On the linear and non-linear evolution of dust density perturbations with MOND
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Twin matter formalism

6.1 Introduction

Milgrom (2009a) has recently proposed a generalization of MOND into the relativistic context by means of a family of theories that involves two metric fields $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$. The family is defined by the following action:

$$I = -\frac{1}{16\pi G} \int \left[ \beta g^{1/2} R + \alpha \hat{g}^{1/2} \hat{R} - 2(g\hat{g})^{1/4} f(\kappa) a_0^2 M(\bar{\Upsilon}/a_0^2) \right] d^4x + I_M + \hat{I}_M,$$

(6.1)

where $\kappa \equiv (g\hat{g})^{1/4}$ and $\bar{\Upsilon}$ is a collection of scalar variables that is formed by contraction of tensors $C^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} - \hat{\Gamma}^\alpha_{\beta\gamma}$, where $\Gamma^\alpha_{\beta\gamma}$ and $\hat{\Gamma}^\alpha_{\beta\gamma}$ are the Levi-Civita connections of the two metrics. The action includes two matter terms $I_M$ and $\hat{I}_M$. The first one is the standard matter action which couples only to the metric $g_{\mu\nu}$. The other one was added to take into account the possibility that exist a matter component associated to the metric $\hat{g}_{\mu\nu}$.

Milgrom has shown that, for a particular choice of $\bar{\Upsilon}$, exists a particular gauge in which a perturbation of the two metrics over a Minkowsky background can be written as:

$$g_{\mu\nu} = \eta_{\mu\nu} - 2\phi \delta_{\mu\nu}$$

(6.2)

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} - 2\hat{\phi} \delta_{\mu\nu}.$$  

(6.3)

For the symmetric case ($\alpha = \beta$), the quasistatic limit gives the following field equations:

$$\nabla^2 \tilde{\phi} = 4\pi G(\rho + \hat{\rho})$$

(6.4)

$$\nabla \cdot \mathcal{M}' \left[ \left( \frac{\nabla \tilde{\phi}}{a_0} \right)^2 \nabla \tilde{\phi} \right] = 4\pi G(\rho - \hat{\rho}),$$

(6.5)

where the potentials $\tilde{\phi}$ and $\hat{\phi}$ are related to the perturbations on the metrics in the following way:

$$\phi = \tilde{\phi} + \zeta \hat{\phi}$$

$$\hat{\phi} = -\tilde{\phi} + \zeta \hat{\phi},$$

(6.6)
where \( \zeta = (2/3)^{-1} \) and the free function \( \mathcal{M} \) was chosen such that
\[
\mathcal{M}'(x^2) = \mu(x).
\] (6.7)

In the non-relativistic limit, the equations of motion for matter and twin matter particles are:
\[
\dot{x}^i = p^i \hspace{1cm} \dot{\hat{x}}^i = \hat{p}^i
\] (6.8)
\[
\dot{p}^i = -\frac{\partial \phi}{\partial x^i} \hspace{1cm} \dot{\hat{p}}^i = -\frac{\partial \hat{\phi}}{\partial \hat{x}^i}.
\]

The action presented above has two possible interpretations. One of them consists in assuming that the second metric included in the Lagrangian is no more than a mathematical device necessary to create MOND. The second approach, proposed in Milgrom (2010a), consists in assuming that there is in fact a second matter component associated to the metric \( \hat{g}_{\mu\nu} \), which will be known as the twin matter component. The forces on each component will be attractive and hence, both will be able to condense into objects. On the other side, the interaction between the two components will be through a repulsive force. Consequently, the cosmological evolution should locate the objects of the real matter component on the voids of the twin component and vice versa. The aim of this chapter is to test whether this conjecture is viable in the context of non-linear cosmological evolution.

The chapter is structured as follows. Section 6.2 will propose a set of field equations equivalent to eqs.6.4 and 6.5 but in a cosmological context. Section 6.3 refers to the N-body code that was employed during the calculations. The code is based on the one described in previous chapter. Several modifications had to be made in order to track the second metric. They will be briefly described, as well as the test that were made. The cosmological runs will be described in section 6.4 and their analysis and results will be discussed in section 6.5. Section 6.6 will summarize the conclusions. Finally, the chapter will be closed with an appendix, where a promising solution will be proposed for the cosmological problems for MOND that were found in this chapter and the previous one.

### 6.2 The equations

The weak field limit equations presented in Milgrom (2010a) (eqs.6.4 and 6.5) are the result of applying perturbation theory over a Minkowsky background and hence they are not the appropriate equations to do cosmology. In order to obtain analogous expressions in the context of cosmology, the same procedure discussed in section 2.1.1 will be employed: the background will be assumed to evolve according to the FRW solution and a change of coordinates will be made on the field equations given over Minkowsky.

The change to comoving variables is the following:
\[
x^i = \frac{r^i}{a},
\] (6.9)
which implemented in eqs.6.4 and 6.5 gives:
\[
\nabla^2 \tilde{\phi} = 4\pi G a^2 \left[ (\rho + \hat{\rho}) - (\langle \rho + \hat{\rho} \rangle) \right]
\] (6.10)
\[
\nabla \cdot \left[ \mu \left( \frac{\nabla \tilde{\phi}}{a a_0(a)} \right) \nabla \tilde{\phi} \right] = 4\pi G a^2 \left[ (\rho - \hat{\rho}) - (\langle \rho - \hat{\rho} \rangle) \right],
\] (6.11)
6.2: The equations

where the mean value of the sources were subtracted to obtain equations that can be treated using periodic boundary conditions (see section 2.1.1 for a detailed discussion on this topic). The fact that the mean value of the source of the field equations must be 0 is not just a formal mathematical subtlety without much importance. Potential solvers based of Fourier methods obtain the solution making basically additions and hence, they will always be able to provide a result. On the other side, iterative solvers like the one used for MOND fail to converge in case that this requirement is not fulfilled.

Section 2.1.1 states that it is customary to write the source of the field equation as a function of the overdensity \( \delta = \delta \rho / \rho \) (see eq.2.19). In the case of the twin formalism, as the two mean densities are subtracted in eq.6.11, to assume identical quantities of matter and twin matter will make the overdensity to be ill-defined; therefore this change will not be implemented on the equations.

To conclude the derivation of the equations, one more change has to be made, which is related to the way in which the densities are calculated. During the N-body simulations, the estimation of the density will be made through the following expression:

\[
\rho_{i,j,k} = \frac{1}{h^3} \sum_p m_p W(x_p, x_{i,j,k}),
\]

(6.12)

where \( \rho_{i,j,k} \) is the density in the cell \((i,j,k)\), the sumation is over all the particles, \( m_p \) is the mass of the particle \( p \) and \( W \) is a smoothing kernel. The size of the cells \( h \) is given by:

\[
h = \frac{B}{n},
\]

(6.13)

where \( B \) is the size of the box and \( n \) is the number of cells per dimension. The definition of the quantity \( B \) has to be taken with care. Let us suppose a set of particles distributed on a uniform grid. As the distribution is uniform, the forces will be zero and the coordinates of the particles will not change on time. If the box size is defined, for instance, as the distance at fixed time between a particle on one side of the box and a particle on the opposite side, with coordinates \((t, 0, 0, 0)\) and \((t, b, 0, 0)\) respectively, one has:

\[
B = \int (g_{ab}T^aT^b)^{1/2} dl = a(t)b,
\]

(6.14)

where the metric \( g_{ab} \) was assumed to be

\[
ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2).
\]

(6.15)

In other words, in comoving coordinates the box is expanding. Assuming \( a = 1 \) at redshift \( z = 0 \), eq.6.13 can be written as:

\[
h = a \frac{B_0}{n},
\]

(6.16)

where \( B_0 \) is the box size at redshift 0. Taking this into account and writing the mean densities as a function of its values at redshift \( z = 0 \):

\[
\langle \rho \rangle_z = \frac{\langle \rho \rangle_{z=0}}{a^3}
\]

(6.17)
chapter 6: Twin matter formalism

(see eq.1.20), one gets a final expression for the field equations given by:

\[
\nabla^2 \tilde{\phi} = \frac{4\pi G}{a} \left[ (\rho + \dot{\rho}) - (\langle \rho + \dot{\rho} \rangle)_{z=0} \right]
\]

(6.18)

\[
\nabla \cdot \left[ \mu \left( \frac{\nabla \tilde{\phi}}{aa_0(a)} \right) \nabla \tilde{\phi} \right] = \frac{4\pi G}{a} \left[ (\rho - \dot{\rho}) - (\langle \rho - \dot{\rho} \rangle)_{z=0} \right],
\]

(6.19)

where the densities are now assumed to be calculated using the size of the cells given at redshift \( z = 0 \) according to:

\[
h_0 = \frac{B_0}{n}.
\]

(6.20)

The time dependence that was included on \( a_0 \) in eq.6.19 is motivated by the fact that when the initial power spectrum is normalized in the proper way (i.e. using CMB observations at a redshift close to recombination), MOND over-predicts the normalization at redshift \( z = 0 \) by around one order of magnitude (see, for instance, results in section 5.2 or Nusser (2002)). A possible way to go around the problem is to graduate the intensity of the MOND effect by giving this explicit dependence of \( a_0 \) with \( a \) (see appendix 6.A for an alternative solution). During this chapter the experimentation will be made with a power law with the form:

\[
a_0(a) = a^{3/2} a_0(a = 1).
\]

(6.21)

Note that the power proposed here is higher than the one used in section 5.2. Thus, a lower normalization for the density perturbations should be obtained at redshift \( z = 0 \).

The equations of motion for the particles in an expanding context are:

\[
\begin{align*}
\dot{x}^i &= \frac{p^i}{a^2} \\
\dot{\tilde{x}}^i &= \frac{\tilde{p}^i}{a^2} \\
\dot{p}^i &= -\frac{\partial \phi}{\partial x^i} \\
\dot{\tilde{p}}^i &= -\frac{\partial \tilde{\phi}}{\partial \tilde{x}^i}
\end{align*}
\]

(6.22)

for the matter and twin components (see discussion on this topic at the beginning of section 2.1.1).

6.3 The code

The code used for the simulations that will be described in next section is the particle mesh code presented in section 5.2.2. In order to run simulations under the twin matter formalism, the code was modified in a way that it reads two sets of initial conditions for the particles, which will correspond to the matter and twin matter components. All the quantities associated to the matter component (densities, potentials, positions and velocities) were duplicated to take into account the twin component. The coupling between the two field equations (eqs.6.18 and 6.19) was also implemented as well as the new definition for the source term, which is now a function of perturbations instead of overdensities. The expansion is given by only one of the metrics. As the simulations were run assuming equal quantities of matter and twin matter, it is irrelevant at this moment which component is chosen.
6.4: The runs

The aim of the chapter is to investigate the modifications that are induced by the presence of a twin component over the MONDian non-linear evolution. In other words, the
Table 6.1: Parameters corresponding to the two sets of initial conditions. In all cases a value for the Hubble parameter of $h = 0.75$ was employed. The column $z_{\text{init}}$ shows the initial redshift and $\rho_{\text{max}}$ is the maximum linear overdensity on a grid of 256 nodes per dimension. The last column specifies the gravitational model used during the linear evolution. See text for explanation.

<table>
<thead>
<tr>
<th>label</th>
<th>$\Omega_b$</th>
<th>$\Omega_{dm}$</th>
<th>$\Omega_{\Lambda}$</th>
<th>$z_{\text{init}}$</th>
<th>$\rho_{\text{max}}$</th>
<th>Linear evolution</th>
</tr>
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<td>0.26</td>
<td>0.7</td>
<td>52.24</td>
<td>0.89</td>
<td>Newton</td>
</tr>
<tr>
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<td>4.93</td>
<td>0.1</td>
<td>BM-MOND</td>
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Table 6.2: Model parameters used for the simulations. The initial conditions are defined in table 6.1. See text for explanation.

<table>
<thead>
<tr>
<th>label</th>
<th>Ics</th>
<th>$\zeta$</th>
<th>Gravitational model</th>
</tr>
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<tr>
<td>$\Lambda CDM$</td>
<td>A</td>
<td>-</td>
<td>Newtonian</td>
</tr>
<tr>
<td>$\Lambda$ CDM</td>
<td>B</td>
<td>-</td>
<td>Bekenstein-Milgrom equation</td>
</tr>
<tr>
<td>Twin-0</td>
<td>B and B rotated</td>
<td>0</td>
<td>Twin formalism</td>
</tr>
<tr>
<td>Twin-1/2</td>
<td>B and B rotated</td>
<td>1/2</td>
<td>Twin formalism</td>
</tr>
</tbody>
</table>

evolution described by the Bekenstein-Milgrom (BM) equation:

$$\nabla \cdot \left[ \mu \left( \frac{\nabla \phi}{a a_0(a)} \right) \nabla \phi \right] = \frac{4\pi G}{a} \delta \rho$$

(6.23)

will be compared to the one predicted by the twin matter formalism defined by eqs.6.18 and 6.19. A set of four simulations was run, whose parameters and those of their initial conditions are summarized in tables 6.1 and 6.2. There is a Newtonian and a MONDian simulation that were used as reference. The two twin simulations, Twin-0 and Twin-1/2, evolved the same set of initial conditions as the MONDian one, but taking into account the presence of a twin component which was generated as a spatial rotation of the matter component. The difference between this two twin simulations resides in the value used for the coupling constant $\zeta$. The box size is the same for all the runs (128 Mpc/h) and 256$^3$ particles were employed for each set of initial conditions. Thus, the runs using the twin formulation will evolve 2 $\times$ 256$^3$ particles. The $\mu$ function employed is the simple version (eq.1.62) in all the cases.

In total, two sets of initial conditions were computed. The initial power spectrum was obtained using the code CMBFAST (Seljak & Zaldarriaga 1996) at redshift $z = 1000$ and 500 for the set A and B respectively. The reason for using a lower value for the model B is that the matter-radiation equality epoch of the associated cosmological model is at a redshift of around 700. In order to make a fair comparison between Newtonian and MONDian simulations, the same type of normalization must be employed. As the MONDian linear evolution is still not well understood, it is safer to avoid a normalization based on the linearly extrapolated value of $\sigma_8$ at redshift $z = 0$. Thus, the two sets of initial conditions were normalized according to CMB values.

Also with the aim of running all the simulations under the same circumstances, the two sets of initial conditions were generated using the same technique, which is the one
described in section 5.2.2. The input power spectra used for the Newtonian case (model A in table 6.2) were obtained with the analytic solutions of the standard growth equation. For the MONDian initial conditions (labeled as B in table 6.2), the self consistent solutions of the standard growth equation coupled to the MOND field equation were employed (see section 5.2.4).

Regarding the twin simulations, initial conditions for matter and twin matter must be provided. It is not possible to use the same set for both components because this will give a null source in equation 6.19 and hence, no MOND effect will be obtained. For the matter component, the model B was employed. The twin component could be obtained generating another set of initial conditions as B, but with different initial seed for the random numbers generator. Nevertheless, there is a simpler alternative, that consist in using the same distribution of particles but making a spatial rotation of it. Consequently, in order to obtain initial conditions for the twin component, the particle distribution that corresponds to the model B was rotated according to the following Euler angles:

\[
\begin{align*}
\phi &= \frac{\pi}{2} \\
\theta &= -\frac{\pi}{2} \\
\psi &= 0
\end{align*}
\] (6.24)

(in the convention of Goldstein (1950)), which corresponds to the following permutation of the coordinates \((y, z, x)\). Naturally, the velocities were also rotated. It is interesting to note that, since the same particle distribution is employed, the power spectrum of the matter and twin components is exactly the same. The required modifications for the twin components appear only in the phase space through the rotation, but the properties in the integrated Fourier space are the same. This simplifies the relation between the two sets of initial conditions and permits of make a better interpretation of the final result of the simulations. In other words, any difference between the two components that could appear in Fourier space at redshift \(z = 0\) will be originated by the interaction between the two fluids and not as a result of cosmic variance.

The initial redshifts of the two set of initial conditions is shown in the fifth column on table 6.1. The values chosen for the MONDian runs could seem too small when compared with the values commonly used. The reason to employ such low values can be seen in the last column on the same table, where the maximum linear overdensity is shown at the initial redshift as given on a grid with 256 nodes per dimension, which is the same grid that will be used during the simulations. As the linear power spectrum in the MONDian case has a very steep slope of around -4, it is necessary to go that far to reach overdensities of the order of 1. To increase the redshift of this initial conditions will give a set of particles whose associated density is dominated by rounding errors. Naturally, to add more particles will permit to start at higher redshits, but it will also increase considerably the computing time needed to run the simulations.

### 6.5 Results

The simulations will be analysed in two different ways. In first place the evolution between Newtonian gravity and standard BM-MOND will be compared and taken as a reference point. Since the time dependence on \(a_0\) employed in this simulations (eq.6.21) is different with respect those used in previous chapters, this comparison is interesting in itself. In second place, the effects of the presence of the twin component over the MONDian evolution will be studied.
A first clue of the modifications that the presence of a twin component induce on the evolution of matter can be obtained before to run any simulation. The twin field equations in the cosmological case are eqs.6.18 and 6.19. In the simplest model given by $\zeta = 0$, the coupling between the two metrics in eq.6.6 disappears and the solution of eq.6.19 is the responsible for the movement of the matter particles. This equation differs with the one used in the standard version of MOND (eq.6.23) only in the fact that the perturbation on the twin component is subtracted to the perturbation in the matter component. As the interaction between this two components is repulsive, it is likely that overdense regions on the matter component will correspond to underdense regions on the twin component. Consequently, the absolute value of the source of the field equation (eq.6.19) will increase. This will increase the forces and hence, it will lead to a faster evolution. In other words, in the case that the initial conditions for the MONDian and twin simulations are the same, the twin simulation should show a farther state of evolution at redshift $z = 0$. The amplitude of this effect can be measured only by means of the simulations.

The other question that we want to consider is the following one: the last paragraph showed that the presence of a twin component should increase the MOND effect. Can this help to solve the problem found in previous section regarding the lack of power at large frequencies by giving a gentler slope to the power spectrum?

Final normalization

The most straightforward measurement of the state of evolution is the variance of the density perturbations $\sigma_8^2$. This quantity was calculated from the particle distributions by taking the mean value over a large number of spheres with random positions and a radius of 8 Mpc/h of the squared of the enclosed perturbed mass:

$$\sigma_8^2 = \frac{1}{n} \sum_s \delta m_{8,s}^2,$$

where $\delta m_{8,s}$ is the perturbed mass on each individual sphere and $n$ is the number of spheres. The value of $\delta m_{8,s}$ was calculated by estimating the density perturbations on a uniform grid by means of equation 1.49 and using the following expression:

$$\delta m_{8,s} = \frac{\rho}{M_S} \sum_{i \in s} \delta \rho,$$

where $\delta \rho$ is the perturbation on the density obtained from the grid and the index $i$ runs over all the nodes that are contained on the sphere $s$. The quantity $M_S$ is the mass of

<table>
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<th>$\sigma_{8,m}$</th>
<th>$\sigma_{8,tm}$</th>
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<tr>
<td>$\Lambda$CDM</td>
<td>1.16</td>
<td>-</td>
</tr>
<tr>
<td>MOND</td>
<td>1.24</td>
<td>-</td>
</tr>
<tr>
<td>Twin-0</td>
<td>1.79</td>
<td>1.84</td>
</tr>
<tr>
<td>Twin-1/2</td>
<td>1.81</td>
<td>1.87</td>
</tr>
</tbody>
</table>

Table 6.3: Model parameters used for the generation of the initial fluctuation spectra. In all cases a value for the Hubble parameter of $h = 0.75$ was employed.
an homogeneous sphere of 8 Mpc/h, which is given by:

\[ M_8 = \frac{4}{3} \pi \rho_0 8^3, \quad (6.27) \]

where \( \rho_0 \) is the background density defined by each particular cosmological model. Table 6.3 summarize the values obtained from the simulations. The two measurements given for the twin simulations correspond to the values for matter and twin matter components respectively.

The Newtonian simulations exceed the observed value in around 20 percent. This is due to the fact that the initial conditions were not normalized to force a particular value at redshift \( z = 0 \) and that pressure effects were neglected, resulting on an overestimation of \( \sigma_8 \). Cosmic variance should be taken also as responsible for this discrepancy, but taken into account the size of the box, it should be a minor effect. In the standard way of normalizing, all this effects are hidden because the normalization is defined at redshift \( z = 0 \) instead of \( z = 1000 \) as we did here. Far from being a problem, this is the reason why this special normalization was used for the Newtonian simulations. To use the standard normalization for the Newtonian run, will give spurious differences with respect to the MONDian simulations, which, as was mentioned before, can not be normalized at redshift \( z = 0 \). It is important to emphasize that the aim of the chapter is to make a relative comparison between simulations and that the observed values, while very important to fix the framework, should not be taken as rigid.

Owning to the time dependence employed for \( a_0 \), the value of \( \sigma_8 \) for the MONDian run approaches the Newtonian value. The extreme difference found in previous chapter, is reduced here to a 6%. Regarding the twin runs, the expected over-prediction on the normalization that was described in previous paragraph was found to be of the order of 40%. The twin matter component has consistent values with those of the matter component. This shows that both components are tightly coupled. No major differences
were found when changing the coupling constant $\zeta$, which gives an indication that this is not a critical parameter, at least in the region of the power spectrum tested with the box size and resolution employed here.

Figure 6.2 shows the time dependence of $\sigma_8$ for the run MOND and the matter component of the run Twin-1/2. Both models evolve together from the initial redshift to a redshift of about $z = 3$, when the differences described before between the two formalism start to be important and the perturbations in the matter that is evolved under the effects of the twin component start to grow faster.

**Power spectrum**

Fig.6.3 shows the final power spectrum at redshift $z = 0$ for the runs $\Lambda$CDM and MOND. The dots are the power spectrum obtained from SDSS data (Tegmark et al. 2004) and the thin line is the Newtonian linear power spectrum as given by the code CMBFAST. The Newtonian simulation has the typical excess of power on small scales related to the fact that pressure was neglected. The MONDian curve can be directly compared with the data presented in Fig.5.19, which shows results from another simulation that was run also with MOND. The box size is the same for both simulations and the initial conditions were generated also in the same way, using the same linear theory. The only difference between this two MONDian simulations is the time dependence in $a_0$. In both cases a power law form was used, but here the exponent is larger. A strong problem is found related to this time dependence. While previous paragraph shows that the time dependence employed here solves the problem with the final normalization, it gives a very steep slope to the power spectrum, preventing MOND to for galaxy sized objects. In other words, the prescription used for MOND in this simulations, which consist basically on field equations derived through Newtonian arguments plus initial conditions generated with standard linear theory coupled to MOND, can be forced to give the right final normalization, but the solution proposed gives the wrong slope of the power spectrum.
Solving problems on one side of the frequency domain of the power spectrum increase the problems on the other side.

Note that this problems were not found in the simulations described in chapter 2 because in that case the initial power spectrum had a power law form that was obtained by adding a dark matter component. The simulations presented here (an in previous chapter) do not include dark matter and hence, MOND is the only way that we have to increase the power on small scales that was suppressed due to Silk dumping on the early universe. The addition neutrino type dark matter which is under discussion to improve the fitting on clusters and at the CMB will not solve this problems. The free streaming of neutrinos give a cut to the initial power spectrum similar to the one that we have here. Consequently MOND will be responsible to form galaxies also in that context. Appendix will 6.A propose a different approach to solve this difficulties that should be carefully studied in the future. The MOND theory employed in the simulation presented in Fig.5.19 is the quMOND version. Intensive testing made when running this simulations show that the differences found here in the slope of the power spectrum are due to the time dependence in $a_0$ and do not depend much on the underlying MOND theory.

Fig.6.4 shows the power spectra of matter and twin matter components at redshift $z = 0$ for the two twin runs (Twin-0 and Twin-1/2). The excess in $\sigma_8$ found previously is not concentrated in a particular range of frequencies. The twin spectra are translated up without presenting large differences in its dependence with frequency. In previous section it was mentioned that the initial conditions for the twin component were specially designed to have the same properties in Fourier space that the matter component. The plot show that the interaction between the two fluids does not have any effect in this sense. While the normalization of the curves corresponding to the twin simulations is higher, the relation between matter and twin matter curves does not change at all: all the curves lie in top of each other. In other words, the interaction between the two fluids
Figure 6.5: Particle’s distributions at redshift 0 for the simulations MOND (upper row) and ΛCDM (lower row). In the MONDian case, two different projections of the same box are shown. See text for explanation.
does not induce transfer of power between different frequencies. As in previous paragraph, the coupling constant $\zeta$ does not show to have a large impact on the evolution.

**Snapshots at redshift $z = 0$**

Fig.6.5 shows the particle distributions at redshift $z = 0$ of all the simulations. The different gray levels are related to the density. A trained eye will see that in the Newtonian case (lower panel), the size of the objects is too big. This is related to the fact that the simulation was run without using refinements and hence the potential wells are not deep enough inside the objects to keep the particles concentrated. One should keep in mind that the simulations were not designed to study the properties of collapsed objects. The aim of the simulations is to make for first time a run that tracks the two metrics, to expose the technical issues and to give a reference point to more elaborated simulations that should be made with bigger codes such as Ramses.

The MOND simulation is shown in the upper row in two different projections. The left panel shows a projection on the plane $x-y$, while the right one shows the same plane after making the permutation implemented to generate the initial conditions for the twin matter component (see eq.6.24). The aim of presenting the data in this way is to facilitate the comparison with the twin model that is shown in the second row. The general patterns on the simulation MOND are the same that for the Newtonian case: a web of filaments defining voids and containing objects in their intersections. The panels can be compared to the left panel on Fig.5.15, which shows the particle distribution for a MONDian simulation made with the same box size but a weaker time dependence on $a_0$. The size of the objects and width of the filaments in Fig.6.5 is much smaller, showing that the normalization of the power spectrum is better using this time dependence. Nevertheless, the lack of small structure discussed in previous paragraph is evident in a catastrophic way.

The middle row shows matter (left) and twin matter (right) components of the model Twin-1/2, also at redshift $z = 0$. While there are differences with respect to the corresponding snapshots in the MOND simulation, the lack of small structure make difficult to say anything about the interaction between the two components, especially the idea proposed in the introduction that the objects in one component will lie in the voids of the other and vice versa. The problem with the normalization of the power spectrum must be solved in a better way before to discuss this kind of phenomena.

**Distribution of the ratio between densities**

While the snapshots do not show large differences between the positions of the objects when a matter distribution is evolved with standard MOND or under the effects of a twin component, it is still interesting to associate a number to this interaction and see if it exists at all. A possible way to measure the interaction between the two fluids is through the distribution of the ratio between matter and twin matter for the twin simulations and to compare it with the distribution of the ratio of the MOND simulation and the spatial rotation of if used to generate twin initial conditions. In case that no interaction is present at all, both distributions should be the same. Fig.6.6 presents the result at redshift $z = 0$ as obtained when the densities are calculated in a grid of 64 nodes per dimension. The distributions are symmetric in logarithmic space, showing a form close to a log-normal distribution.
A clear difference exist between the histograms. In the MOND case, as the two components do not see each other, the width of the histogram is basically a measurement of the cosmic variance that is produced by the rotation. The curve corresponding to the twin run presents larger winds, which shows that the interaction exist and that both components are avoiding each other. The lower side of the distribution corresponds to voids of matter. The broadening can be understood in this case by the fact that in the twin formalism, more empty spaces in the matter component are hosting more collapse objects of the twin component. With an analog reasoning, one can see that the higher side of the domain of the distribution corresponds to voids on the twin component, which are dominantly populated by collapsed matter objects (in opposition to matter voids).

There is one important point that should be taken into account. It was discussed that the twin simulations present a further evolutionary state at redshift $z = 0$. In order to confirm the result presented here, the distributions should be calculated at a moment in which both simulations (MOND and Twin) have same state of evolution. To this end, the histogram corresponding to the twin simulation was recalculated at redshift $z = 0.33$, which corresponds to a moment in which $\sigma_8$ for that simulation is equal to the one of the MONDian simulation (see fig.6.2). The result is presented also in Fig.6.6. The histogram is slightly more concentrated that the curve at redshift $z = 0$, but it is still broader that the one corresponding to the MOND simulation. This shows that the interaction exist and can be measured in simulations. In order to calculate the probability to find a twin matter halo in the surroundings of the Milky Way that could be eventually observed better simulations must be made.

### 6.6 Conclusions

A new prescription for MOND was proposed recently in the context of bi-metric theories for gravity (Milgrom 2009a). Furthermore, the second metric was interpreted in Milgrom
(2010a) as associated to a second matter component, which interacts with the standard matter component through a repulsive gravitational force. The aim of the chapter was to test the differences in cosmological non-linear evolution that exist between the standard formulation of MOND and this new one.

After a brief description of the equations that constitute the new theory, a set of equations to be used in the context of cosmology (that were not provided in the original paper) was constructed. The procedure employed to get the cosmological equations is the same that was employed in previous chapters for other formulations of MOND: the equations given in a Minkowsky background were translated to a cosmological context by means of a change of variables to the comoving frame.

In order to test the consequences of adding a twin component on non-linear evolution a set of simulations was run, which includes a Newtonian and MONDian simulation made as comparison and two twin simulations made with different coupling constants. A new time dependence on $a_0$ was tested to solve problems on the final normalization of density perturbations found for MOND in previous chapter. It was found that, while this helps in solving the problem that exist with the final normalization of the density perturbations, it greatly suppress the presence of small structure, avoiding the formation of galaxies. A better solution must be provided to the normalization problem. Regarding the evolution under the twin formalism, it was found that both components evolve together and the new formalism over-predicts the final normalization of the power spectrum in around 45% with respect to standard MOND treated under the same conditions.

The interaction between the two components was quantified by means of the distribution of the ratio between matter and twin matter components and analog distributions for the MONDian run. It was found that the interaction exist, but due to the lack of small structure, it is still difficult to give predictions about the number of objects in the twin component that should be interacting the the matter objects. The results presented in this chapter show that the twin formalism does not help on solving the problems found for MOND in cosmological evolution. Before to study in more detail the viability of the twin matter formalism, a solution for the cosmological problems for MOND must be found. Appendix 6.A provides a possible solution that should be taken into account for future work.

### Appendix 6.A Is this the end of the story? Non-standard $\mu$ functions as solution to cosmological problems for MOND

The results presented in this chapter and in section 5.2 seems to be very conclusive. When a realistic power spectrum is used at the CMB, MOND has a severe problem: there is an over-prediction of the power at large scales at redshift $z = 0$ and at the same time, a lack of small structure. A possible cure for the large scales problem is to give a time dependence to $a_0$, but it was found that while useful to bring down the normalization, this increments the problem on small scales. Next chapter will propose possible solutions that should be investigated in future work. This section will test one of this solutions with promising results.

The new solution that we propose here consist in changing the asymptotic behavior of
the $\mu$ function. While there are many forms of the interpolating function in the literature, all of them have in common their asymptotic definition:

$$\mu(x) = \begin{cases} 
  x & \text{if } x \ll 1 \Rightarrow \text{Galaxies/cosmology} \\
  1 & \text{if } x \gg 1 \Rightarrow \text{Solar system}
\end{cases}$$ (6.28)

Nevertheless, this function was still not measured in all its domain. The lowest accelerations observed on galaxies are those on low surface brightness (LSB) galaxies; the values are of the order of $0.1 \times a_0$ (e.g. Swaters et al. 2010). The behavior of the $\mu$ function for lower accelerations, as those present on cosmological scales, is still open to discussion. Here, a different $\mu$ function is proposed, which conserves the standard definition on galaxies, but has a different asymptotic definition:

$$\mu(x) = \begin{cases} 
  1 & \text{if } x \ll 1 \Rightarrow \text{Cosmology} \\
  x^n & \text{if } x \sim 1 \Rightarrow \text{Galaxies} \\
  1 & \text{if } x \gg 1 \Rightarrow \text{Solar system}
\end{cases}$$ (6.29)

Three accelerations regimes are defined. For very low and very high accelerations (with respect to $a_0$), the theory behaves as the Newtonian one. The MOND effect appears only for the intermediate accelerations regime. Additionally, a freedom was given to the exponent on the MONDian regime. The case $n = 1$ is the original definition of MOND, which predicts the Tully-Fisher relation. The idea behind this new function is that at very high redshift, the accelerations will lie in the intermediate regime and thus MOND will permit to rise the perturbations that were dumped by Silk dumping at small scales (assuming a universe populated only with baryons). At later epochs, the accelerations will drop (see for instance section 5.1.3) and will reach the lower Newtonian regime, which will prevent the excess of collapse previously found. At the same time, the galaxies will
6.A: Is this the end of the story? Non-standard $\mu$ functions

<table>
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<th>$\Omega_{dm}$</th>
<th>$\Omega_\Lambda$</th>
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</thead>
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<td>0.26</td>
<td>0.7</td>
</tr>
<tr>
<td>MOND</td>
<td>0.04</td>
<td>0.0</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Table 6.A1: Parameters used for the linear runs made to test the new $\mu$ function. In the two cases a value for the Hubble parameter of $h = 0.75$ was employed.

still lie in the intermediate regime and thus, their rotation curves will be dictated by MOND.

A rising $\mu$ function towards low accelerations could seem dangerous because of unicity problems (the function is not invertible). However, one should not forget that what is needed to be invertible is the relation between the forces. In consequence, the function that must be invertible is not $\mu$ but $x\mu$. This gives the necessary freedom to define the $\mu$ function as in eq.6.29.

There are already indications in the literature that the $\mu$ function should be such that Newtonian gravity is recovered at very low accelerations. For instance, Hoekstra et al. (2001) have found by means of weak lensing analysis that the dark matter halos should be truncated at a finite radius. Famaey et al. (2007) gave theoretical motivations in the context of TeVeS. Furthermore, the fittings of rotations curves of LSBs show increasing residuals towards $g \lesssim 0.1a_0$ (Swaters et al. 2010), which is also and indication that the predicted MOND effect is too strong at low accelerations.

A simple reasoning in the context of cosmology gives one more motivation for introducing a new type of $\mu$ function. In standard MOND (defined by eq.6.28), it is commonly assumed that an homogeneous universe does not experience MOND effects. In that case, Newtonian physics is valid and the expansion is given by the standard Friedmann equations. The problem is that, it is enough to add a very tiny perturbation to the universe for it to suddenly fall into the deep MOND regime. This sets a discontinuous transition between two very different descriptions that seems difficult to be interpreted. When a $\mu$ function as in eq.6.29 is employed, a very slightly perturbed universe is not subjected to MOND effects until the perturbations are large enough to reach the MOND regime from the lower side. This gives a continuous transition from an unperturbed universe regulated by Newton (Friedmann equations) to a perturbed one dominated by MOND.

A toy model for this kind of function is the following:

$$
\mu(x) = \begin{cases} 
1 & \text{if } x < a \Rightarrow \text{Cosmology} \\
Bx^{1/5} & \text{if } x \in (a, b) \Rightarrow \text{Galaxies} \\
\frac{1}{2}x^{7/4} & \text{if } x \in (b, c) \Rightarrow \text{Galaxies} \\
\frac{1}{2}x^{3/2} & \text{if } x \in (c, d) \Rightarrow \text{Galaxies} \\
1 & \text{if } x > d \Rightarrow \text{Solar system}
\end{cases}
$$

(6.30)

where $a = .095 < b < c < d$ and the values of $b$, $c$, $d$ and $B$ can be determined asking to the function to be continuous. The values of the exponents and the constant $a$ were determined empirically. The powers $7/4$ and $3/2$ were chosen to emulate the simple $\mu$ function given by:

$$
\mu(x) = \frac{x}{1 + x}
$$

(6.31)
for values of $x$ greater than $b$ (a broken function was employed for simplicity during the calculations). The part of the function with logarithmic slope $1/5$ interpolates between the low accelerations Newtonian regime and the MONDian one given by the approximated simple $\mu$ function. While the function has non continuous derivatives, it is able to show the effect that is under discussion here. Better behaved functions from the point of view of Calculus should be studied in the future. Figure 6.A1 compares the relation between the Newtonian and MONDian forces for the new function and the simple one. The definition of the forces was made, only for the plot, using the simple MOND formula:

$$\mu \left( \frac{|g_{\text{MOND}}|}{a_0} \right) g_{\text{MOND}} = g_{\text{Newton}}.$$  \hspace{1cm} (6.32)

The vertical lines show the places were the new function is broken.

The viability of this new function in the context of cosmological evolution was tested in the linear regime by solving the standard growth equation coupled to the MOND equation. The equations were solved in a self consistent manner as proposed in section 5.2.4. The version of MOND used is given by the conservative quMOND formalism (Appendix D). No time dependence in $a_0$ was employed. This means that only one $a$ factor appears in the argument of the $\mu$ function, which is related to the change of variables to the comoving frame. In other words, the MONDian potential was defined as given by eqs.D.6 and D.7 with no further changes.

Two linear runs were made with Newtonian and MONDian models. The parameters are summarized in table 6.A1. A box of 128 Mpc/h was employed with a grid with 128 nodes per dimension. The model $\Lambda$CDM was run with standard gravity, while the MOND run was made using quMOND with the new $\mu$ function. The time step is uniform in $a$ space and was calculated assuming 500 time steps between the initial redshift ($z=500$) and $z=0$. Section 5.2.4 has shown that this number of time steps gives a slight underestimation of the final power spectrum. As the Newtonian model used for comparison was run with the same number of time steps, the bias will be in both runs.
and thus, they can be directly compared. In any case, this bias is much smaller than the effect that we want to measure. The initial power spectrum was calculated in both cases using the code CMBFAST (Seljak & Zaldarriaga 1996). The normalization was made at $z = 500$, with no reference to the values at $z = 0$. In other words, the models are not forced to give the same normalization at $z = 0$.

Figure 6.A2 shows the power spectrum for the two models at different redshifts. The Newtonian model that the typical scale free evolution, presenting only changes in the normalization. The MONDian model starts with a very steep power spectrum which is a consequence of the Silk damping acting during the radiation dominated era. Initially the universe is in the MOND regime, so MOND can bring the power spectrum close to a power law form. At late times, when the large scales accelerations drop, the scales corresponding to $k \sim 0.1$ fall into the lower Newtonian regime and evolve parallel to the Newtonian model, preventing the excess on the final normalization.

While the function presented here is a toy model, with discontinuous derivative, it shows that to add a Newtonian regime at very low accelerations is a very promising route to solve the problems on cosmological scales for MOND. Future observational studies should be directed to give constrains on the form of the $\mu$ function, not on galaxies but at much lower accelerations, in the context of voids.