+t-1} \theta(k) \theta(2s+2t-2u-2-k) A^k(s+t-u, \lambda) \\ & \times \hat{i}^{2s-2} \frac{(2s+2t-2u-2+[u])!}{(2s+2t-2u-1)!} F_{st}^u(\lambda) \\ & \times \left\{ \left[[2s-u]^{[2u]_2} + \hat{i} (-)^{2s} [2t-u]^{[2u]_2} \left(\frac{1-u}{2s+2t-u-2} \right)^{|2u+1|_2} \right] \right. \\ & \quad \times \delta(s+t-u-1-\frac{1}{2}k) \\ & \quad \left. + \left[[2s-u]^{[2u]_2} + [2t-u]^{[2u]_2} \left(\frac{1-u}{2s+2t-u-2} \right)^{|2u+1|_2} \right] \right. \\ & \quad \left. \times [(-1)^{2s} - \hat{i}] \delta(s+t-u-\frac{3}{2}-\frac{1}{2}k) \right\}. \quad (4.17) \end{aligned}$$

We now multiply this equation by $(-\hat{i})^{2s-2}[(-)^{2s} + \hat{i}]$ and take its imaginary part. This cancels the term proportional to $[(-)^{2s} - \hat{i}]$ on the right-hand side of (4.17) and leads directly to an expression for $F_{st}^u(\lambda)$,

$$\begin{aligned}
 F_{st}^u(\lambda) &= (-)^{[s+t-u-1]} \frac{(2s+2t-2u-2)!}{(2s+2t-[u]-3)!} \\
 &\times \sum_{i=0}^{2s-2} \sum_{j=0}^{2t-2} \delta([i/2] - [s+t-u-1-\frac{1}{2}j]) A^i(s, \lambda) A^j(t, \lambda) \\
 &\times \left\{ \frac{(2\lambda - [s])_{[s-1-i/2]}}{(-2\lambda - [s] + 2)_{[s-1-i/2]}} |i+1|_2 |2s+2t-2u-j-1|_2 \right. \\
 &\quad + \frac{(2\lambda - [s-\frac{1}{2}])_{[s-1-i/2]}}{(-2\lambda - [s-\frac{1}{2}] + 1)_{[s-1-i/2]}} |i|_2 (-)^{2s+1} \\
 &\quad \left. \times \{|2s+2t-2u-j-1|_2 + (-)^j |2s+2t-2u-j|_2\} \right\}. \quad (4.18)
 \end{aligned}$$

Using the relation (A.20) derived in appendix A, the above result can be written as

$$\begin{aligned}
 F_{st}^u(\lambda) &= (-)^{[s+t-u-1]} \frac{(2s+2t-2u-2)!}{(2s+2t-[u]-3)!} \sum_{i=0}^{2s-2} \sum_{j=0}^{2t-2} \delta(i+j-2s-2t+2u+2) \\
 &\times A^i(s, \tfrac{1}{2} - \lambda) A^j(t, \lambda) (-)^{2s+2t(s+t-u)}. \quad (4.19)
 \end{aligned}$$

This result, which satisfies relation (4.14), is very similar to the corresponding result (3.9) for the bosonic algebra. As the qualitative features are also the same, we refrain from giving further details here. We should note, however, that the $F_{st}^u(\lambda)$ satisfy a number of identities owing to the $N=2$ supersymmetry. These identities are not manifestly satisfied by the representation (4.19), but do not take a particularly illuminating form.

Finally, we can express the structure constants for the $W_\infty(\lambda)$ algebra in terms of the constants $F_{st}^u(\lambda)$. Again the result of the commutators can be written in the form (4.4), with the constants $F_{st}^u(\lambda)$ replaced by

$$f_{st}^u(\lambda) = F_{st}^u(\lambda) + (-)^{[-u]+4(s+u)(t+u)} F_{st}^u(\tfrac{1}{2} - \lambda), \quad (4.20)$$

where we made use of eq. (4.14). For $\lambda = \frac{1}{4}$ the structure constants vanish whenever $[-u] + 4(s+u)(t+u)$ is odd. This shows that the Lie algebra can be

consistently truncated to the generators with $s = 2n$ or $s = 2n - \frac{1}{2}$, where n is a positive integer, in accord with previous arguments based on the anti-automorphism (2.6). The second (non-supersymmetric) truncation is not directly obvious in this supersymmetric formulation. Another truncation, which can be implemented for $\lambda = 0, \frac{1}{2}$, shall be discussed in sect. 5.

5. Currents

It is possible to define a field-theoretic representation of the super- $W_\infty(\lambda)$ algebra in terms of an operator product expansion for quasi-primary field operators. For specific values of λ such representations have been defined in terms of free boson or fermion fields [25, 28, 29]. For general value of λ such a representation can be based on two conformal superfields, a commuting field B and an anticommuting field C , with conformal weights λ and $\frac{1}{2} - \lambda$, respectively. When subjected to their field equations, these fields decompose according to $B(z, \theta) = \beta(z) + \theta b(z)$ and $C(z, \theta) = c(z) + \theta \gamma(z)$. Since θ has weight $-\frac{1}{2}$, we find that b , c , β and γ have conformal weights $\lambda + \frac{1}{2}$, $-\lambda + \frac{1}{2}$, λ and $-\lambda + 1$. The supersymmetric action equals [24]

$$S = \frac{1}{\pi} \int d^2z \, d^2\theta \, B \bar{D}C = \frac{1}{\pi} \int d^2z \{ \beta \bar{\partial} \gamma + b \bar{\partial} c \}. \quad (5.1)$$

The generators of the super- $W_\infty(\lambda)$ algebra are then related to the following conserved currents

$$\begin{aligned} V_\lambda^{(s)+}(z) &= \sum_{i=0}^{s-1} a^i(s, \lambda) \partial^{s-1-i} \{ (\partial^i \beta) \gamma \} + \sum_{i=0}^{s-1} a^i(s, \lambda + \tfrac{1}{2}) \partial^{s-1-i} \{ (\partial^i b) c \}, \\ V_\lambda^{(s)-}(z) &= -\frac{s-1+2\lambda}{2s-1} \sum_{i=0}^{s-1} a^i(s, \lambda) \partial^{s-1-i} \{ (\partial^i \beta) \gamma \} \\ &\quad + \frac{s-2\lambda}{2s-1} \sum_{i=0}^{s-1} a^i(s, \lambda + \tfrac{1}{2}) \partial^{s-1-i} \{ (\partial^i b) c \}, \\ Q_\lambda^{(s)\pm}(z) &= \sum_{i=0}^{s-1} \alpha^i(s, \lambda) \partial^{s-1-i} \{ (\partial^i \beta) c \} \mp \sum_{i=0}^{s-2} \beta^i(s, \lambda) \partial^{s-2-i} \{ (\partial^i b) \gamma \}. \end{aligned} \quad (5.2)$$

Using

$$\gamma(z) \beta(w) \sim \frac{1}{z-w}, \quad c(z) b(w) \sim \frac{1}{z-w}, \quad (5.3)$$

with all other independent contractions vanishing, one can then evaluate the operator products of these currents. To establish the correspondence with the previous representation of the algebra, one considers the products of the currents with some of the fields. For instance, the operator product expansions of $V^{(s)+}(z)\beta(w)$ and $V^{(s)+}(z)b(w)$ take the form

$$\begin{aligned} V_\lambda^{(s)+}(z)\beta(w) &\sim \sum_{i=0}^{s-1} a^i(s, \lambda) \partial_z^{s-1-i} \left(\frac{1}{z-w} \right) \partial^i \beta(w), \\ V_\lambda^{(s)+}(z)b(w) &\sim \sum_{i=0}^{s-1} a^i\left(s, \lambda + \frac{1}{2}\right) \partial_z^{s-1-i} \left(\frac{1}{z-w} \right) \partial^i b(w). \end{aligned} \quad (5.4)$$

The commutator of $B(z, \theta)$ with the charge

$$V^{(s)+} = \frac{1}{2\pi i} \oint dz \Lambda^{(s)+}(z) V^{(s)+}(z), \quad (5.5)$$

then leads directly to the differential operator $L_\lambda^{(s)+}$ defined in (2.5) acting on the superfield $B(z, \theta)$.

It is instructive to write out explicitly some of the lower-spin currents,

$$\begin{aligned} V_\lambda^{(2)+} &= (\partial\beta)\gamma - \lambda\partial(\beta\gamma) + (\partial b)c - \left(\lambda + \frac{1}{2}\right)\partial(bc), \\ Q_\lambda^{(2)\pm} &= (\partial\beta)c - 2\lambda\partial(\beta c) \mp b\gamma, \\ V_\lambda^{(1)+} &= \beta\gamma + bc, \\ V_\lambda^{(1)-} &= -2\lambda\beta\gamma + (1 - 2\lambda)bc, \\ Q_\lambda^{(1)} &= \beta c. \end{aligned} \quad (5.6)$$

The currents $(Q^{(2)+}, V^{(2)+})$ reproduce the $N=1$ super-Virasoro algebra (cf. (2.1)). The currents $(Q_\lambda^{(s)+}, V_\lambda^{(s)+})$ and $(V_\lambda^{(s)-}, Q_\lambda^{(s+1)-})$ constitute non-decomposable representations of $\text{osp}(1, 2)$. In superfield notation the currents (5.2) take the form

$$\begin{aligned} W_\lambda^{(s)+}(z, \theta) &= \frac{1}{2} Q_\lambda^{(s)+}(z) + \theta V_\lambda^{(s)+}(z) \quad (s \geq 2), \\ W_\lambda^{(1)+}(z, \theta) &= Q_\lambda^{(1)+}(z) + \theta V_\lambda^{(1)+}(z), \\ W_\lambda^{(s)-}(z, \theta) &= V_\lambda^{(s)-}(z) - \theta Q_\lambda^{(s+1)-}(z), \end{aligned} \quad (5.7)$$

and can be expressed in terms of the superfields $B(z, \theta)$ and $C(z, \theta)$. The normalization is such that the operator product expansion of the currents with the

B, C superfields reproduces the superspace differential operators given in eq. (2.19). For the lower-spin currents (5.6), this leads to

$$\begin{aligned} W_\lambda^{(2)+}(z, \theta) &= \lambda B(D^2 C) - \frac{1}{2}(DB)(DC) - \frac{1}{2}(1 - 2\lambda)(D^2 B)C, \\ W_\lambda^{(1)-}(z, \theta) &= -2\lambda B(DC) + (1 - 2\lambda)(DB)C, \\ W_\lambda^{(1)+}(z, \theta) &= BC. \end{aligned} \quad (5.8)$$

The corresponding expressions for currents of arbitrary spin will be presented shortly.

It is well known [24] that the supersymmetric bc system actually has a $N=2$ superconformal invariance. Indeed the currents $(V^{(1)-}, Q^{(2)\pm}, V^{(2)+})$ define an $N=2$ super-Virasoro algebra. In the Neveu–Schwarz sector it contains a finite $\text{osp}(2, 2)$ subalgebra, which was already discussed in sect. 4, generated by the current components corresponding to $(L_0^{(1)-}, G_{1/2}^{(2)\pm}, G_{-1/2}^{(2)\pm}, L_0^{(2)+}, L_\pm^{(2)+})$. All currents thus fit into $N=2$ supermultiplets with respect to this $\text{osp}(2, 2)$ subalgebra. The resulting combinations are

$$\begin{aligned} (V^{(s)-}, Q^{(s+1)\pm}, V^{(s+1)+}) \quad s = 1, 2, 3, \dots, \\ (Q^{(1)}, V^{(1)+}), \end{aligned} \quad (5.9)$$

where $(Q^{(1)}, V^{(1)+})$ constitutes a so-called $N=2$ scalar multiplet.

In this field-theoretic representation the equivalence of $W_\infty(\lambda)$ and $W_\infty(\frac{1}{2} - \lambda)$ is related to the interchange of the two superfields B and C . To be more specific, consider the effect of the replacements

$$\begin{aligned} c &\rightarrow \varepsilon_1 \beta, & \beta &\rightarrow -i\varepsilon_2 c, \\ \gamma &\rightarrow -\varepsilon_1 b, & b &\rightarrow i\varepsilon_2 \gamma, \end{aligned} \quad (5.10)$$

where ε_1 and ε_2 are real anti-commuting parameters*. Using the resummation formulae (A.8)–(A.10) given in the appendix, we can prove that the substitution of (5.10) into the expressions for the currents yields

$$\begin{aligned} V_\lambda^{(s)\pm} &\rightarrow \pm (-)^s \alpha_{12} V_{1/2-\lambda}^{(s)\pm}, \\ Q_\lambda^{(s)\pm} &\rightarrow (-)^s \alpha_{12} Q_{1/2-\lambda}^{(s)\pm}, \end{aligned} \quad (5.11)$$

where we introduced the notation $\alpha_{12} = i\varepsilon_1 \varepsilon_2$. The above result can easily be verified for the low-spin generators listed in (5.6).

* In the purely bosonic case one interchanges the fields β and γ .

The above substitution can also be applied to products of currents. First note that the contractions (5.3) change under the substitutions (5.10) into $-\alpha_{12}(z-w)^{-1}$. For a product of two currents one makes the substitution (5.10) in one current and the same substitution, but now with ε_1 and ε_2 replaced by ε_3 and ε_4 , respectively, in the second current. This then leads to the same restrictions on the structure constants as found before.

One can show that a truncated version of the operator algebra must exist for $\lambda = 0, \frac{1}{2}$. For instance, for $\lambda = 0$ the conformal weights of b and c are equal to $\frac{1}{2}$ while the conformal weights of β and γ are equal to 0 and 1, respectively*. In the operator expansion one can then replace γ by $\partial\phi$, where ϕ has conformal weight 0. The truncation can now be implemented by identifying b with c and β with ϕ . To see the effect of this identification we rewrite the currents (5.2) as

$$\begin{aligned}
 V_\lambda^{(s)+}(z) &= \sum_{i=0}^{s-1} \tilde{a}^i(s, \lambda) (\partial^i \beta) (\partial^{s-1-i} \gamma) + \sum_{i=0}^{s-1} \tilde{a}^i(s, \lambda + \tfrac{1}{2}) (\partial^i b) (\partial^{s-1-i} c), \\
 V_\lambda^{(s)-}(z) &= -\frac{s-1+2\lambda}{2s-1} \sum_{i=0}^{s-1} \tilde{a}^i(s, \lambda) (\partial^i \beta) (\partial^{s-1-i} \gamma) \\
 &\quad + \frac{s-2\lambda}{2s-1} \sum_{i=0}^{s-1} \tilde{a}^i(s, \lambda + \tfrac{1}{2}) (\partial^i b) (\partial^{s-1-i} c), \\
 Q_\lambda^{(s)\pm}(z) &= \sum_{i=0}^{s-1} \tilde{\alpha}^i(s, \lambda) (\partial^i \beta) (\partial^{s-1-i} c) \mp \sum_{i=0}^{s-2} \tilde{\beta}^i(s, \lambda) (\partial^i b) (\partial^{s-2-i} \gamma). \quad (5.12)
 \end{aligned}$$

Here we made use of the resummation formulae (A.11)–(A.13) given in appendix A; the coefficients $\tilde{a}^i(s, \lambda)$, $\tilde{\alpha}^i(s, \lambda)$ and $\tilde{\beta}^i(s, \lambda)$ are defined in (A.14). Substituting $b = c$, $\beta = \phi$ and $\gamma = \partial\phi$, one verifies, using (A.15), (A.19) and the fact that $\tilde{\alpha}^0(s, 0) = 0$ for $s > 1$, that $Q^{(s)+} = 0$ for s even and $Q^{(s)-} = 0$ for $s > 1$ odd. Furthermore one uses (A.21) to show that all operators $V^{(s)\pm}$ with odd $s > 1$ vanish as well. The operators with $s = 1$ require special attention. While $V^{(1)-}$ vanishes identically for $\lambda = 0$, the remaining operators are given by $V^{(1)+} = \phi \partial\phi$ and $Q^{(1)} = \phi c$. These are the only operators that depend on ϕ and not exclusively on derivatives of ϕ . Therefore, the $s = 1$ generators are never generated in the operator products of the $s > 1$ generators (actually, these generators are no longer conserved). Consequently we may consistently reduce the operator algebra to the $s > 1$ operators, so that we are left with the operators $V^{(s)\pm}$ and $Q^{(s)-}$ with $s = 2, 4, \dots$ and $Q^{(s)+}$ with $s = 3, 5, \dots$. This algebra, which exhibits $N = 1$ supersymmetry, coincides with the super- $W_{\infty/2}$ algebra of ref. [25]. The bosonic subalgebra

*For $\lambda = \frac{1}{2}$ the weights of b , c , β and γ are equal to 1, 0, $\frac{1}{2}$ and $\frac{1}{2}$, respectively, so that the same arguments can be applied.

bra of super- $W_{\infty/2}$ equals $W_{\infty/2} \oplus W_{(1+\infty)/2}$, where $W_{\infty/2}$ and $W_{(1+\infty)/2}$ are the truncated versions of W_∞ and $W_{1+\infty}$ discussed in ref. [6]. Observe that the identification of b with c and γ with $\partial\beta$ causes additional factors of 2 in the operator product expansions. However, these factors can again be re-absorbed into the definition of the currents so that the structure constants of the algebra remain the same. Of course, the central charge will be different when we reduce the number of degrees of freedom.

As already discussed several times, there exist two other truncations of super- $W_\infty(\lambda)$ for $\lambda = \frac{i}{4}$. In that case the conformal weights of b , c , β and γ are equal to $\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{3}{4}$, respectively. The remaining generators are $V^{(s)+}$ with $s = 2, 4, \dots$, $V^{(s)-}$ with $s = 1, 3, \dots$ and the fermionic operators $Q^{(s)\pm}$ with s restricted to either $s = 1, 3, \dots$ or to $s = 2, 4, \dots$.

The field-theoretic realization allows an explicit determination of the central extension of super- $W_\infty(\lambda)$. Our starting point are the following expressions for the currents, which follow from (5.7) and (5.12),

$$\begin{aligned} W_\lambda^{(s)+} &= \frac{1}{2} \sum_{i=0}^{s-1} \tilde{\alpha}^i(s, \lambda) (\partial^i B) (\partial^{s-i-1} C) - \frac{1}{2} \sum_{i=0}^{s-2} \tilde{\beta}^i(s, \lambda) (\partial^i DB) (\partial^{s-i-2} DC), \\ W_\lambda^{(s)-} &= -\frac{s-1+2\lambda}{2s-1} \sum_{i=0}^{s-1} \tilde{a}^i(s, \lambda) (\partial^i B) (\partial^{s-i-1} DC) \\ &\quad + \frac{s-2\lambda}{2s-1} \sum_{i=0}^{s-1} \tilde{a}^i(s, \lambda + \tfrac{1}{2}) (\partial^i DB) (\partial^{s-i-1} C). \end{aligned} \quad (5.13)$$

The operator product of the superfields $B(z, \theta)$ and $C(z, \theta)$ is equal to [24]

$$C(z_1, \theta_1) B(z_2, \theta_2) \sim \frac{\theta_{12}}{z_{12}}, \quad (5.14)$$

with $\theta_{12} = \theta_1 - \theta_2$ and $z_{12} = z_1 - z_2 + \theta_1 \theta_2$. Note that $D_1 z_{12} = -\theta_{12}$ and $D_1 \theta_{12} = 1$. The central charges are defined by the coefficient of the leading term in the operator product expansion of two currents,

$$\begin{aligned} W^{(s)+}(z_1, \theta_1) W^{(t)+}(z_2, \theta_2) &\sim c^{++}(s, t; \lambda) \frac{1}{(z_{12})^{s+t-1}} + \dots, \\ W^{(s)\pm}(z_1, \theta_1) W^{(t)\mp}(z_2, \theta_2) &\sim c^{\pm\mp}(s, t; \lambda) \frac{\theta_{12}}{(z_{12})^{s+t}} + \dots, \\ W^{(s)-}(z_1, \theta_1) W^{(t)-}(z_2, \theta_2) &\sim c^{--}(s, t; \lambda) \frac{1}{(z_{12})^{s+t}} + \dots. \end{aligned} \quad (5.15)$$

It satisfies the condition

$$\begin{aligned} c^{\pm\pm}(s, t; \lambda) &= (-)^{s+t} c^{\pm\pm}(t, s; \lambda) = -c^{\pm\pm}\left(t, s; \frac{1}{2} - \lambda\right), \\ c^{+-}(s, t; \lambda) &= (-)^{s+t+1} c^{-+}(t, s; \lambda) = -c^{-+}\left(t, s; \frac{1}{2} - \lambda\right). \end{aligned} \quad (5.16)$$

A straightforward calculation, using the two-point functions given in (5.14), yields explicit expressions for the central charge. For instance, we find

$$\begin{aligned} c^{++}(s, t; \lambda) &= \frac{1}{4}(-)^{s-1} \sum_{i=0}^{s-1} \sum_{j=0}^{t-2} (s+j-i-1)! (t+i-j-2)! \tilde{\alpha}^i(s, \lambda) \tilde{\beta}^j(t, \lambda) \\ &\quad + \frac{1}{4}(-)^{s-1} \sum_{i=0}^{s-2} \sum_{j=0}^{t-1} (s+j-i-2)! (t+i-j-1)! \tilde{\beta}^i(s, \lambda) \tilde{\alpha}^j(t, \lambda), \end{aligned} \quad (5.17)$$

and one may obtain similar expressions for $c^{\pm\mp}$ and c^{--} .

Note that the superwedge algebra is an anomaly-free subalgebra. To show this, one multiplies the operator-products (5.15) with the corresponding parameters $\Omega^{(s)\pm}(z, \theta)$, integrated along an appropriate loop in holomorphic superspace. After writing the first term in the operator-expansion as a multiple superspace derivative of (5.14) and performing a suitable integration by parts, one can show that it vanishes by virtue of eq. (4.6).

For comparison, we evaluate the central charge for the super-Virasoro subalgebra. Using (5.17) we find

$$c^{++}(2, 2; \lambda) = -\frac{1}{2}(1 - 4\lambda). \quad (5.18)$$

This result coincides with that of ref. [24].

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Appendix A

For convenience we list some useful formulae related to the coefficients $a^i(s, \lambda)$, $\alpha^i(s, \lambda)$ and $\beta^i(s, \lambda)$, defined in (2.5). These coefficients are not independent, and

we note the relations

$$\alpha^i(s, \lambda) = \frac{2(s-1)}{s+i-1} a^i(s, \lambda), \quad (\text{A.1})$$

$$\beta^i(s, \lambda) = \frac{(2\lambda+s-2)(s+i-1)}{(2\lambda+i)(2s-3)} a^i(s-1, \lambda). \quad (\text{A.2})$$

Furthermore

$$a^{i+k}(s, \lambda) = \frac{(i+1-s)_k}{(i+1)_k} \frac{(s+i)_k}{(2\lambda+i)_k} a^i(s, \lambda), \quad (\text{A.3})$$

$$\alpha^{i+k}(s, \lambda) = \frac{(i+1-s)_k}{(i+1)_k} \frac{(s+i-1)_k}{(2\lambda+i)_k} \alpha^i(s, \lambda), \quad (\text{A.4})$$

$$\beta^{i+k}(s, \lambda) = \frac{(i+2-s)_k}{(i+1)_k} \frac{(s+i)_k}{(2\lambda+i+1)_k} \beta^i(s, \lambda). \quad (\text{A.5})$$

In sums we often encounter expressions that involve hypergeometric series, such as the Gauss function

$${}_2F_1[a, b; c; z] = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!}. \quad (\text{A.6})$$

For a or b equal to some negative integer, this becomes a finite series. The Gauss summation theorem states that (see for instance ref. [30])

$${}_2F_1[a, b; c; 1] = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \quad (\text{A.7})$$

The above results can be used to obtain the following resummation formulae,

$$\sum_{i=0}^{s-1} a^i(s, \lambda) \partial^i ((\partial^{s-1-i} A) B) = \sum_{i=0}^{s-1} (-)^{s-1-i} a^i(s, 1-\lambda) (\partial^{s-1-i} A) (\partial^i B), \quad (\text{A.8})$$

$$\sum_{i=0}^{s-1} \alpha^i(s, \lambda) \partial^i ((\partial^{s-1-i} A) B) = \sum_{i=0}^{s-1} (-)^{s-1-i} \alpha^i(s, \frac{1}{2}-\lambda) (\partial^{s-1-i} A) (\partial^i B), \quad (\text{A.9})$$

$$\sum_{i=0}^{s-2} \beta^i(s, \lambda) \partial^i ((\partial^{s-2-i} A) B) = \sum_{i=0}^{s-2} (-)^{s-2-i} \beta^i(s, \frac{1}{2}-\lambda) (\partial^{s-2-i} A) (\partial^i B). \quad (\text{A.10})$$

Another set of coefficients $\tilde{a}^i(s, \lambda)$, $\tilde{\alpha}^i(s, \lambda)$ and $\tilde{\beta}^i(s, \lambda)$ are defined by the expansion

$$\sum_{i=0}^{s-1} a^i(s, \lambda) \partial^{s-1-i} ((\partial^i A) B) = \sum_{i=0}^{s-1} \tilde{a}^i(s, \lambda) (\partial^i A) (\partial^{s-1-i} B), \quad (\text{A.11})$$

$$\sum_{i=0}^{s-1} \alpha^i(s, \lambda) \partial^{s-1-i} ((\partial^i A) B) = \sum_{i=0}^{s-1} \tilde{\alpha}^i(s, \lambda) (\partial^i A) (\partial^{s-1-i} B), \quad (\text{A.12})$$

$$\sum_{i=0}^{s-2} \beta^i(s, \lambda) \partial^{s-2-i} ((\partial^i A) B) = \sum_{i=0}^{s-2} \tilde{\beta}^i(s, \lambda) (\partial^i A) (\partial^{s-2-i} B). \quad (\text{A.13})$$

Using again eq. (A.7) these coefficients can be evaluated,

$$\begin{aligned} \tilde{a}^i(s, \lambda) &= \binom{s-1}{i} \frac{(-)^i}{(s)_{s-1}} (2\lambda - s)_i (2 - 2\lambda - s)_{s-1-i}, \\ \tilde{\alpha}^i(s, \lambda) &= 2 \binom{s-1}{i} \frac{(-)^i}{(s)_{s-1}} (2\lambda - s + 1)_i (2 - 2\lambda - s)_{s-1-i}, \\ \tilde{\beta}^i(s, \lambda) &= \binom{s-2}{i} \frac{(-)^i}{(s)_{s-2}} (2\lambda - s + 1)_i (2 - 2\lambda - s)_{s-2-i}. \end{aligned} \quad (\text{A.14})$$

They satisfy the identities

$$\begin{aligned} \tilde{a}^i(s, \lambda) &= (-)^{s-1} \tilde{a}^{s-1-i}(s, 1 - \lambda), \\ \tilde{\alpha}^i(s, \lambda) &= (-)^{s-1} \tilde{\alpha}^{s-1-i}(s, \tfrac{1}{2} - \lambda), \\ \tilde{\beta}^i(s, \lambda) &= (-)^{s-2} \tilde{\beta}^{s-2-i}(s, \tfrac{1}{2} - \lambda). \end{aligned} \quad (\text{A.15})$$

Three other identities that involve coefficients for different values of λ , are

$$\tilde{\alpha}^{i+1}(s, \lambda) = \frac{i+1+2\lambda}{i+1} \frac{s-1-2\lambda}{s-1+\lambda} \tilde{\beta}^i(s, \lambda + \tfrac{1}{2}), \quad (\text{A.16})$$

$$-\tfrac{1}{2} \beta^i(s, \lambda) = a^i(s, \lambda + \tfrac{1}{2}) - a^i(s, \lambda), \quad (\text{A.17})$$

$$\beta^i(s+1, \lambda) = \frac{s-2\lambda}{2s-1} a^i(s, \lambda + \tfrac{1}{2}) - \frac{s-1+2\lambda}{2s-1} a^i(s, \lambda). \quad (\text{A.18})$$

From eq. (A.16) we derive

$$\tilde{\alpha}^{i+1}(s, 0) = \tilde{\beta}^i(s, \tfrac{1}{2}), \quad (\text{A.19})$$

an identity that we used in sect. 5. From eqs. (A.17)–(A.18) we find a relation for the coefficients $A^i(s, \lambda)$ that we used in the derivation of (4.19)

$$A^{2i+1}(s, \tfrac{1}{2} - \lambda) + A^{2i}(s, \tfrac{1}{2} - \lambda) = \left(\frac{2\lambda + [s] - 1}{2\lambda - [s] - 1} \right)^{|2s|_2} A^{2i}(s, 1 - \lambda). \quad (\text{A.20})$$

Finally we note two more identities, which follow directly from the definition (A.14)

$$\begin{aligned} \tilde{a}^{s-i}(s, 0) &= (-)^s \tilde{a}^i(s, 0), \\ \tilde{a}^{s-1-i}(s, \tfrac{1}{2}) &= (-)^{s-1} \tilde{a}^i(s, \tfrac{1}{2}). \end{aligned} \quad (\text{A.21})$$

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