2

Theoretical aspects of proton dosimetry

2.1 Proton interaction with matter

2.1.1 Introduction

For an understanding of the dose distribution produced by protons, a knowledge of their energy loss and scattering is needed. Protons traversing matter lose energy through successive collisions with the atoms and molecules of the material. With respect to energy loss, the most important interaction is between the proton and the atomic or molecular electrons. The interaction between a proton and an atomic nucleus effects the proton flux (nuclear reactions), and the proton trajectory ((in)elastic scattering).

The most important parameter characterizing the energy loss of an incident proton is the stopping power, which is mean energy loss per unit path length in a material. A full description of the proton energy loss process, however, requires more detailed information than is provided by the stopping power alone. The amount of energy transferred from a proton to an atomic electron, as well as the number of interactions that occur per unit path length has a probability distribution. This causes statistical fluctuations in the energy deposition. Moreover, there is a certain probability that very energetic electrons are produced (\(\delta\)-electrons or \(\delta\)-rays) which can travel a considerable distance before their energy is deposited.

The most important contribution to proton scattering comes from the electromagnetic interaction with the nucleus. This gives rise to small scattering angles, but since there are a large number of collisions, the effect can be considerable. If the impact parameter\(^1\) is small also the hadronic interaction contributes to elastic scattering. In addition inelastic interactions can occur: these can either be an inelastic scattering process during which the incident proton transfers energy to the nucleus (which will then be in an excited state and decay by \(\gamma\)-emission) or a nuclear reaction process (such as (p,n), (p,d), (p,2p) or (p,3p) ) where the incident proton will disappear. In case of scattering of protons by very light nuclei, such as protons in hydrogen, also the recoil nucleus can travel a considerable length before its energy is fully deposited.

\(^1\)distance of closest approach between proton and nucleus.
In the second part of this chapter the influence of the detection equipment on the measured shape of the dose distribution is discussed. Because radiation detectors have a finite size, and have in general a composition different from the medium one is interested in, they effect the outcome of the measurement. This is the subject of cavity-chamber theory. This is especially important for the comparison of experimental data with Monte Carlo calculations and for the determination of conversion factors between different detector types. Because protons can produce secondary particles ($\beta$-electrons, nuclear reaction products or recoil nuclei) as a result from energy loss processes there can be a difference between the local energy loss and the energy absorbed by the medium. This also has consequences for the measurement of dose distributions.

2.1.2 Proton interaction with electrons I: energy loss

Within the energy range of importance in proton therapy (from stopping protons to $\approx 250$ MeV) it is convenient to consider 2 energy intervals separately:

- Low energy: below $\approx 0.5$ MeV protons can pick up orbital electrons and form hydrogen. Also energy can be lost to atomic nuclei due to electromagnetic interactions (nuclear stopping power). These are complicated processes, but fortunately they only play a role at the very last microns of a proton track. It is important for the subject of microdosimetry, which deals with the energy loss process on a microscopic scale (for example: the study of the effect of ionizing radiation on DNA). For our purpose (the macroscopic dosimetry of proton beams), however, its relevance is not so large so that we will not discuss it further.

- High energy: for proton energies between $\approx 0.5$ MeV and 250 MeV the atoms in the stopping medium can be excited or ionized. The collision process is well understood and in principle the stopping power can be calculated theoretically.

The mean energy loss per proton $S$ can be described by the Bethe theory [18] which leads to the following expression:

$$\frac{1}{\rho} S = \frac{1}{\rho} \frac{dE}{dz} = K \frac{1}{A} \frac{Z}{\beta^2} L(\beta)$$

with:

$$K = \frac{2\pi r_e^2 mc^2 N_{av}}{\varepsilon_0} \approx 0.1535 \ \text{MeV cm}^2 \ \text{g}^{-1}$$

where $r_e = e^2/4\pi\varepsilon_0 mc^2$ is the classical electron radius, $\varepsilon_0$ the permittivity of the vacuum (which is introduced by the use of SI units), $mc^2$ is the electron rest mass energy, $N_{av}$ Avogadro’s number, $\beta$ is the particle velocity in units of the velocity of light ($\beta^2 = 1 - (\frac{m c^2}{E mc^2})^2$), $Mc^2$ is the proton rest mass $\approx 938.3$ MeV, $E$ the proton kinetic energy, $Z$ and $A$ are the atomic number and relative atomic mass of the target target.
atom. The quantity $L(\beta)$ takes into account the fine details of the energy loss process and is written as the sum of three terms:

$$L(\beta) = L_0(\beta) + L_1(\beta) + L_2(\beta)$$  \hspace{1cm} (2.3)$$

The first term is given by:

$$L_0(\beta) = \ln \left( \frac{2m^2 \beta^2 T_{\text{max}}}{1 - \beta^2} \right) - 2 \beta^2 - 2 \ln I - 2 \frac{C}{Z} - \delta$$  \hspace{1cm} (2.4)$$

where $I$ is the average excitation potential of the atoms of the medium, $C/Z$ the shell correction and $\delta$ the density-effect correction. The shell correction plays a role at low velocities and deals with effects due to the finite speed of the proton compared to the velocity of bound electrons. It can be as high as 10% but is usually implicitly taken into account by the choice of the excitation potential $I$. The density correction takes the polarization of the medium due to the passage of the projectile proton into account. It can be neglected for proton energies below 500 MeV ($\ll 0.1 \%$). $T_{\text{max}}$ is the largest possible energy loss in a single collision with a free electron, given by:

$$T_{\text{max}} = \frac{2m^2 \beta^2}{1 - \beta^2} \left[ 1 + \frac{2m}{M \sqrt{1 - \beta^2}} + \left( \frac{m}{M} \right)^2 \right]^{-1}$$  \hspace{1cm} (2.5)$$

where $c$ is the velocity of light, $m$ the electron mass and $M$ the proton mass. The error obtained by setting the factor between square brackets to 1 is smaller than 0.2% for proton energies below 250 MeV. The $L_1$-term in equation (2.3) is known as the Barkas correction and is responsible for the slightly different stopping power for positively and negatively charged particles. It does play a role at low energies, but is usually taken together with the shell correction and implicitly included into the average excitation potential $I$. The $L_2$-term is known as the Bloch correction and only important for relativistic energies. When one deals with a compound medium, which is usually the case in proton therapy, it is possible to use Bragg’s rule: the $S$ value for compounds can be found by averaging the $S$ over each atomic element in the compound weighted by the fraction of electrons belonging to each element. In this way we can define effective values for $Z$, $A$ and $I$.

The average excitation potential $I$ is the average of the excitation energies over all atomic states weighted by their transfer probability. These probabilities, which are called optical dipole oscillator strengths, are for most materials other than hydrogen unknown. It is therefore easier to determine values for the average excitation potential $I$ by fitting the stopping power formula to experimental range-energy data. In this way also simultaneously the shell correction $C/Z$ and Barkas correction $L_1$ can be determined. Unfortunately there is a still disagreement on which data to use. For instance, the data of H.H. Andersen et al. obtained in 1967 [6], differs by 2% from the ones obtained in 1981 [7]. Although 2% is a small number, it has the consequence that
the uncertainty in the dosimetry chain (see figure 1.7 in section 1.2) already begins with 2%. The ICRU has therefore decided to tabulate stopping powers for the most frequently used compounds and elements [67] in order to achieve at least consistency. For convenience a small summary is given in table 2.1, for the materials water, air (of interest for ionization chamber dosimetry) and gadolinium (which is the most important element in our scintillator screen system). In this table also the maximum energy loss in a single collision \( T_{\text{max}} \) is tabulated.

The range of the protons \( R \) in a medium can be determined, by integrating the stopping power from 0 to \( E \):

\[
R = \int_0^E - \left( \frac{dE}{dz} \right)^{-1} dE
\]  

(2.6)

This is called the continuous slowing down approximation (CSDA). In practice however not all protons that start with the same energy will have the same range. This is caused by the statistical fluctuation in the energy loss process, which is described in the next
It is possible to fit the relation between CSDA range (in g/cm$^2$) and initial energy (in MeV) by:

$$R = AE^p$$  \hspace{1cm} (2.7)

The factor $A$ is approximately proportional to the square root of the effective atomic mass of the medium, $\sqrt{A_{\text{eff}}}$. Relation (2.7) is known as the Bragg-Kleeman rule [25]. In figure 2.1 a comparison between the fit using the parameters from table 2.2 and the ICRU ranges (table 2.1) can be seen.

The largest difference between the fit and the ICRU ranges is 0.1 g/cm$^2$, that is in the order of magnitude of the uncertainty of the underlying theory.

**Table 2.2:** Parameters for the fit of relation (2.7) to the ICRU data (table 2.1). $R$ is expressed in g/cm$^2$, $E$ in MeV.

<table>
<thead>
<tr>
<th></th>
<th>water</th>
<th>air</th>
<th>Gadolinium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (g/cm$^2$ MeV$^{-p}$)</td>
<td>$2.56 \times 10^{-3}$</td>
<td>$2.73 \times 10^{-3}$</td>
<td>$5.96 \times 10^{-3}$</td>
</tr>
<tr>
<td>$p$ (dimensionless)</td>
<td>1.74</td>
<td>1.75</td>
<td>1.70</td>
</tr>
</tbody>
</table>
2.1.3 Proton interaction with electrons II: energy loss distribution

The amount of energy loss of a proton in a medium is subject to two sources of fluctuations. The number of proton-electron collisions can fluctuate, and at the same time the energy lost in each collision varies statistically. Both distributions are characterized by a Poisson-like behavior. In most of the proton-electron collisions only a small amount of energy is transferred from the proton to electron, due to the large ratio of proton to electron mass. There is however a small, but finite probability that a collision occurs where the energy transfer approaches $T_{\text{max}}$ and the atomic electron is dissociated from the atom. This extracted electron is called a δ-electron or δ-ray. Depending on the application it is possible to use a macroscopic, statistical description of the energy loss or a microscopic description, in which the δ-electrons are treated separately. With the statistical description the probability of occurrence of a certain energy loss $\epsilon$ in a medium layer with thickness $t$ as a function of mean energy loss $S$ and proton velocity $\beta$ can be calculated. With the microscopic description the δ-electron spectrum per incident proton can be calculated.

The macroscopic energy loss distribution function depends on the scale one is considering. In a thick layer a large number of collisions occur and the energy loss is expected to be distributed according to a Gaussian. For a small layer of material, however, the probability of a collision with an energy transfer close to $T_{\text{max}}$ occurs remains constant, but its relative contribution to the total energy loss becomes much larger. This means that larger fluctuations can occur for small layers. The parameter which describes the collision regime is called the skewness parameter $\kappa$, which relates the energy loss in a medium with thickness $t$ as a function of mean energy loss $S$ and proton velocity $\beta$ can be calculated:

$$\kappa = \frac{\xi}{T_{\text{max}}}$$ (2.8)

with

$$\xi = \frac{Z}{\beta} \frac{1}{\sqrt{\beta}} K \cdot t$$ (2.9)

The symbols are as in equations (2.1), (2.2). In case $\kappa < 1$, $\xi$ has the following physical meaning, as will be explained later using equation (2.12): when a proton passes through a medium with thickness $t$, there will occur, on the average, one collision with an atomic electron in which the proton loses an amount of energy greater than $\xi$. For $\kappa \ll 1$ the energy distribution is described by the Landau theory [74] where the distribution has a large tail towards high energy loss. In case $\kappa > 1$ ($\xi$ becomes larger than $T_{\text{max}}$ for thick media) the energy distributions approaches a Gaussian. Vavilov has treated the energy loss distribution in a more general way, which includes the Gaussian and Landau distribution as a limiting case [133]. The Vavilov energy distribution function $\phi_v$ is a function of $\beta$, $\kappa$ and the scaled energy loss $\lambda$:

$$\lambda = \frac{\epsilon - \bar{\epsilon}}{\xi} - \beta^2 - 1 + 0.577 \ldots - \ln \kappa$$ (2.10)
2.1 Proton interaction with matter

\[ \kappa = 10 \]
\[ \kappa = 5 \]
\[ \kappa = 1 \]
\[ \kappa = 0.1 \]

\( \lambda \)

\( \phi ( \lambda ) \)

Figure 2.2: The Vavilov distribution function \( \phi \) as a function of the scaled energy loss \( \lambda \) for 200 MeV protons (using algorithm of [117]).

where \( \epsilon \) is the actual energy loss, \( \bar{\epsilon} \) is the mean energy loss \((= \rho \cdot S \cdot t)\), 0.577... is Euler's constant and the other symbols are as previous. The actual expression for \( \phi_\kappa (\lambda, \beta, \kappa) \) is very complicated and is easiest evaluated numerically, for example using the algorithm of Schorr\(^2\) [117]. In the limit \( \kappa \to 0 \) the Vavilov theory approaches the Landau theory under the assumption that \( \xi \) remains much larger than the mean excitation energy \( I \). This is necessary in order to treat the atomic electrons as free particles.

In case of thick absorbers the energy loss distribution approaches a Gaussian with a width \( \sigma \) given by [18]:

\[ \sigma_E^2 = \xi T_{max} \left( 1 - \frac{\beta^2}{2} \right) \]  

(2.11)

\( \xi \) is given by equation (2.9), \( T_{max} \) is given by equation (2.5). The Gaussian approximation is valid if the proton energy can be assumed to be constant during the passage through the absorber. This does not hold anymore for very thick absorbers. In that case one has to divide the absorber into several smaller slabs and sum the \( \sigma_E^2 \) of the individual slabs.

In figure 2.2 a comparison of the Vavilov energy distribution function for different skewness values \( \kappa \) is shown. This calculation has been performed for 200 MeV protons \((\beta^2 = 0.32)\) but because of the small dependence on \( \beta \) it is also valid for lower energies. To use the distribution in different energy ranges one has to adapt the scaled energy loss \( \lambda \).

\( ^2 \) Also contained in the CERN program library, routine names: vviset and vviden (G116)
It can be seen that the Vavilov distribution transfers into the Landau distribution for small values of $\kappa$ while for large values the Gaussian distribution is approximated. For small $\kappa$-values there is a difference between the most probable energy and the mean because of the asymmetry towards the high $\lambda$ values.

It is interesting to apply the results of sections 2.1.2, 2.1.3 to our cell model (using table 1.2). The mean energy loss of a 200 MeV proton passing a cell is 4.5 keV, $\xi$ is 0.26 keV and the skewness value $\kappa$ of a cell is $5 \times 10^{-4}$, which is clearly in the Landau region. The result is that the most probable energy loss ($\lambda = -0.22$) is 2.7 keV but 5% of the protons deposits 8.4 keV or more.

In the case the energy transfer from the proton to the electron $T$ is much larger than the mean excitation potential $I$, it is also possible to use a microscopic description and explicitly consider $\delta$-electrons. The number of $\delta$-electrons $N$ as a function of electron energy $T$ per unit thickness can be calculated using the Bhabha cross section [19] which can be written as the product of the classical Rutherford cross section and a quantummechanical correction for spin-$\frac{1}{2}$ particles (factor between square brackets):

$$
\frac{d^2N}{dTdz} = K \cdot \frac{\rho Z}{A} \cdot \frac{1}{\beta^2} \cdot \frac{1}{T^2} \left[ 1 - \beta^2 \cdot \frac{T}{T_{\text{max}}} + \frac{T^2}{2(E + Mc^2)^2} \right] \quad (2.12)
$$

where the symbols are the same as in equations (2.1), (2.2). The difference between the classical Rutherford cross section and equation (2.12) increases for increasing proton energy and energy transfer, up to 40% for 250 MeV protons and an energy transfer of $T_{\text{max}}$. The meaning of $\xi$ can also be illustrated using the integral of equation (2.12) without the quantummechanical correction factor: this is the probability that one $\delta$-electron with an energy $\xi$ is produced:

$$
N(\xi, t) = K \cdot \frac{\rho Z}{A} \cdot \frac{1}{\beta^2} \frac{t}{\xi} = 1 \quad (2.13)
$$

In figure 2.3 the $\delta$-electron spectrum per incident proton according to equation 2.12 can be seen for a number of proton energies.

For a number of topics such as the biophysical effect of radiation (section 1.1.2), ionization chamber conversion factors (section 2.2) or scintillator efficiencies (section 4.1.2) it is useful to establish a threshold energy $T_{\text{cut}}$ below which the energy loss is assumed to be local and above which $\delta$-electrons are produced. The component of the stopping power which originates from energy transfers smaller than $T_{\text{cut}}$ is usually called the restricted stopping power. It can be calculated using the restricted Bethe-Bloch equation which has only contributions from energy transfers below $T_{\text{cut}}$. It can be written in the form of equation (2.1) but with a different $L_0(\beta)$-factor [67]:

$$
L_0(\beta) = \ln \left( \frac{2mc^2\beta^2T_{\text{cut}}}{1 - \beta^2} \right) - \beta^2 (1 + \frac{T_{\text{cut}}}{T_{\text{max}}}) - 2 \ln I - \frac{C}{Z} - \delta \quad (2.14)
$$
2.1 Proton interaction with matter

The integral from $T_{\text{cut}}$ to the kinematical limit $T_{\text{max}}$ of the product of the number of $\delta$-electrons (equation (2.12)) and the $\delta$-electron energy $T$ corresponds with the contribution of $\delta$-electrons to the total stopping power. The choice of $T_{\text{cut}}$ depends on the scale of the problem, but a usual choice is 10 keV (10 keV electrons have a CSDA range of 2.5 $\mu$m in water and 2.4 mm in air). This implies that for 200 MeV protons the contribution of $\delta$-electrons is 22% of the total stopping power, for 100 MeV 20% and for 50 MeV 17%. The energy loss fluctuation are caused by fluctuation in the number of $\delta$-electrons and by fluctuations in the $\delta$-electron energy. The energy loss fluctuation below $T_{\text{cut}}$ can be described by the Vavilov/Landau theory, although also in that case the condition $T' \gg 1$ has to be fulfilled.

The energy straggling caused by the energy distribution described in this section causes also a distribution in the proton range, which is called range straggling. The range is in first approximation also distributed according to a Gaussian with a width $\sigma_R$. The mean square fluctuation in the range $\sigma_R^2$ at a depth $R$ depends in the following way on the mean square fluctuation in the energy $\sigma_E^2$ [18]:

$$\sigma_R^2(R) = \int_0^R dz \frac{\sigma_E^2}{dz} \left( \frac{dE}{dz} \right)^{-2} dz$$

(2.15)

For $\sigma_R^2$, we can use equation (2.11) which yields a complex integral. In [18] it is shown that the error we make by using the Bohr approximation: $\sigma_E^2 = \xi T_{\text{max}}$ ($\xi$ is given by equation (2.9), $T_{\text{max}}$ is given by equation (2.5)) is small for proton energies
above 10 MeV. Together with the empirical range-energy relation (2.7) this results in a simple expression for $\sigma_R^2$:

$$\sigma_R^2 = K \cdot 2mc^2 \cdot \frac{Z}{A} \left( \frac{p^2 A^{2/p}}{3 - 2/p} \right) R^{1 - 2/p}$$

(2.16)

where the symbols are the same as in equations (2.1), (2.2), (2.7).

### 2.1.4 Proton interaction with nuclei I: scattering

A proton will experience a deflection as it passes in the neighborhood of a nucleus. This deflection is the result of the combined interaction with the Coulomb- and hadronic field of the nucleus (the deflection caused by collisions with electrons can be neglected because of the mass ratio). The Coulomb interaction has been derived by Rutherford [18]:

$$d\sigma(\theta) = \left( \frac{e^2}{16\pi\varepsilon_0} \right)^2 \frac{Z^2}{E^2} \frac{1}{\sin^2 \frac{\theta}{2}} 2\pi \sin \theta \cdot d\theta$$

(2.17)

where $E$ is the proton kinetic energy (in MeV) and $d\sigma(\theta)$ the differential cross section in mb ($\equiv 10^{-31}$ m$^2$), the other symbols as in equations (2.1), (2.2). This cross section decreases very rapidly with increasing angle, and with increasing energy. The consequence is that most particles are only slightly deflected. The hadronic interaction will only play a role when the distance between the proton and the nucleus becomes very small ($\approx$ diameter of the nucleus $\approx 1.3 \times 10^{-13} A^{1/3}$ m). Although the mechanism of the hadronic interaction is very complicated, it manifests itself experimentally with deviations from the Rutherford scattering cross section, which can be parametrized using optical models [105]. In figure 2.4 the ratio between the Rutherford cross section and observed elastic cross section (so including the hadronic interaction) can be seen as a function of angle in case of 180 MeV protons on $^{16}$O. The elastic cross section has been calculated using the DWUCK$^3$ code, with parameters for oxygen from [105].

In addition to this elastic hadronic interaction, also inelastic interactions occur, which are described in the next section.

Only for large scattering ($> 4^\circ$) angles the difference becomes significant. The probability that a scattering occurs with an angle more than $4^\circ$ in 1 cm H$_2$O is less than 0.05 % (this can be calculated using equation (2.17)). So in practice only the small angle deflections, which are caused by distant collisions, contribute.

However because of the large number of interactions the total effect is considerable. A multiple scattering theory has been derived by Molière [91] and later on improved by Bethe [18] which is valid for scattering angles $\theta \lesssim 30^\circ$. Analog to the energy distribution function it is possible to derive macroscopically an angular deflection distribution function. According to Molière this distribution function can be expressed as

---

$^3$DWUCK4 version 10/10/81 by P.D. Kunz, J. Comfort and M.N. Harakeh
2.1 Proton interaction with matter

Figure 2.4: Solid line (scale on right axis): ratio between total elastic cross section of 180 MeV protons incident on $^{16}$O (using DWUCKA) and Coulomb contribution (using equation (2.17)) as a function of deflection angle. Dotted line: ratio = 1. Dashed line (scale on left axis): Coulomb contribution.

a series expansion which involves complicated functions. The limiting case for many collisions again is a Gaussian distribution:

$$f(\theta)d\theta = \frac{1}{\sqrt{2\pi}\theta_0^2} \exp\left[-\left(\frac{\theta_{\text{plane}}}{2\theta_0}\right)^2\right] d\theta$$ \hspace{1cm} (2.18)

For the calculation of the width $\theta_0$ (the mean squared scattering angle projected on a plane $\theta_{\text{plane}}^2$, which is $\frac{1}{2}\theta_{\text{space}}^2$) of the Gaussian several approximations exist. In [52] is shown experimentally that the Highland formula gives the best results for protons:

$$\theta_0 \approx \frac{14.1 \text{ MeV}}{\beta^2(E + Mc^2)} \sqrt{\frac{t}{L_R}} \left[1 + \frac{1}{9} \log_{10}\left(\frac{t}{L_R}\right)\right] \text{ rad}$$ \hspace{1cm} (2.19)

where $t$ is the thickness of the medium, $E$ the proton kinetic energy, $Mc^2$ the proton rest mass (expressed in MeV) and $L_R$ the radiation length of the material, which is the distance over which the electron energy is reduced to a factor 1/e due to bremsstrahlung only (this is tabulated e.g. in [77], for water $L_R = 36.1$ g/cm$^2$). Although the radiation length is a material property derived for electrons, this formula turns out to fit the experimental data with protons well [52]. For small angles we can approximate $\theta_{\text{space}}^2 \approx (\theta_{\text{plane}}^2 + \theta_{\text{plane}}^2)$, The deflection in $x$ and $y$ - direction are independent and identical distributed. The Highland formula works under the assumption that the proton kinetic energy remains constant during the passage, which means that the thickness...
The pathlength travelled by the proton will increase due to the multiple scattering process. This means that the average penetration depth $Z$ of the protons will be shorter than their CSDA range $R$. The difference between the lengths can be calculated using [18]:

$$\langle R - Z \rangle = \int_0^{E_0} \frac{E}{(1 - \cos \theta)} \left( \frac{dE}{dz} \right)^{-1} dE$$  \hspace{1cm} (2.20)

where $\langle R - Z \rangle$ is the mean difference and $\langle 1 - \cos(\theta) \rangle$ the mean deflection angle. In [67] they are tabulated for different materials: it turns out that for protons in water the effect is negligible above 1 MeV.

### 2.1.5 Proton interaction with nuclei II: nuclear reactions

Because the energy of the proton in radiotherapy applications is much higher than the Coulomb-barrier, protons will have a probability of reacting with the nucleus other than by elastic or inelastic scattering. This causes a decrease of the proton flux with depth, already long before the end of the proton range. When we assume the CSDA...
Figure 2.6: Proton flux reduction due to inelastic nuclear reactions for a 80 and a 180 MeV beam in a water medium calculated using equation (2.21) and the cross sections in figure 2.5.

approximation (see equation (2.6)), this can be described by:

$$
\Phi = \Phi_0 \exp \left( -\frac{\rho N_{av}}{A_{eff}} \int_E^{E_0} \sigma_{inelas}(E') \frac{dE'}{S(E')} \right) \tag{2.21}
$$

where $\Phi$ is the flux of protons with energy $E$, $\Phi_0$ the initial flux, $E_0$ the start energy, $S(E)$ the stopping power and $\sigma_{inelas}(E)$ the inelastic nuclear reaction cross section. For radiotherapy applications (where water is the reference material) we are mainly interested in the cross section with oxygen because the reaction cross section with hydrogen is negligible [71]. The $^{16}$O cross section has been measured by Carlsson et al. [28] and by Renberg et al. [109]. Figure 2.5 shows the experimental points together with a fit that has been made by Seltzer [16] using the GNASH-code [141].

With respect to the reaction products the situation is more complicated. The secondary particles can be neutrons, protons and recoil fragments. The energy transferred to the recoil fragments will be deposited locally, but secondary protons can travel a considerable distance before stopping. The secondary neutrons will either escape from the medium or produce another nuclear reaction, in which tertiary particles can be produced. In general it is not possible to make an analytical calculation of the contribution of nuclear reactions to the energy deposition as a function of depth. Berger [16] uses the estimate that 60 % of the initial proton energy (i.e. before the reaction) is deposited locally while the other 40 % escapes from the medium in the form of neutrons and $\gamma$-s. Monte Carlo calculations, however, in which the secondary particles are separately followed are not in agreement with this and show that a depth dependent contribution is
deposited (see section 3.4). The result for the flux reduction of the primary protons using equation (2.21) can be seen in figure 2.6. The steepness of flux reduction is slightly larger for the 80 MeV proton beam, due to the slightly higher cross section.

### 2.1.6 Proton dose distributions

When we summarize the results of the previous sections, it is possible to calculate the proton dose (=energy lost to the medium) as a function of position. We start with the dose dependence in the direction parallel to the beam. We define the energy fluence \( \Psi \) at depth \( z \) by:

\[
\Psi(z) = \Phi(z)E(z)
\]

where \( \Phi \) is the proton flux, and \( E \) the proton kinetic energy as a function of depth \( z \). The energy loss by the protons as a function of depth is then given by:

\[
D(z) = -\frac{1}{\rho} \frac{d\Psi(z)}{dz} = -\frac{1}{\rho} \left( \Phi(z) \frac{dE(z)}{dz} + C \frac{d\Phi(z)}{dz}E(z) \right)
\]

The first term describes the energy lost to the atomic electrons (see sections 2.1.2, 2.1.3). In first approximation we assume that this energy is deposited locally (this applies when the medium is water, for air this is not the case, see section 2.2). The second term describes the energy lost by the flux reduction due to nuclear interactions. Here the assumption of local deposition is no longer valid. The best thing we can do analytically is to use the approximation that a fraction \( C \) of 0.6 is deposited locally, while the remaining energy escapes in the form of neutrons and \( \gamma \)'s (see however sections 2.1.5, 3.4). Using these assumptions, the energy loss \( D(z) \) in equation (2.23) is equal to the deposited dose.

Using the empirical relation between \( E \) and \( R \) given in relation (2.7), it’s possible to simplify the calculation considerably.

The energy \( E(z) \) at a depth \( z \) is determined by the residual range \( R_0 - z \) which the protons traverse before stopping:

\[
E(z) = \frac{1}{A^{1/p}}(R_0 - z)^{1/p}
\]

From this we can determine \( dE/dz \):

\[
\frac{dE}{dz} = -\frac{1}{pA^{1/p}}(R_0 - z)^{1/p-1}
\]

With respect to the flux reduction due to nuclear reactions, we can use equation (2.21), but in [76] it is shown that the error resulting from ignoring the energy dependence of the cross section \( \sigma_{inelas} \) and using a straight line fit:

\[
\Phi(z) = \Phi_0 \frac{1 + B(R_0 - z)}{1 + BR_0}
\]
is small compared to the uncertainty on the position of energy deposition of the secondary particles (see also figure 2.6). Therefore:

\[
\frac{d\Phi}{dz} = -\Phi_0 \frac{B}{(1 + BR_0)}
\]

(2.27)

where \( B \) is a medium dependent parameter and \( \Phi_0 \) the initial flux.

In order to incorporate the range straggling caused by the energy straggling in the medium and the initial energy spread of the proton beam, the relevant distribution functions have to be folded into equation (2.23). Bortfeld has shown [23] that it is possible to do so, with a limited number of simplifications to the proton energy spectra:

for the range straggling he uses the Bohr approximation (2.16) and for the initial beam spread the sum of a Gaussian and a small ‘tail’ extended towards low energies. The result is a rather complicated expression:

\[
D(z) = \Phi_0 \frac{e^{-\zeta^2/\sigma^2/\Gamma(1/\rho)}}{\sqrt{2\pi\rho\sigma^2\Gamma(1/\rho)}} \times \left[ \frac{1}{\sigma} D_{-1/\rho}(-\zeta) + \left( \frac{B}{\rho} + C + \frac{\varepsilon}{R_0} \right) D_{-1/\rho-1}(-\zeta) \right]
\]

(2.28)

where \( D \) is the parabolic cylinder function, \( \Gamma \) is the gamma function (both tabulated in [3]) and \( \zeta = (R_0 - z)/\sigma \) where \( \sigma \) is the total width of the range straggling. The parameter \( \varepsilon \) describes the low energy tail of the initial proton spectrum. This tail contains a small fraction of the initial flux, but is influenced by the way a proton beam is produced. In the calculations \( \varepsilon \) has been set to zero. The total width consists of a term \( \sigma_{\text{mono}}^2 \) caused by the energy straggling in the medium and a term \( \sigma_{\text{Eo}}^2 \) caused by the initial beam spread. The total \( \sigma \) can be calculated with:

\[
\sigma^2 = \sigma_{\text{mono}}^2 + \sigma_{\text{Eo}}^2 \left( \frac{dR_0}{dE_0} \right)^2 = \sigma_{\text{mono}}^2 + \sigma_{\text{Eo}}^2 A^2 p^2 E_0^{(2p-2)}
\]

(2.29)

\( \sigma_{\text{mono}} \) is the range straggling that is present for a monoenergetic beam and can be calculating using equation (2.16). The other, medium dependent parameters are determined by equations (2.7), (2.23), (2.27). A summary of the parameters is given in table 2.3 together with the values in case the medium is water.

Although expression (2.28) looks complex, it is easy to evaluate numerically. In figure 2.7 we see dependence of the height of the Bragg peak as a function of initial energy spread \( \sigma_{\text{Eo}} \) both for a 80 and a 175 MeV proton beam. Also the position of the Bragg peak changes. The definition for the term ‘Bragg peak’ subsequently used in this thesis is the position where the maximal dose occurs. In figure 2.7 and subsequent figures we plot the dose per flux instead of the dose, which means that to obtain a dose this value has to be multiplied with the number of protons per cm².

The range \( R_0 \) for 180 MeV protons is 21.64 cm. It can be seen in figure 2.7 that this equals the depth at which the dose has dropped to 80 \pm 1 \% of its maximum value.
Table 2.3: The parameters which determine equation (2.28) together with their numerical values in case of a water medium. The last two parameters are beam dependent.

<table>
<thead>
<tr>
<th>description</th>
<th>value for water</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$ exponent of range-energy relation</td>
<td>1.74</td>
<td>1</td>
</tr>
<tr>
<td>$A$ proprotionality factor of $R_0$</td>
<td>2.56×10^{-3}</td>
<td>cm MeV^{-p}</td>
</tr>
<tr>
<td>$R_0$ range</td>
<td>$AE_0^p$</td>
<td>cm</td>
</tr>
<tr>
<td>$B$ slope parameter of flux reduction</td>
<td>0.012</td>
<td>cm^{-1}</td>
</tr>
<tr>
<td>$C$ fraction of energy released locally</td>
<td>0.6</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_{\text{mono}}$ width of range straggling</td>
<td>0.012 $R_0^{0.935}$</td>
<td>cm</td>
</tr>
<tr>
<td>$\sigma_{E_0}$ initial beam width $\rightarrow$ beam dependent</td>
<td>$\approx 0.01E_0$</td>
<td>MeV</td>
</tr>
<tr>
<td>$\varepsilon$ contribution to tail $\rightarrow$ beam dependent</td>
<td>$\approx 0.0-0.2$</td>
<td>1</td>
</tr>
</tbody>
</table>

beyond the Bragg peak, independent of the initial energy spread. This is known as the $z_{80} = R_0$ empirical rule of thumb [16].

With respect to the dose distribution in the lateral direction we can make use of the Gaussian approximation of the multiple scattering theory as is described in section 2.1.4. The total width of the proton beam is given by the quadratic sum of the width caused by the initial divergence of the proton beam and the (depth dependent) broadening caused by multiple scattering. The result for a parallel beam (with no initial divergence) can be seen in figure 2.8. The multiple scattering increases in the Bragg peak, due to the $1/E$ dependence of the scattering angle. For two or more dimensional problems it is in general not possible to use analytical tools. To calculate the distribution in figure 2.8 we have therefore used a numerical method: the Monte Carlo code PTRAN [16]. In chapter 3 we present more details on the this code, together with comparisons between experimental data, equation (2.28) and other Monte Carlo codes.
2.1 Proton interaction with matter

Figure 2.7: The dose per fluence as a function of depth in water calculated with (2.28) for 80 and 180 MeV proton beams, with an initial energy spread $\sigma$ increasing from 0% to 1.5% in steps of 0.25%. Entrance dose per fluence is for a 180 MeV beam 5.78 MeV cm$^{-2}$/g and for a 80 MeV beam 9.32 MeV cm$^{-2}$/g. $R_0$ is shown as the dashed line.

Figure 2.8: Contour plot of dose of a 160 MeV 2 cm radius parallel proton beam in water. The contour lines go from 1% to 99% of the maximum dose. Calculated with PTRAN [16]. At zero depth and on the beam axis the dose is 21% of the dose in the Bragg peak.
2.2 Proton dose measurement

2.2.1 Introduction

A measurement of the absorbed dose distribution necessitates the introduction of a radiation-sensitive device into the medium. Normally, this device will differ from the medium in both atomic number and density and it therefore constitutes a discontinuity, which will be referred to as a cavity, see figure 2.9.

In this section we will examine the influence of the detector properties on the result of the dose measurements. We start with the most simple situation where the detector has the same material properties as the medium, we extend this to other material properties (Bragg-Gray theory) and to non-local energy deposition (Spencer-Attix theory). Finally we examine the relation between the energy absorbed in the medium and the dosimeter signal in case the dosimeter is an ionization chamber. The case where the dosimeter is a scintillator will be discussed in sections 3.5.4, 4.1.2.

2.2.2 Influence of detector geometry

In case the detector material is equal to the medium material, we can assume that the influence of boundaries can be neglected. Since the detector is thin, we neglect the change in proton flux caused by nuclear interactions. The stopping power $S(E)$ is energy dependent, therefore we have to integrate over the complete energy spectrum. The dose at a detector position $z$ is then given by:

$$D_{cav}(z) = \frac{1}{\rho} \int_0^{E_0} \Phi(E, z) \cdot S(E) \cdot dE$$

(2.30)

where is $D(z)$ the dose, $\Phi(E, z)$ the proton flux at position $z$ and $S(E)$ the proton stopping power. In first approximation we assume that the flux $\Phi(E, z)$ does not change under influence of the detector and that the energy lost by the proton is deposited locally. In reality the detector is not infinitesimal thin, which means that the

![Figure 2.9: A schematic overview of a cavity in a medium which has the purpose of measuring dose distributions.](image)
2.2 Proton dose measurement

The influence of detector thickness for a water equivalent detector on the measurement of the Bragg peak in water for a monoenergetic 80 and 180 MeV proton beam is illustrated in Figure 2.10. Infinitesimal thin and \( t = 0.1 \) mm detector thicknesses are almost indistinguishable both for 80 and 180 MeV. The dose distribution for an infinitesimal thin detector has been calculated using the analytical model (2.28).

The stopping power \( S(E) = \frac{dE}{dx} \) has to be replaced by \( \frac{\Delta}{t} \) where \( \Delta \) is the mean energy loss in the detector with thickness \( t \):

\[
D_{\text{cav}}(z) = \frac{1}{\rho} \int_0^{E_0} \Phi(E, z) \frac{\Delta}{t} dE
\]  

(2.31)

This averaging can cause major distortions in the observed Bragg peak, especially for low energy (\( \leq 80 \) MeV, mainly used for ocular melanoma) beams where the detector \( \rho \cdot t \) is not small anymore compared to the width of the Bragg peak. The observed Bragg peak will be shifted to the left and broadened compared to the actual dose distribution given by equation (2.30). This is illustrated in figure 2.10 for a 80 and a 180 MeV proton beam (calculated using the analytical model (2.28)). It is clear that the smaller the cavity size the less distortions in the measured dose distribution occur.

2.2.3 Bragg-Gray theory

The main interest, however, is not the dose in the cavity, but rather the dose in the medium as it would be in absence of the detector. Therefore one has to account for differences in material property between the medium and detector material. In first approximation we can multiply equation (2.31) with the spectrum averaged proton stopping power ratios [18]:

\[
D_{\text{med}}(z) = D_{\text{cav}}(z) \frac{\rho_{\text{cav}}}{\rho_{\text{med}}} \frac{1}{E_0} \int_0^{E_0} S_{\text{med}}(E) \Phi(E, z) dE
\]  

\[
\frac{1}{E_0} \int_0^{E_0} S_{\text{cav}}(E) \Phi(E, z) dE
\]  

(2.32)
This relation is known as the Bragg-Gray relation, and was originally derived for X-rays by Gray [53]. X-rays are indirectly ionizing, which means that the energy is deposited by the electrons produced by the photo-electric effect, Compton scattering or pair-creation. These electrons are called primary electrons. Therefore in the Bragg-Gray relation for X-rays the stopping power ratio for the primary electrons has to be used. The fundamental assumptions of the Bragg-Gray cavity theory are:

1. the cavity is so small that the flux $\Phi(E, x)$ is unchanged.
2. the energy lost is deposited locally.
3. the energy loss is continuously. For this an electron equilibrium is necessary, which means that the same number of electrons will enter and exit the cavity (from the medium or by backscattering).

Assumption 2 clearly is not valid when energetic, $\delta$ electrons are produced in a small cavity. For X-rays the Bragg-Gray equation (2.32) has been extended by Spencer and Attix, which is described in the next section. For protons the situation is less clear. Although protons do have direct (local) energy loss, they can also produce energetic $\delta$-electrons which are not absorbed locally (see section 2.1.3). We have made studies of this effect using Monte Carlo methods together with experimental data. This will be presented in section 3.5.

The Bragg-Gray relation (2.32) alone shows already a large influence of the cavity material on the shape of the observed Bragg peak. Because the mean excitation potential is present in the logarithmic term of equation (2.4), the ratio of stopping powers is energy dependent (see table 2.1). The result for 80 and 180 MeV proton beams when the detector material is air, silicium, carbon or gadolinium can be seen in figure 2.11 (using proton flux spectra $\Phi(E, z)$ that have been calculated with the PTRAN code, see section 3.2).

It can be seen that the difference in distortion between a 80 and a 180 MeV beam is smaller than the difference in distortion caused by the finite detector size (cf. figure 2.10). It can also be seen that the error made by neglecting differences in material in case of air or carbon detectors is very small, 1 %.

### 2.2.4 Spencer-Attix theory

For X-rays deviations between equation (2.32) and experimental data (for example using water calorimetry, see next section) have been found. Spencer and Attix have derived a theory which accounts for these deviations by taking the creation of secondary $\delta$-electrons into account [121]. For X-rays this secondary electrons are created by the primary electrons (which themselves have an energy comparable to the incident X-rays). The creation of secondary electrons in case the primary particle is a proton has been treated in section 2.1.3. In the Spencer-Attix theory, the secondary electrons
Figure 2.11: Depth-dose curves for different detector materials. Solid lines represent curves for water, air and carbon which are indistinguishable both for 80 and 180 MeV. Dash-dotted line: gadolinium and dashed line: silicium. The entrance dose of the air, carbon, silicium and gadolinium is normalized to the entrance dose of water.

are separated into two groups, divided at a cut-off energy $\Delta$ that corresponds approximately to the energy of an electron that can just cross the cavity. A typical air gap in an ionization chamber is in the order of 2 mm $\approx 0.24$ mg/cm$^2$, which is slightly below the range of 10 keV electrons. A typical choice of $\Delta$ is therefore 10 keV. A ‘slow’ secondary electron, with an energy less than $\Delta$, is considered to dissipate its energy at the point at which it is generated. The ‘fast’ secondary electrons (the $\delta$-electrons in section 2.1.3), with an energy greater than $\Delta$, are also considered as primary particles that have a continuous energy loss, and their energy is not considered to have been dissipated until they have dropped below $\Delta$. Since the fast secondary electrons are accounted for separately, their energy should not be included in the primary stopping power, which is now restricted to energy losses less than $\Delta$. It is assumed that slow electrons released into the cavity from the chamber wall are in equilibrium with other slow electrons released from the gas that impinge upon the chamber wall, so that there is no net energy transfer. The energy interval just above $\Delta$ deserves special attention, because for that interval the particles can originate from outside the cavity, but stop inside. This contribution is called the track-end term $TE$ [2,9,65,84]. When we apply this to protons, relation (2.32) has to be split into a proton and a $\delta$-electron term:

$$D_{med}(z) = D_{cav}(z) \cdot \frac{\rho_{cav}}{\rho_{med}} \left[ \int_{\Delta}^{E_0} \Phi_p(E) S^{\delta}_{med}(E) \Phi_p(E, z) dE + T E_{med}^{p}(z) + \cdots \right.$$

$$+ \left. \cdot \cdot \cdot + \int_{\Delta}^{E_{max}} \Phi_p(E) S^{\delta}_{cav}(E) \Phi_p(E, z) dE + T E_{cav}^{p}(z) + \cdots \right]$$

(2.33)
Table 2.4: Electron stopping powers and ranges according to ICRU 37 [66]. $\rho_{H_2O}=1.00$ g/cm$^3$, $\rho_{air}=1.20$ mg/cm$^3$ (weight fractions: 75.5% N, 23.2% O, 1.28% Ar, 0.01% C) $I_{H_2O}=75.0$ eV $I_{air}=85.7$ eV

<table>
<thead>
<tr>
<th>$E$ (keV)</th>
<th>$\frac{dS}{dE}$ (MeV cm$^2$/g)</th>
<th>$#_{air}$</th>
<th>$R$ (mg/ cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_2O$</td>
<td>air</td>
<td>$H_2O/air$</td>
</tr>
<tr>
<td>10</td>
<td>22.56</td>
<td>19.76</td>
<td>1.142</td>
</tr>
<tr>
<td>20</td>
<td>13.18</td>
<td>11.58</td>
<td>1.138</td>
</tr>
<tr>
<td>30</td>
<td>9.657</td>
<td>8.495</td>
<td>1.137</td>
</tr>
<tr>
<td>40</td>
<td>7.781</td>
<td>6.852</td>
<td>1.136</td>
</tr>
<tr>
<td>50</td>
<td>6.607</td>
<td>5.822</td>
<td>1.135</td>
</tr>
<tr>
<td>100</td>
<td>4.119</td>
<td>3.637</td>
<td>1.133</td>
</tr>
<tr>
<td>150</td>
<td>3.242</td>
<td>2.865</td>
<td>1.132</td>
</tr>
<tr>
<td>200</td>
<td>2.798</td>
<td>2.474</td>
<td>1.131</td>
</tr>
<tr>
<td>250</td>
<td>2.533</td>
<td>2.241</td>
<td>1.130</td>
</tr>
<tr>
<td>300</td>
<td>2.360</td>
<td>2.089</td>
<td>1.130</td>
</tr>
<tr>
<td>400</td>
<td>2.154</td>
<td>1.908</td>
<td>1.129</td>
</tr>
<tr>
<td>500</td>
<td>2.041</td>
<td>1.809</td>
<td>1.128</td>
</tr>
<tr>
<td>600</td>
<td>1.972</td>
<td>1.751</td>
<td>1.126</td>
</tr>
<tr>
<td>700</td>
<td>1.926</td>
<td>1.715</td>
<td>1.123</td>
</tr>
</tbody>
</table>

where $\zeta^p$ is the restricted proton stopping power (which can be calculated using equation (2.14)). $S^\delta$ is the electron stopping power: in principle the restricted stopping power with energy losses below 10 keV should be used for this, but since the fraction of tertiary $\delta$-electrons generated by electrons below $\approx 0.5$ MeV is very small, the error caused by taking the ordinary, unrestricted stopping powers is very small [85]. To calculate the electron stopping powers we can use a theory similar to the one used in section 2.1.2. Such a theory has been derived by Bethe [18] and later on improved by others [66]. It is outside the scope of this thesis to present this theory, but a summary of its results (from [66]) can be found in table 2.4.

From a comparison between the ratio of water/air stopping powers for electrons in the energy range 10 keV - $T_{max}$ (in table 2.4) and the ratio of water/air stopping powers for protons in the energy range 10 - 250 MeV (in table 2.1) it can be seen that both are very similar: around 1.13. This means that the correction obtained by splitting the continuous energy loss term into proton and $\delta$-electron contributions is small.

In case electrons are the primary particles the track end term may be approximated by [95]:

$$TE^\delta(z) = \Phi_\delta(\Delta, z) S^\delta(\Delta) \Delta$$  (2.34)

For protons the situation is more complicated, because there the contribution from low energy $\delta$-electrons is much higher than in case of primary electrons. We have to account both for the $\delta$-electrons that escape from the cavity but also for the $\delta$-electrons
2.2 Proton dose measurement

that enter through the chamber wall (from the medium or backscattered). In [75] a simplified calculation for a gas chamber with thin walls is presented, resulting in a net loss of 2.3% of the total energy deposited in the cavity due to \(\delta\)-electron escape. When the thickness of the walls is increased (as is the situation for a chamber in a medium) this loss decreases to 0.8%. In addition to this effect, where the wall material was assumed to be equal to the medium material, differences occur when the wall consists of a different material (for example carbon, or PMMA). This also affects the track end terms. Although some preliminary studies have been performed [85, 101], it remains unclear how to calculate these wall correction factors for protons. In the AAPM\(^4\) dosimetry protocol from 1986 [1] the \(\delta\)-electrons have been pointed out to be a major difficulty and in 10 years the situation has not become clearer. Also Monte Carlo calculations are very difficult because of the small energies and distances involved. In section 3.5 we present some results obtained with GEANT.

2.2.5 Ionometry

For the actual measurement of the dose that is deposited in the cavity, one can exploit the experimental fact that the average energy that is needed to create an ion pair is nearly independent of the proton energy. It is also nearly the same for \(\alpha\)-particles, protons and electrons. Moreover, the energy is not very different for different stopping gases and, contrary to naive expectation, is smallest for the inert gases whose ionization potentials are the highest of all atoms. These facts have been explained in a semi-quantitative theory by Fano [18, 39]. Instead of the energy of just the proton, Fano considers the total energy available for ionization, whether it resides in the proton or in a \(\delta\)-electron. In a collision in which the atom is excited (cross section \(\sigma_e\)) by either a proton or a \(\delta\)-electron, the available energy is reduced by the excitation energy \(W_e\). If the atom is ionized and the kinetic energy of the ejected electron is less than the ionization potential \(I_{\text{ion}}\) (cross section \(\sigma_{I_{\text{ion}}1}\)), the total energy given to the atomic electron \(W_{I_{\text{ion}}1}\) is lost from the available energy. However, if an electron of kinetic energy greater than \(I_{\text{ion}}\) is produced (cross section \(\sigma_{I_{\text{ion}}2}\)), its kinetic energy is available for further ionization, and the loss of available energy is considered to be only the ionization potential \(I_{\text{ion}}\). The average amount of (available) energy spent per ion is then:

\[
\bar{w} = \frac{\sigma_e W_e + \sigma_{I_{\text{ion}}1} W_{I_{\text{ion}}1} + \sigma_{I_{\text{ion}}2} I_{\text{ion}}}{\sigma_{I_{\text{ion}}1} + \sigma_{I_{\text{ion}}2}}
\]

Now the ratio of the various cross sections, especially of \(\sigma_e\) and \(\sigma_{I_{\text{ion}}1}\), does not change much with the proton energy. This does also hold for the average energies \(W_e\) and \(W_{I_{\text{ion}}1}\). Since \(\sigma_{I_{\text{ion}}2} \ll \sigma_{I_{\text{ion}}1}\), this explains in a qualitative way the independence of \(\bar{w}\) from proton energy.

The independency of \(\bar{w}\) with proton energy does not hold exactly, especially for gases such as air which have a low ionization potential \(I_{\text{ion}}\) (nitrogen: 15.5 eV, oxygen:

\(^4\)American Association of Physicists in Medicine
12.5 eV) and much energy is ‘wasted’ to excitations. For clinical purposes however the use of air is desirable, to avoid complicated gas systems. The usual way of determining the $w$ value is by comparing a water calorimeter result with an air-filled ionization chamber result [118]. Through the years a large number of measurements of this $w$ value has been performed, which however do not completely agree with each other. A problem is that the experimental value obtained is the product $s_{w, \text{air}} \cdot w$. This means that the result also depends on the stopping power values one is using. Another complication is that the humidity of air influences $w$ in a different way than $s_{w, \text{air}}$. Analog to the stopping power calculations it has been tried to achieve uniformity through recommendations: unfortunately for $w$ there remain two competing values: 34.3 eV in [1] and 35.2 eV in [64, 135, 136]. Until more accurate measurements are available this will be a major source of systematic uncertainty in absolute proton dosimetry.

### 2.2.6 Ion recombination correction

Not all the ions that are created in an ionization chamber will be collected by the electrodes. Some of the ions will meet electrons before they can reach the electrodes. This recombination can be reduced by sweeping the ions out of the chamber more rapidly, either by increasing the field strength or by reducing the distance between the electrodes. In [21] a theoretical description for this recombination is given, in which a collection efficiency $f$ for a parallel plate ionization chamber is derived:

$$f = \frac{1}{(1 + m \cdot q \cdot \frac{d}{U})} \quad (2.36)$$

where $q$ is the ionization rate (in C cm$^{-3}$ s$^{-1}$), $d$ the distance between the electrodes (in mm, for a Markus or NACP chamber 2 mm) and $U$ the chamber voltage (in V). $m$ is a constant, characteristic of the gas at a given temperature and pressure. For air at STP conditions its value is: $m = 6.73 \times 10^{11}$ C$^{-1}$ V$^{2}$ cm$^{-1}$. This can be converted to a correction factor $k_s$ which depends on the doserate $\dot{D}$ (Gy/s):

$$k_s = 1 + m_D \cdot \dot{D} \cdot \frac{d^4}{U^2} \quad (2.37)$$

where the actual dose is $k_s$ times the measured dose, $m_D = 2.4$ Gy$^{-1}$ s$^{-2}$ cm$^{-2}$ for air at STP condition. In the spot scan technique (see section 5.2.1) the most distal spots are in general reached only once. This means that the complete dose (in the order of $\approx 1-2$ Gy) has to be delivered in 10 ms. It can be seen that even in this case the correction is minimal, because high voltages up to 3 kV/cm can be used.