Sensitive polarimetry in a search for the deuteron electric dipole moment
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Chapter 2

Motivation

2.1 Discrete Symmetries in the Standard Model

The search for violations of fundamental symmetries has played an important role in the development of particle physics in the 20th century. The behavior of a physics observable under a transformation is related to a corresponding symmetry. For a true symmetry, physics observables do not depend on the transformation. Consequently, if a physics observable does change when applying one or more transformations, the corresponding (joined) symmetry must be broken. In particular, tests of the discrete symmetries, charge conjugation $C$, parity $P$ and time reversal $T$, have been essential to establish the structure of the SM [3–5]. Parity $P$ is the symmetry related to the reversal of all spatial coordinates, $\vec{r} \rightarrow -\vec{r}$, whereas $T$ is related to the reversal of the time coordinate, $t \rightarrow -t$. Charge conjugation $C$ is the transformation exchanging all particles with antiparticles and vice versa.

Directly associated with the existence of the mentioned symmetries is the CPT theorem [6, 7] that states that any relativistic (Lorentz-invariant) quantum field theory must be invariant under $CPT$, the combination of $C$, $P$ and $T$ [6, 7]. As a consequence of this assumed invariance, probing one of these three symmetries is equivalent to probing the combination of the remaining two. For example, probing some process for $T$ invariance is equivalent to probing it for $CP$ invariance.

Tests of the discrete symmetries have provided essential information in the formulation of the SM in its present form [8]:

\[\text{\ldots} \]
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• The electromagnetic and strong interaction have not yet been shown to violate $C$, $P$ or $T$, or any combination of them.

• The discovery of parity violation in the weak interactions [4, 5] indicated that matter fields should be combined into asymmetric left and right handed chiral multiplets, which constitutes one of the indispensable features of the Standard Model.

• The observation of CP violation in the decay of K$^0$ mesons [3] provided strong evidence for the presence of three quark generations, through the Kobayashi-Maskawa mechanism [9], before experimental indications for the third family had been found.

The search for permanent electric dipole moments (EDMs), which violate both parity and time reversal invariance, is among the most sensitive tests of discrete symmetries. Prior to the observation of parity violation, Purcell and Ramsey proposed considering the EDM of the neutron as a probe for new physics, at that time parity violation. The non-observation of a neutron EDM has, by now, ruled out more speculative models beyond standard theory than any other experimental approach, and established severe constraints on others.

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2.2.1 Theoretical Considerations

For a particle in an electric $\vec{E}$ and magnetic $\vec{B}$ fields, the field dependent part of the Hamiltonian is

$$H = -2(\vec{\mu} \cdot \vec{B} + \vec{d} \cdot \vec{E}) = -2(\mu \vec{B} + d \vec{E}) \cdot \hat{S}$$

(2.1)

where $\vec{S}$ stands for the spin vector of the particle, the only direction defining vector in the rest frame of the particle, and $\mu$ and $d$ are the magnetic and electric dipole moments defined, respectively, as

$$\mu = g(e/2mc)$$

(2.2)
and

\[ d = \eta (e/2m). \]  

(2.3)

Under a \( P \) transformation, \( \vec{E} \) reverses while \( \vec{B} \) and \( \hat{S} \) remain unchanged. Similarly, under a \( T \) transformation, \( \vec{B} \) and \( \hat{S} \) reverse while \( \vec{E} \) remains unchanged. In both situations, the Hamiltonian is modified by the transformation, violating both \( P \) and \( T \) symmetries, unless \( d = 0 \). A graphic illustration of these transformations is given in figure 2.1. Under the assumption of CPT invariance, the

Figure 2.1: Symmetry violation for a non-zero EDM. The darker pattern indicates the charge distribution giving rise to an electric dipole moment and the arrows represent the spin. Under a parity transformation the electric dipole moment changes sign (dark pattern inverted vertically) but the spin does not. Similarly, under a time reversal transformation the electric dipole moment does not change but the spin does (change of direction of the black arrow).

\( T \) and combined \( CP \) symmetries are equivalent. In the current Standard Model, quark mixing leads to \( CP \) violation in the weak interaction, which also maximally violates \( P \) by itself. The SM thus predicts non-zero EDMs.

Such EDMs arise because of radiative corrections, i.e. the interaction with virtual particles. As all three quark generations need to be involved, only weak interaction corrections of third and higher order (or fourth in the case of leptons) contribute. The resulting Standard Model induced EDMs are at the level of \( 10^{-36} \) e.cm for the electron and \( 10^{-32} \) e.cm for the neutron. These are far below the sensitivity of current and planned experiments.

With the maturing of Quantum Chromo Dynamics (QCD), it became clear that also the strong interaction Lagrangian may contain a \( CP \) violating term. The strength of this term is characterized by the dimensionless parameter \( \bar{\theta} \). In the
framework of $SU(3)$ symmetry within the Standard Model, the resulting EDM of the neutron can be calculated to be $d_n = 5 \times 10^{-16}\bar{\theta}$ e.cm.

The current upper limit on the neutron EDM leads to the apparent limit $\bar{\theta} < 10^{-10}$. The not yet understood fine-tuning of the $\theta$-term, which might even be zero, constitutes the so-called strong CP problem of the SM.

Several possible explanations have been proposed for the smallness of $\bar{\theta}$. Some of them involve the existence of a new particle, the axion, which has so far evaded experimental observation [10].

In the Standard Model there is a description of QCD and the electroweak interactions in one single framework. The fine-tuning of $\theta$ can be interpreted as the absence of flavor-conserving CP-violation[8]. This is a totally different situation compared to the flavor-changing weak sector to which all the observed CP-violating effects belong. In this case, the incorporation of CP-violation through the Kobayashi-Maskawa mechanism [9] has been supported by experiments using neutral $K^0$ and $B^0$ mesons[11, 12]. Another difference is the fact that the phase in the CKM mixing matrix does not require any fine-tuning. The predictions for any CP-violating effect in the flavor-conserving channel, such as EDMs, induced by CKM mixing are very small. This makes it hard to detect experimentally.

Searches for other sources of CP-violation, than the CKM mechanisms, constitute a unique probe for physics beyond the SM. Since the first suggestion by Ramsey and Purcell to search for an EDM of a neutron, such measurements have become some of the most sensitive probes to look for new sources of CP-violation. In case no EDM is found, this constrains new speculative models down to a particular level of sensitivity (see Figure( 2.2)). Until now, dedicated searches have not revealed any intrinsic EDM of a fundamental particle or system. With the achieved experimental precision they constitute some of the most stringent tests of the SM.

Looking for new sources of CP-violation is further motivated by the observed matter-antimatter inequality in the Universe. According to Sakharov, this asymmetry could be explained through a combination of $C$, $CP$ and baryon number violation in an universe out of thermal equilibrium [13]. However, the level of $CP$-violation in the SM, through the CKM phase or the $\bar{\theta}$ parameter, is insufficient to explain this crucial asymmetry. This constitutes a strong hint that there
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Figure 2.2: EDM searches and SM model extension that can be constrained by them.

could be additional sources of CP violation at play. These would give rise to EDMs considerably bigger than those predicted by the SM.

Without knowing the new source for CP violation, no "best" system to look for an EDM can be identified a priori. A new CP-violating fundamental interaction manifests itself by giving the quarks and leptons an EDM and by modifying the properties of the other interactions. QCD, nuclear, atomic and molecular theory must be applied to obtain the resulting hadronic, nuclear, atomic and molecular EDMs, respectively. A hypothetical new interaction thus gives rise to a distinct pattern of EDMs of different magnitude in some collection of systems and vice versa. The observation of a single non-zero EDM thus suffices to confirm the presence of physics not contained in the SM. To discriminate between various possible forms of this new physics requires the observation of the aforementioned pattern (assumed to give a characteristic "fingerprint"). Different mechanisms responsible for EDMs are shown in picture 2.3.
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As an example, an EDM arising from $P$ and $T$-odd pion-nucleon couplings $g_i$ can be expressed as

$$d = A_0 g_0 + A_1 g_1 + A_2 g_2.$$  \hspace{1cm} (2.4)

The $A_i$ parameters take different values for different systems. Table 2.1 lists their values for some possible systems.

Table 2.1: Sensitivity to CP-violation for different systems [15].

<table>
<thead>
<tr>
<th>System</th>
<th>$A_0$ [e.fm]</th>
<th>$A_1$ [e.fm]</th>
<th>$A_2$ [e.fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>0.14</td>
<td>-0.14</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.10</td>
<td>0.23</td>
<td>0.00</td>
</tr>
<tr>
<td>$^{129}$Xe</td>
<td>$6 \times 10^{-5}$</td>
<td>$6 \times 10^{-5}$</td>
<td>$12 \times 10^{-5}$</td>
</tr>
<tr>
<td>$^{199}$Hg</td>
<td>$2 \times 10^{-6}$</td>
<td>$2 \times 10^{-4}$</td>
<td>$-3 \times 10^{-5}$</td>
</tr>
<tr>
<td>$^{225}$Ra</td>
<td>-0.06</td>
<td>-0.12</td>
<td>0.11</td>
</tr>
</tbody>
</table>
2.2 Electric Dipole Moments

EDM searches comprise several different approaches and systems which can be distinguished as follows:

1. Searches for EDMs of hadrons, nucleons in particular
2. Searches for EDMs in diamagnetic atoms
3. Searches for EDMs in paramagnetic atoms and molecules.
4. Searches for EDMs in leptons

For each of them the most sensitive experiments to date are on the neutron, mercury and thallium. The electron EDM is calculated from the atomic one. The present limits for each of them are presented in table 2.2. These limits for the different EDMs are comparable despite their apparently different size. They play complementary roles in constraining fundamental CP-violation sources.

Table 2.2: Current limits for different EDM types.

<table>
<thead>
<tr>
<th>Type</th>
<th>System</th>
<th>EDM Limit [ e.cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nucleon</td>
<td>n</td>
<td>$</td>
</tr>
<tr>
<td>Diamagnetic</td>
<td>$^{199}$Hg</td>
<td>$</td>
</tr>
<tr>
<td>Paramagnetic</td>
<td>$^{205}$Tl</td>
<td>$</td>
</tr>
<tr>
<td>Electron</td>
<td>$^{205}$Tl</td>
<td>$</td>
</tr>
</tbody>
</table>

The following sections consider each of the cases in more detail.

2.2.2 Experimental Searches

In all EDM experiments, three stages can be recognized.

1. Polarization: an ensemble of spin polarized particles is prepared;
2. Interaction : this ensemble interacts with an electric field;
3. Polarimetry : the evolution of the spin due to the electric field is measured.
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2.2.2.1 Neutron

Historically, the neutron was the first particle to be considered as a candidate for an EDM experiment [19]. The first experiment, that set the pace for the EDM studies that followed, was performed at the Oak Ridge reactor in 1949 by Smith, Purcell and Ramsey. It set an example for experimental concepts of future EDM experiments, making use of the change in Larmor precession frequency of the particles in a transverse applied electric field $\vec{E}$ [20]. A scheme of the experimental apparatus is depicted in figure 2.4.

![Figure 2.4: Experimental apparatus for the neutron EDM [2]. The neutron beam is polarized by total reflection from a magnetized mirror before entering a region of homogeneous magnetic field from the left side.](image)

A beam of slow neutrons ($E \sim 0.4$ eV) is spin-polarized by reflecting it of a magnetized mirror. Next, they enter a region with collinear electric field $E$ and magnetic field $B$, oriented perpendicular to the beam. In this field, the neutrons will undergo Larmor precession with frequency $\omega = (2\mu_B \pm 2dE)/\hbar$, where the $\pm$ sign corresponds to parallel and anti-parallel $E$ and $B$ fields. An EDM reveals itself through a $E$-dependent shift in $\omega$. The frequency difference $\Delta \omega = \omega(E^+) - \omega(E^-)$ for $E$ parallel and anti-parallel to $B$ is then given by

$$\Delta \omega = \frac{4d_n E}{\hbar}$$

The precession frequency was measured using Ramsey’s method of magnetic resonance with spatially separated oscillating fields, implemented as a set of coils placed at the entrance and exit of the field region [21]. The polarization of the
outgoing neutrons is determined by measuring the neutron flux reflected of a magnetized foil.

More recent experiments [22] make use of ultra-cold neutrons (UCNs), which have a typical energy of 100 meV or less. At these energies and for suitable materials, neutrons undergo total internal reflection at any angle of incidence, so that they can be trapped in closed vessels without any appreciable loss for a long time (100's of seconds). The vessel is placed in a collinear electric and magnetic field and the oscillating fields are separated in time, rather than spatially.

The most recent and precise experiment was performed at the Institute Laue-Langevin (ILL) in Grenoble [16], where the result quoted in table 2.2 was obtained. The precision of this result was mainly determined by the limited number of produced and stored ultra cold neutrons. Several efforts, exploring new methods, are underway to improve this precision [23, 24].

2.2.2.2 Schiff’s Theorem

It has long been consider impracticable to search for an EDM on charged particles. When placed in a strong electric field such a particle would accelerate and quickly leave the measurement region. It was pointed out by Ramsey and Purcell that applying an external electric field on a neutral atom (or molecule) containing a nucleus or unpaired electron with an EDM is also mostly useless [19]. The charged constituents rearrange themselves such that the force exerted by the external electric force is balanced by the internal forces. In the non-relativistic point-charge limit, the only forces are electrostatic. Hence, the net electric field on each charge is zero. This is commonly referred to as Schiff’s theorem. Schiff also pointed out several ways to evade (complete) shielding, making it possible to search for both nuclear and electron EDM using selected atoms.

2.2.2.3 Diamagnetic Atoms : Nuclear EDMs

In diamagnetic atoms shielding of an external electric field is reduced if the nuclear charge and EDM distributions are not the same. In this case, a small residual EDM effect remains in the electron’s potential, proportional to the nuclear EDM. This effect is parameterized via the Schiff moment, which is a vector proportional
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to the nuclear EDM and which depends on the difference between the normalized charge and EDM distributions. Generally speaking, the Schiff moment is largest for heavy nuclei. It may be further enhanced by one or two orders of magnitude in octupole-deformed nuclei such as $^{225}$Ra [25].

The most stringent EDM limit to date was obtained for the spin-1/2 $^{199}$Hg nucleus[17]. A quartz cell containing isotopically enriched $^{199}$Hg vapor was placed in collinear electric and magnetic fields. The sample is polarized by optical pumping. The subsequent precession frequency of the nuclear spin is determined by observing the modulation rate of the absorption of circularly polarized light. Rather than flipping the orientation of the electric field with respect to the magnetic field, the modulation frequency is measured simultaneously in two cells with oppositely oriented electric fields, but with the same magnetic field. The observed frequency difference is then related to the EDM, as given in equation 2.5. The experimental setup is shown schematically in figure 2.5.

Figure 2.5: Simplified diagram of the $^{199}$Hg EDM apparatus showing details of the vapor cell holding vessel and middle cell light beams. The topmost cell is shown inside a cutaway view of the top electrode, while the bottom electrode shows a light access hole for the enclosed cell.
To reach the current precision, great care had to be taken to eliminate or reduce systematic errors. Some examples include the following. The vapor cells are made out of quartz glass with a high resistance to reduce leakage currents that correlate with the reversal of the electric field and thus mimic an EDM. These cells were placed inside a three-layer magnetic shield. Inside this shield a magnetic field of 15 mG was maintained being stable to 25 ppb on a time scale of 100 s. Fluctuations and a gradient in the magnetic field, which would lead to an EDM-like difference in the precession frequency observed by the two vapor cells, was monitored by placing two additional cells above and below the central ones. These cells are not exposed to electric fields and thus allow precise measurements of the magnetic field. The electric and magnetic field reversals and changes were frequently carried out in order to check for systematic effects.

The largest contribution to the $^{199}\text{Hg}$ nuclear EDM is estimated to arise from $P,T$-odd nucleon-nucleon interaction. However, calculations show there is also a contribution from the intrinsic EDM of the proton. Thus, besides setting a stringent limit on the $^{199}\text{Hg}$ nuclear EDM, the experimental result can also be related to a limit on the proton EDM [26].

New developments are being pursued in exotic nuclei possessing an octopole moment [27] believed to have enhanced CP-violating effects. Among these is the Radium atom experiment being carried out at KVI [28–30].

### 2.2.2.4 Paramagnetic Atoms: the Electron EDM

It was shown by Sandars that Schiff’s theorem is also evaded when considering relativistic effects on unpaired electrons in paramagnetic atoms [31]. He showed that the atomic EDM induced by the electronic EDM may actually be enhanced rather than suppressed. For the ground state of alkali atoms or select other atoms, such as thallium, one finds for the ratio of the EDM of an atom $d_{\text{atom}}$ and the EDM of an electron $d_{e}$ [31–34]

$$|R| = \left| \frac{d_{\text{atom}}}{d_{e}} \right| \sim 10^{Z^3 \alpha^2 J(J + \frac{1}{2})(J + 1^2)} d_{e},$$

(2.6)

where $J$ stands for the angular momentum of the electron, $Z$ for the atomic number and $\alpha$ for the fine structure constant.
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For a system with large atomic number $Z$, $R$ may become very large; for thallium, one has calculated $R = -585$ [34]. Atomic paramagnetic systems thus constitute powerful probes to search for an electron EDM. Similar arguments apply to heavy polar diatomic paramagnetic molecules, such as YbF in its ground state or PbO* in one of its metastable states.

The experimental limit on the atomic or molecular EDM can thus be interpreted as a limit on the electron EDM. This assumes that $R$ is known and that there is no contribution from $P,T$-odd nuclear-electron interactions. The $^{205}$Tl atomic beam magnetic resonance experiments [35] have provided the most precise limit on $d_e$. The experimental setup is shown in figure 2.6. The experiment makes use of laser optical pumping to polarize the atomic beam and to perform polarimetry through fluorescence.

As in the mercury experiment, two experiments are performed at once in which two beams are exposed to oppositely oriented $E$ and $B$ fields. When moving through a strong transverse electric field $\vec{E}_\perp$ a motional magnetic field $\vec{B}_\perp = -\gamma \frac{1}{c^2} \vec{v} \times \vec{E}_\perp$ appears in the rest-frame of the particles, where $\vec{v}$ is the velocity of the particles and $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$. Due to the interaction with the magnetic moment of the particle, the spin precession frequency will change. This change reverses when the electric field orientation is reversed and thus mimics a non-zero EDM. Therefore, the direction in which the beams travel, meaning the sign of $\vec{v}$, is altered every few seconds to expose and correct for this effect.

There are many new electron EDM searches in preparation, using a variety of systems, including: atomic cesium, francium or radium; molecular YbF, PbO*; the molecular ion HfF⁺; paramagnetic solids like gadolinium gallium garnet (GGG) or ferri magnetic gadolinium iron garnet (GdIG). It is thus likely that the limit on the electron EDM will be improved considerably in the near future. [2, 31–34, 36–38].
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Figure 2.6: Schematic experimental setup used for the $^{205}$Tl EDM measurement. Atoms leave the trichamber oven thermally distributed among the ground state hyperfine levels. After some collimation they enter the quantizing magnetic field $B$. Laser beams then depopulate the states with nonzero magnetic quantum numbers. The atoms then move into the electric field, nominally parallel or antiparallel to $B$ [39].
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2.2.2.5 Charged Particles: Muon EDM

All of the EDM experimental approaches presented so far involve the study of neutral particles. To enhance an EDM related signal as much as possible, the system under study must be placed in an electric field that is as strong as possible. In such a field an isolated charged particle experiences a strong force, which is not balanced by internal forces as was the case when the particle was embedded in an atom or molecule. It will thus accelerate and quickly leave the observation region, precluding precise measurement of the precession frequency.

As pointed out by Schiff, this argument does not hold when other than electrostatic forces are present. This is, for example, the case for fast charged particles stored in a magnetic storage ring, where the particles are contained using the Lorentz force. The magnetic field in the laboratory frame gives rise to an electric field in the rest frame of the particle, \( \vec{E}_\perp = \gamma \vec{v} \times \vec{B} \). This gives rise to an additional term in the spin precession rate proportional to the EDM, in addition to the terms related to the magnetic moment.

The most stringent EDM limit measured directly on a charged particle has been reported for the muon, \( |d_\mu| < 1.9 \times 10^{-19} \text{ e.cm (95\% C.L.)} \) [40, 41]. This limit was obtained in experiment E821 at Brookhaven National Laboratory, which was aimed at measuring the muon anomalous magnetic moment \( a_\mu \). A highly uniform magnetic dipole field of 1.45 T, combined with an electric quadrupole field, were used to store polarized muons with a momentum of 3.1 GeV/c. At this so-called "magic" momentum, which satisfies \( \beta^2 \gamma^2 = 1/a \) with \( \beta = \frac{v}{c} \), the spin is insensitive to the electric field. The spin precession, relative to the particle momentum, is then given by

\[
\vec{\omega} = \vec{\omega}_a + \vec{\omega}_c = -\frac{e}{m} \left[ a \vec{B} + \frac{n}{2} \beta \times \vec{B} \right], \tag{2.7}
\]

with \( a \) the magnetic moment anomaly \( a = (g - 2)/2 \) for \( \mu = g e^2 mc \) and similarly \( d = \frac{n}{2} \). The precession frequency is

\[
\omega = \omega_a \sqrt{1 + (\eta \beta/2a)^2} \approx \omega_a \left[ 1 + \frac{1}{2} \left( \frac{\eta \beta}{2a} \right)^2 \right], \tag{2.8}
\]

The frequency shift is second order in \( \eta \) and thus immeasurably small. The radial component of \( \vec{\omega} \), which is proportional to the EDM, gives rise to an oscillatory
vertical spin component that is out of phase with the magnetic dipole precession. It is from the limit on the magnitude of this component, that the quoted limit on the muon EDM was derived. Because of the magnetic moment, the spin rapidly

![Diagram](image-url)

Figure 2.7: Scheme of the orientation of the vectors in the storage ring search for a muon EDM. The angular velocity $\omega_a$ is collinear with B, while $\omega_e$ is normal to the $\beta \cdot B$ plane. The unit vector $\hat{s}$ represents the spin quantization axis of the muon and its precession is shown. It precesses in the plane perpendicular to $\omega = \omega_a + \omega_e$ resulting in an oscillating vertical component in the laboratory frame.

precesses. Therefore, the interaction between the EDM and the electric field cannot build up a large vertical spin component with storage time.

In most theoretical models, including the Standard Model, the electron, muon and tau EDMs are approximately proportional to their masses [42]. From the existing electron EDM limit, one would then predict $d_\mu < 3 \times 10^{-25}$ e.cm, many orders of magnitude below the direct limit. However, plausible models exist in which the mass scaling is quadratic or even cubic, giving rise to a several orders of magnitude larger muon EDM [43, 44]. This possibility motivates $d_\mu$ searches at the $10^{-24}$ e.cm. level [45].
2.3 Storage Ring Techniques for Charged Particle EDM Searches

In the muon EDM experiment described above it was apparent that the effect of an EDM on the spin precession of a particle stored in an ordinary electromagnetic storage ring is extremely small compared to the effect of the magnetic dipole moment. The fast spin precession caused by the magnetic moment prevents the buildup of an EDM induced signal. Several possible schemes have been proposed to overcome this limitation.

2.3.1 Thomas-BMT Equation

In a frame rotating with a particle moving in an electric field $E$ and magnetic field $B$, the spin precession, including only a magnetic moment, is given by [46]

$$\frac{d\vec{S}}{dt} = \frac{e}{m} \vec{S} \times \left[ a\vec{B} - \frac{a\gamma}{\gamma + 1}(\vec{\beta} \cdot \vec{B})\vec{\beta} + \left( \frac{1}{\gamma^2 - 1} - a \right) \vec{\beta} \times \vec{E} \right].$$  \hspace{1cm} (2.9)

This expression is usually referred to as the Thomas-BMT equation, where BMT stands for Bargman-Michel-Telegdi and can be written as

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_{T-BMT} \times \vec{S}$$ \hspace{1cm} (2.10)

with

$$\vec{\Omega}_{T-BMT} = -\frac{e}{m} \left[ a\vec{B} - \frac{a\gamma}{\gamma + 1}(\vec{\beta} \cdot \vec{B})\vec{\beta} + \left( \frac{1}{\gamma^2 - 1} - a \right) \vec{\beta} \times \vec{E} \right].$$  \hspace{1cm} (2.11)

Including a non-zero EDM results in [47]

$$\vec{\Omega} = \vec{\Omega}_{T-BMT} + \vec{\Omega}_{EDM}$$

$$\quad = \vec{\Omega}_{T-BMT} - \frac{e\eta}{2m} \left[ \vec{E} - \frac{\gamma}{\gamma + 1}(\vec{\beta} \cdot \vec{E})\vec{\beta} + \vec{\beta} \times \vec{B} \right]$$ \hspace{1cm} (2.12)

Based on the classical description of the spin motion in combined electric and magnetic fields several experimental methods can be derived to measure or limit a possible EDM of a charged particle moving in those fields.
2.3 Storage Ring Techniques for Charged Particle EDM Searches

2.3.2 Parasitic Method

In the aforementioned muon g-2 experiment, a homogeneous magnetic dipole field was used. At the "magic" momentum the effect of the interaction between the electric field and the magnetic moment \(1/(\gamma^2 - 1) - a = 0\) is canceled and the muon polarization is perpendicular to the magnetic field \((\vec{\beta} \cdot \vec{B} = 0)\). In this case, equation 2.12 reduces to

\[
\vec{\Omega} = -\frac{e}{m} \left[ a\vec{B} + \frac{\eta}{2} \vec{\beta} \times \vec{B} \right].
\] (2.13)

As was already discussed in some detail above, a non-zero EDM \((\eta \neq 0)\) will lead to a vertical spin component that oscillates with frequency \(\Omega\) and relative amplitude \(\eta \beta/2a\).

2.3.3 Frozen Spin Method

In [48], it is proposed to apply a radially oriented electric field, in addition to a vertical magnetic field. In the proper combination, this eliminates the spin precession caused by the magnetic moment, given by

\[
\vec{\Omega}_{T-BMT} = -\frac{e}{m} \left[ a\vec{B} + \left( \frac{1}{\gamma^2 - 1} - a \right) \vec{\beta} \times \vec{E} \right].
\] (2.14)

This term is zero for

\[
E_r = \frac{aB_z\beta}{1 - (1 + a)\beta^2}.
\] (2.15)

In the absence of an EDM, the spin will no longer precess. Hence the name "frozen spin". For a non-zero EDM, the remaining spin precession is

\[
\vec{\Omega} = -\frac{en}{2m} \left[ \vec{E} + \vec{\beta} \times \vec{B} \right].
\] (2.16)

Both \(\vec{E}\) and \(\vec{\beta} \times \vec{B}\) are oriented radially. Spin precession about \(\vec{\Omega}\) will cause the polarization to oscillate between longitudinal and vertical.

The (relative) strengths of the magnetic and electric fields needed to freeze the spin depend on the velocity and anomalous magnetic moment of the particle being studied. In most cases the maximum attainable \(E\) will be the limiting
2.3 Storage Ring Techniques for Charged Particle EDM Searches

factor. Using equation 2.15 for a completely frozen spin, the spin precession rate can be rewritten as

\[ \Omega = \frac{e\eta E}{2m}\frac{a + 1}{a\gamma^2} \] (2.17)

\( \Omega \) is proportional to the EDM and will thus be extremely small. Therefore, in an experiment only a small fraction of a precession cycle can be observed. For a beam with originally longitudinally polarization \( P_0 \), the vertical polarization component \( P_z \) will evolve as

\[ P_z = P_0 \sin \Omega t \simeq P_0 \Omega t = P_0 \frac{e\eta E}{2m} \frac{a + 1}{a\gamma^2} t. \] (2.18)

In this approximation, the vertical growth of the polarization component is proportional to the EDM. The strength of the electric field necessary to freeze the spin is proportional to the anomalous magnetic moment. As this strength is usually the limiting factor, this method is only suitable for a particle with a small magnetic anomaly. Furthermore, the effective electric field is inversely proportional to \( \gamma^2 \). The experiment should thus be done at modest energy to avoid reducing the sensitivity.

2.3.4 Electrostatic Method

In the muon g-2 experiment, the fact that at the magic momentum the spin precession is insensitive to the electric field was used to measure the anomalous magnetic moment with great precision. This suggests a third possibility to search for an EDM, namely the use of a purely electrostatic storage ring operating at the magic momentum. For \( a > 0 \), the magic momentum is determined as

\[ \gamma_{\text{magic}} = \sqrt{\frac{a + 1}{a}}. \] (2.19)

In this case,

\[ \overline{\Omega} = -\frac{e\eta E}{2m}. \] (2.20)

In the absence of a magnetic field, the particles need to be contained solely by the electric field, which must thus be oriented radially inward. This method therefore only works for positive values of \( a \). Furthermore, no amplification of the electric field occurs. Other than that, this method is equivalent to the frozen spin method.
2.3 Storage Ring Techniques for Charged Particle EDM Searches

2.3.5 Resonance Method

The drawback of the parasitic method is the lack of accumulation of the EDM induced spin precession. This is caused by the fact that the torque causing this precession, proportional to

\[ \vec{S} \times (\vec{\beta} \times \vec{B}) \]

is zero on average because the spin rapidly precesses due to the magnetic moment. However, the electric field need not be static. In [49] a method, reminiscent of the magnetic resonance method, was proposed in which the velocity of a particle (\( \beta \)) moving in a static magnetic dipole field is modulated at the spin precession frequency (\( \omega = e/maB \)), hence the name "resonance method". In this case,

\[ \langle \vec{S} \times (\vec{\beta} \times \vec{B}) \rangle = S \cdot \delta \beta \cdot B \cdot \cos \phi, \]

with \( \delta \beta \) the modulation amplitude of \( \beta \) and \( \phi \) the phase between the spin precession and velocity modulation. This will cause a steadily growing vertical polarization component given by

\[ P_z = P \frac{\epsilon \eta}{2m} \delta \beta B t \cos \phi, \]

in addition to the oscillating vertical spin component, which is proportional to \( \beta \) as in the parasitic method.

This method is in principle applicable to all particles. However, modulation of the particle velocity, also known as synchrotron oscillations, requires a phase-locked accelerating cavity placed in the storage ring. To obtain a large modulation depth \( \delta \beta \), the electric field in this cavity must be as strong as possible. The frequency at which the cavity operates is dictated by the spin precession frequency. For a practical cavity, this frequency must be as small as possible. This limits this method to particles with small anomaly, possibly combined with a modest magnetic field strength. The latter reduces the sensitivity.

2.3.6 Rehash

The methods described only address techniques to enhance the interaction between the EDM and an electric field. Further criteria for a sensitive EDM experiment include the availability of an intense source of highly polarized particles.
This drastically limits the number of possible candidates for either of these methods. Finally, a method must exist to track the evolution of the spin, i.e. to perform sensitive polarimetry.

The parasitic method will be exploited again in the planned muon g-2 experiment in preparation at Fermilab [50]. The frozen spin method is being studied as a method to search for an EDM on the positron [51], muon [52] or deuteron [53]. The exploitation of an electrostatic ring is studied for an EDM search on the proton [54]. Of these particles, the deuteron is a unique system, as it is the only nucleus, adding CP-odd parts in the nucleon-nucleon interaction as a possible source for a nuclear EDM in addition to intrinsic EDMs of free particles [55].

2.4 The Deuteron EDM

2.4.1 Origin

Until now, the most stringent bounds on flavor-diagonal CP-violation in the hadronic sector have been established by the EDM limits on the neutron, mercury atom and electron. The sensitivity of each system for existing or new sources of CP violation depends on the details and the dynamics of the constituents. The deuteron falls, in terms of complexity, in between the neutron and heavy atoms. It is the simplest system in which the $P$-odd, $T$-odd nucleon-nucleon interaction contributes to the EDM. Finding or establishing a limit for its EDM would provide important complementary information to the neutron and atomic systems.

It was found that at the level of $10^{-29}$ e.cm, the deuteron EDM is one to two orders of magnitude more sensitive than any present EDM limit and within the predictions of specific SUSY models[56, 57]. Besides the improvement on sensitivity the deuteron has the advantage of having smaller theoretical uncertainties [58]. At the nuclear level the deuteron EDM can be a result of a combination of the proton and neutron EDMs and also of meson exchange between the nucleons with $CP$ odd couplings such that,

$$d_D = d_n + d_n + d_{pNN}^N.$$  \hspace{1cm} (2.24)
2.4 The Deuteron EDM

It has been shown that the error for the calculation of $d^e_{NN}$ can be systematically reduced using realistic wave functions for the deuteron [58]. In more complex nuclei, such as $^{199}$Hg, large uncertainties are possible because of the complex nuclear structure.

2.4.1.1 Strong CP Violation

The calculation of $d_D$ as a function of the strong CP-violating $\bar{\theta}$ parameter, has, for example, been calculated using QCD sum-rules [59]. Numerically, the dependence can be written as

$$d_D \simeq -[(7.0 \pm 2.8) + (2.8 \pm 0.8)] \times 10^{-17} \bar{\theta} \text{e.cm.} \quad (2.25)$$

For the neutron,

$$d_n = 1.2 \times 10^{-16} \bar{\theta} \text{e.cm} \quad (2.26)$$

These results evaluated for the predicted sensitivity of the neutron EDM of $10^{-28}$ e.cm and the deuteron EDM of $10^{-29}$ e.cm leads to a limit for $\bar{\theta}$ of

$$|\bar{\theta}| < 8 \times 10^{-12}, \quad (2.27)$$

for the neutron and

$$|\bar{\theta}| < 3 \times 10^{-13}, \quad (2.28)$$

for the deuteron case. This seems to indicate that the projected EDM limit on the deuteron is at least one order on magnitude more constraining on $\bar{\theta}$ than the one projected for a future neutron EDM.

2.4.1.2 Quark Chromo EDMs

An example of a New Physics contribution, EDMs induced by color EDMs at the quark level are considered. Chromo EDMs (or quark-color EDMs) can arise from SUSY loops and give rise to regular quark EDMs. A comparison can be established between the neutron and the deuteron, in terms of sensitivities [56–58].

$$d_n \approx 1.4(d_d - 0.25d_u) + 0.83e(d_d^c + d_u^c) + 0.27e(d_d^c - d_u^c) \quad (2.29)$$

$$d_D \approx (d_d + d_u) + 6e(d_d^c - d_u^c) - 0.2e(d_d^c + d_u^c) \quad (2.30)$$
2.4 The Deuteron EDM

where $d_q$ stand for the regular electromagnetic quark EDMs and $d_{c}^{q}$ for the chromo EDMs of a quark of $q$ flavor. Rewriting the previous expressions, only as a function of the chromo EDMs it comes [56] [58],

$$d_n \approx -0.01d_d^c + 0.49d_u^c,$$

(2.31)

$$d_D \approx -4.67d_d^c + 5.22d_u^c.$$

(2.32)

These two EDMs probe different linear combinations of the up and down quark contributions, signaling once more the complementarity between possible future experiments. Using a comparative approach, the simple numerical comparison between the coefficients for the different quark contributions for the neutron and the deuteron, the latest appears to be more sensitive by one to two orders of magnitude, at least in this case.

2.4.1.3 Dimensional Analysis

An estimate can be made of the scale of new physics that EDM experiments are sensitive to. Defining the EDM of a particle as [60]

$$d_i \approx \frac{m_i}{\Lambda^2} e \sin \phi$$

(2.33)

where $m_i$ is the mass of the particle, $\sin \phi$ corresponds to the CP-violating phase and $\Lambda$ the new physics energy scale. For a quark mass of $\sim 10$ MeV and $\sin \phi$ of order 1/2 it becomes [60]

$$d_D = 10^{-22} \left( \frac{1|\text{TeV}|}{\Lambda} \right)^2 e.\text{cm}. $$

(2.34)

This means that for $d_D = 10^{-29} e.\text{cm}$ the new physics scale is probed at $\Lambda \sim 3000$ TeV. The competitiveness in terms of sensitivity, together with the relative theoretical simplicity of its wave function and consequent physical interpretation, the experimental availability of polarized samples and its small anomalous magnetic moment, establishes the deuteron as the ideal candidate for an EDM search on charged particles in a storage ring.
2.4.2 Proposed Experiment

For the proposed deuteron EDM experiment presented to the Brookhaven National Laboratory in 2008 [60] each of the three distinct steps introduced in section 2.2.2 are addressed.

2.4.2.1 Polarization

Polarization is the initial stage of the experiment and it consists of preparing an ensemble of spin polarized particles. This is achieved by using polarized ion sources which are widely available and well understood. Examples of such sources are the POLIS at KVI [61] and the polarized $H^-$ source at BNL [62] which provide very high polarizations at the level of 70-90%.

2.4.2.2 Interaction

The polarized sample, delivered by the ion source, interacts with an electric field in a storage ring. The method found most suitable is the frozen spin method. For its application, a conceptual EDM ring was designed [63]. The ring parameters of relevance for this study are the choice of deuteron momentum acceptable for efficient deuteron polarimetry, $p_D = 0.7$ GeV/c, the magnitude of the electric field determines the precession frequency and horizontal emittance at the chosen 5 cm distance between electrodes $E_R = 3.5$ MV/m and the free space available between lattice determines the size and numbers of polarimeters.

2.4.2.3 Polarimetry

Monitoring of the spin behavior under the effect of the applied electric field needs to be performed in a continuous and highly efficient manner. Deuteron polarimetry in storage rings has previously been carried out attaining efficiencies of the order of $10^{-6}$ [64].

The success of the experiment depends on the combination of all three stages described here. The development and research of a new highly efficient deuteron polarimeter concept constitute the main subject of this dissertation.