Gas–Liquid Mass Transfer Coefficient in Stirred Tank Reactors

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Knowledge of gas–liquid mass transfer rate and therefore volumetric gas–liquid mass transfer coefficient (kₐ) is very important in designing or in the scale-up of gas–liquid stirred tank reactors (STR). In the literature kₐ is correlated either by using dimensionless groups (Smith and Wamioeskerken, 1985; Smith, 1991) or energy input criterion (Smith et al., 1977; van’t Riet, 1979; Hickman, 1988; Whitton and Nienow, 1993) based on Kolmogoroff’s theory. Correlations based on dimensionless groups are of the form kₐ = f(Fr, Frᵢ, D/T, etc.) while correlations based on energy input criterion are of the form kₐ = f(P/V)ᵢ (V)ᵢ⁰.

In our earlier work (Yawalkar et al., 2002), it has been shown that gas hold-up (eᵢ) in STR can be reliably predicted by a correlation based on a relative dispersion parameter, N/Nₐₑ over wide range of geometric conditions and operating parameters. Gas hold-up data available in the literature (Greaves and Barigou, 1990; Smith, 1991; Rewatkar et al., 1993) for larger tanks (T = 0.57 m to 2.7 m) equipped with either radial flow impeller (standard six-blade disc turbines (DTs)) or axial flow impellers (pitched bladed turbines (PBTs)) of different sizes and different sparger designs show unanimity with the proposed correlation.

The present work it has been shown that similar to gas hold-up (eᵢ), volumetric gas–liquid mass transfer coefficient (kₐ) can be correlated reliably over a wide range of geometric configurations and operating parameters (impeller speed/power input and gas rate) by a correlation based on the relative dispersion term N/Nₐₑ.

Nₐₑ represents the minimum impeller speed at which all the liquid in a tank is in contact with the sparged gas. Extensive studies are available on Nₐₑ in the literature (Nienow et al., 1977; Chapman et al., 1983; Rewatkar and Joshi, 1993). For any economical gas–liquid mixing operation in STR, minimum impeller speed for complete dispersion of the gas (Nₐₑ) must be known, because at Nₐₑ complete dispersion of the gas in liquid phase is obtained (Nienow et al., 1977; Chapman et al., 1983). Also, the mixing process at any impeller speed below Nₐₑ results in incomplete dispersion of the gas. In other words, at an impeller speed below Nₐₑ, there is very little or no gas in the region below the impeller. Therefore, the lower part of the tank is wasted, resulting in poor performance of the STR.

If the gassed power number (NₑP) or the ratio of power numbers under gassed conditions and ungassed condition (NₑP/Nₑ₀) versus the flow number (Frₑ) curve is plotted, a minimum occurs in the gassed power number at Nₑₑ (Nienow et al., 1977; Bakker and van den Akker, 1994) (Figure 1). At Nₑₑ the growing gas cavity covers the whole of the rear face of the blade, which causes drop in power input (Chapman et al., 1983). Studying the large amount of data generated from the experimental work incorporating wide range of system configurations (flat/dished bottom tanks with T = 0.29 to 1.83 m, D/T = 0.125 to 0.5, C/T = 0.25 to 0.5, ring spargers, pipe spargers and point spargers) and operating

Volumetric gas–liquid mass transfer coefficient (kₐ) data available in the literature for larger tanks (T = 0.39 m to 2.7 m) have been analyzed on the basis of relative dispersion parameter, N/Nₑₑ. It was observed that at a given superficial gas velocity (Vₑ), kₐ values were approximately the same irrespective of geometric configuration (size of the tank, type and size of the impellers, type of the sparger, etc.) at a particular N/Nₑₑ. A single correlation based on N/Nₑₑ is presented which shows satisfactory agreement with the kₐ data of different workers.

Les données sur le coefficient de transfert de matière gaz–liquide volumétrique (kₐ) fournies par la littérature scientifique pour des réservoirs de grande taille (T = 0.39 m à 2,7 m), ont été analysées relativement au paramètre de dispersion relatif, N/Nₑₑ. On a observé qu’à une vitesse de gaz superficielle donnée (Vₑ), les valeurs de kLa sont approximativement les mêmes peu importe la configuration géométrique (taille du réservoir, type et taille des turbines, type d’aérateur, etc.) pour un N/Nₑₑ donné. On présente une corrélation simple basée sur N/Nₑₑ qui montre un accord satisfaisant avec les données de kₐ de différents chercheurs.

Keywords: STR, kₐ, Nₑₑ, relative dispersion (N/Nₑₑ), Vₑ.
conditions \((P/V_L\text{ up to } 8200 \text{ W/m}^3 \text{ and } V_G = 0.2 \text{ to } 4.6 \times 10^{-2} \text{ m/s})\), Nienow et al. (1977) have given a correlation for \(N_{cd}\) in an STR equipped with a standard six-blade disc turbine (DT).

The correlation proposed by Nienow et al. (1977) for DT is:

\[
N_{cd} = \frac{4(Q_c)^{0.5}(r)^{0.25}}{(D)^2}.
\]

Equation (1) is very reliable for predicting \(N_{cd}\) for disc turbines (Yawalkar et al., 2002). Chapman et al. (1983) studied \(N_{cd}\) for different impellers such as four-blade mixed flow impellers pumping up (MFUs) and pumping down (MFDs), six-blade MFUs and MFDs, axial flow impellers pumping up (AFUs) and pumping down (AFDs), disc turbines (DTs) and angular-bladed disc turbines (ADTs). Rewatkar and Joshi (1993) have extensively studied \(N_{cd}\) in an STR equipped with six-blade downflow pitched turbines. \(N_{cd}\) can be determined by simple methods such as visual observation, from the power input curve, and from mixing time studies (Rewatkar and Joshi, 1993).
bubbles break up when the hydrodynamic stresses (Reynolds et al., 1959; Hinze, 1955; Hughmark 1974; Miller, 1974). The drag on the bubble and hence the bubble size (Calderbank 1958).

Turbulent fluctuations in the gas–liquid dispersion control the relative gas dispersion (Joshi, 1993). The last two are objective methods but show good agreement with the first, which is subjective (Rewatkar and Joshi, 1993). The last two are objective methods but show good agreement with the first, which is subjective (Rewatkar and Joshi, 1993).

Relative Gas Dispersion ($N/N_{cd}$)

Turbulent fluctuations in the gas–liquid dispersion control the drag on the bubble and hence the bubble size (Calderbank et al., 1959; Hinze, 1955; Hughmark 1974; Miller, 1974). The bubbles break up when the hydrodynamic stresses (Reynolds stresses) outweigh the surface tension force (Hinze, 1955; Calderbank et al., 1959; Walter and Blanch, 1986). Walter and Blanch (1986) gave the correlation of maximum stable bubble size on the assumption that bubble break-up is caused by the eddies of the same scale as the maximum stable bubble size. Parthasarathy and Ahmed (1991) have discussed the process of bubble break-up in a turbulent field using the criterion of power dissipation according to Kolmogoroff’s theory. Accordingly, they concluded that bubbles are broken by eddies of the inertial subrange.

The turbulence increases the drag on the bubbles (Hughmark, 1974) thereby reducing the bubble rise velocity. These bubbles with reduced rise velocity can be more easily entrained in the downward liquid flow generated by the impeller. Thus, the overall effect of increasing turbulence results in increasing gas hold-up ($e_g$) and therefore gas–liquid interfacial surface area ($a$) in STR.

At $N/N_{cd}$ complete dispersion of the sparged gas is achieved. Therefore, from the above discussion it can be concluded that at $N/N_{cd}$, the bubble size generated in the vicinity of the impeller has a buoyancy lesser than the downward drag caused by the downward liquid flow and the bubbles are pulled down in the lower

### Table 2. Correlations proposed by different workers to estimate volumetric gas–liquid mass transfer coefficient ($k_{La}$) in STR and used in the present work.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Correlation Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calderbank (1958), van’t Riet (1979)</td>
<td>$k_{La} = 0.026 (P/V)0.04(V_g)^{0.5}$</td>
</tr>
<tr>
<td>Smith et al. (1977)</td>
<td>$k_{La} = 0.01 (P/V)^{0.475}(V_g)^{0.4}$</td>
</tr>
<tr>
<td>Smith and Warmoeskerken (1985)</td>
<td>BLC regime: $k_{La} = 1.1 \times 10^{-7}(P)^{0.16}(Re_g)^{1.1} (N)$ ALC regime: $k_{La} = 1.6 \times 10^{-7}(P)^{0.42}(Re_g)^{0.02} (N)$</td>
</tr>
<tr>
<td>Linek et al. (1987)</td>
<td>$k_{La} = 4.95 \times 10^{-3} (P/V)^{0.593}(V_g)^{0.4}$</td>
</tr>
<tr>
<td>Hickman (1988)</td>
<td>For $T = 0.60 m$ $k_{La} = 0.043 (P/V)^{0.4}(V_g)^{0.57}$ For $T = 2 m$ $k_{La} = 0.027 (P/V)^{0.54}(V_g)^{0.68}$</td>
</tr>
<tr>
<td>Smith (1991)</td>
<td>$k_{La} = 1.25 \times 10^{-6}(D/T)^{2.8}$ $(P)^{0.04}(Re_g)^{0.7}$ $(P)^{0.435}(D/T)^{0.5}$</td>
</tr>
<tr>
<td>Whitton and Nienow (1993)</td>
<td>$k_{La} = 0.57 (P/m)^{0.4}(V_g)^{0.55}$</td>
</tr>
<tr>
<td>Zhu et al. (2001)</td>
<td>$k_{La} = 0.031 (P/V)^{0.4}(V_g)^{0.5}$</td>
</tr>
</tbody>
</table>

### Table 3. Hughmark’s (1980) correlation for impeller power input under aerated condition ($P_{O}/P_{D}$) for standard six bladed disc turbine is based on following experimental details.

<table>
<thead>
<tr>
<th>Tank diameter, $T$ (m)</th>
<th>0.21 to 3.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D/T$</td>
<td>0.33 to 0.576</td>
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<tr>
<td>$C/T$</td>
<td>0.33 to 0.67</td>
</tr>
<tr>
<td>$H/T$</td>
<td>0.75 to 1.87</td>
</tr>
<tr>
<td>$V_g$ (max.)</td>
<td>0.053 m/s</td>
</tr>
<tr>
<td>Blade width $(w), m$</td>
<td>0.2D</td>
</tr>
<tr>
<td>$\mu$ (mPa·s)</td>
<td>0.8 to 28</td>
</tr>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>870 to 1600</td>
</tr>
<tr>
<td>$\sigma$ (g/s$^2$)</td>
<td>25 to 72</td>
</tr>
<tr>
<td>$P_{O}/P_{D}$</td>
<td>0.31 to 0.8</td>
</tr>
</tbody>
</table>

### Table 4. Correlations obtained for the $k_{La}$ data of different workers based on relative dispersion parameter, $N/N_{cd}$.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Correlation based on the term, $N/N_{cd}$</th>
<th>$R^2$</th>
<th>Standard error</th>
<th>Number of data analyzed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calderbank (1958), van’t Riet (1979)</td>
<td>$k_{La} = 2.76 (N/N_{cd})^{1.14}(V_g)^{0.97}$</td>
<td>1</td>
<td>-</td>
<td>7</td>
</tr>
<tr>
<td>Smith et al. (1977)</td>
<td>$k_{La} = 2.42 (N/N_{cd})^{1.38}(V_g)^{0.96}$</td>
<td>0.98</td>
<td>0.07</td>
<td>41</td>
</tr>
<tr>
<td>Chandrasekharan and Calderbank (1981)</td>
<td>$k_{La} = 2.68 (N/N_{cd})^{1.13}(V_g)^{0.96}$</td>
<td>0.99</td>
<td>0.04</td>
<td>11</td>
</tr>
<tr>
<td>Smith and Warmoeskerken (1985)</td>
<td>$k_{La} = 12.63 (N/N_{cd})^{1.34}(V_g)^{1.27}$</td>
<td>0.99</td>
<td>0.02</td>
<td>15</td>
</tr>
<tr>
<td>Linek et al. (1987)</td>
<td>$k_{La} = 5.16 (N/N_{cd})^{1.69}(V_g)^{1.09}$</td>
<td>1</td>
<td>-</td>
<td>7</td>
</tr>
<tr>
<td>Hickman (1988)</td>
<td>$k_{La} = 4.28 (N/N_{cd})^{1.35}(V_g)^{1.04}$</td>
<td>0.97</td>
<td>0.12</td>
<td>21</td>
</tr>
<tr>
<td>Smith (1991)</td>
<td>$k_{La} = 6.48 (N/N_{cd})^{1.44}(V_g)^{1.12}$</td>
<td>0.99</td>
<td>0.02</td>
<td>32</td>
</tr>
<tr>
<td>Whitton and Nienow (1993)</td>
<td>$k_{La} = 3.50 (N/N_{cd})^{1.17}(V_g)^{1.0}$</td>
<td>0.99</td>
<td>0.02</td>
<td>13</td>
</tr>
<tr>
<td>Bakker and van den Akker (1994)</td>
<td>$k_{La} = 0.030 (N/N_{cd})^{1.42}$</td>
<td>0.93</td>
<td>0.10</td>
<td>16</td>
</tr>
<tr>
<td>Zhu et al. (2001)</td>
<td>$k_{La} = 3.31 (N/N_{cd})^{1.14}(V_g)^{0.97}$</td>
<td>1</td>
<td>-</td>
<td>15</td>
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</table>
approximately proportional to impeller speed $N$. Thus, $k_{La}/k_{La_d} = f(N/N_{cd})$. The ratio $N/N_{cd}$ represents relative dispersion of the gas and, therefore, relative volumetric gas–liquid mass transfer coefficient, $k_{La}/k_{La_d}$, in STR for a given system configuration and gas input rate.

As discussed in our earlier work (Yawalkar et al., 2002), $N/N_{cd}$ represents relative amount of gas retained in the liquid at impeller speed $N$ with respect to that at impeller speed $N_{cd}$. Therefore, it seems logical to define volumetric gas–liquid mass transfer coefficient ($k_{La}$) in terms of a relative dispersion parameter $N/N_{cd}$ (Jadhav and Pangarkar, 1991) have shown that particle–liquid mass transfer coefficient ($k_{Ld}$) data of different workers show unanimity when compared on the basis of

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Type of the impeller</th>
<th>$T$, m</th>
<th>$D/T$</th>
<th>$V_G \times 10^2$, m/s</th>
<th>$N_{cp}$, rev/s</th>
<th>$N/N_{cd}$</th>
<th>$P/V_\nu$, W/m$^3$</th>
<th>$k_{La} \times 10^2$, 1/s (observed)</th>
<th>$k_{La} \times 10^2$, 1/s ($k_{La_d} = f(u/u_{cd})$)</th>
<th>$k_{La} \times 10^2$, 1/s (Equation 4)</th>
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<tr>
<td>Smith and Warmoeskerken (1985)</td>
<td>6DT</td>
<td>0.46</td>
<td>0.81</td>
<td>3.76</td>
<td>0.89</td>
<td>1.72 (ALC)</td>
<td>3.11 (BLC)</td>
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<td>Smith (1991)</td>
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</table>
relative particle suspension parameter, \(N/N_{cd}\). \(N_{cd}\) is the minimum impeller speed for complete suspension of the particles.

In the present work, \(k_a\) data available in the literature for large STRs (\(T = 0.39\) m to 2.7 m) equipped with standard disc turbines for which \(N_{cd}\) values can be obtained from Equation (1) were studied. Also, \(k_a\) data for a lightnin A315 hydrofoil impeller and a pitched bladed turbine (PBT) for which \(N_{cd}\) values can be obtained from the power input versus flow number plots, provided by respective authors, were analyzed on the basis of \(N/N_{cd}\).

**Data Available on \(k_a\) in the Literature for Larger Stirred Tank Reactors and Used in the Present Study**

Table 1 summarizes the experimental details of the studies on \(k_a\) in larger STRs whose data have been used in the present work. Table 2 gives the correlations proposed by these workers for predicting \(k_a\) in an air–water system. \(k_a\) data of different workers were generated by applying the respective correlation strictly over respective geometric configuration and operating parameters used in that specific study.

Most of the workers correlated their \(k_a\) data either in terms of dimensionless groups or power input criterion. While Chandrasekharan and Calderbank (1981) and Bakker and van den Akker (1994) have given their \(k_a\) data for different geometric and operating conditions in tabular and graphical forms, respectively.

**Studies Based on Dimensionless Groups**

Plotting the Froude number \((Fr)\) versus gas flow number \((Fl_G)\) Smith and Warnaeskerken (1985) and Smith (1991) defined flow regimes in gas–liquid STRs, such as vortex clinging cavities regime (high \(Fr\) and low \(Fl_G\)), large cavities regime (high \(Fr\) and high \(Fl_G\)), flooded region (low \(Fr\) and high \(Fl_G\)) and gas recirculation regime (very high \(Fr\) and low \(Fl_G\)).

Smith and Warnaeskerken (1985) considered their gas hold-up data and \(k_a\) data separately in two groups: (i) clinging cavity regime or before large cavity (BLC) regime; and (ii) higher gas loading regime or after large cavity (ALC) regime in STR. Smith (1991) in his systematic study on gas dispersion behavior, studied gas hold-up behavior and volumetric gas–liquid mass transfer rates in STRs equipped with DT. Smith (1991) observed that his \(k_a\) data for \(T = 0.60\) m, 2.4 m and 2.7 m can be correlated satisfactorily by a correlation based on dimensionless groups.

The \(N/N_{cd}\) versus \(k_a\) data for both these studies were directly derived by applying the correlations over specific range of geometric configurations and operating parameters for a respective study.

**Studies Based on Power Input Criterion**

Smith et al. (1977) studied \(k_a\) in large STRs, \(T = 0.60, 0.91\) and 1.83 m equipped with different impellers with majority of the data based on DT of different sizes. Their correlation for air–water system is of the form \(k_a = f\left(P/V_{L}^a\right)\left(V_{G}^{b}\right)\) with maximum deviation of \(\pm 20\%\). Since reliable correlation for \(N_{cd}\) for such large-scale STRs equipped with DT is available (Equation 1), \(k_a\) data of Smith et al. (1977) for DTs could be converted to the \(N/N_{cd}\) form.

van’t Riet (1979) carefully studied the available data on \(k_a\) and indicated that correlation of data from various reports on mass transfer for disc turbines is possible with power input per unit volume and gas superficial velocity. In his correlation treatment for coalescing media (air–water) van’t Riet (1979) used large tank data from the work of Calderbank (1958) and Smith et al. (1977). In this work \(k_a\) data of Calderbank (1958) were obtained using van’t Riet’s correlation.

Linek et al. (1987) critically reviewed inconsistencies in various dynamic methods for measuring volumetric gas–liquid mass transfer coefficient and suggested a correct dynamic method. However, they showed that for coalescing systems (air–water) various incorrect techniques give practically the same \(k_a\) values (Linek et al., 1987). Linek et al. (1987) correlated their \(k_a\) data on \(T = 0.29\) m using power input criterion.

Hickman (1988) used a steady-state technique to determine \(k_a\) by measuring rate of decomposition of hydrogen peroxide to oxygen in the liquid phase. He correlated \(k_a\) data on STRs of \(T = 0.60\) m and 2 m using the power input criterion.

Whitton and Nienow (1993) studied gas hold-up behavior and gas–liquid mass transfer rates in STRs of diameter \(0.61\) m and \(2.67\) m with disc turbine of \(D = T/3\). They correlated gas hold-up and \(k_a\) data through power input per unit mass and superficial gas velocity.

Recently, Zhu et al. (2001) studied the mass transfer performance of several commonly used impellers for an air–water system with \(T = 0.39\) m. They concluded that data for DTs as well as other impellers such as A315, PBT and a concave blade turbine can be reasonably correlated by an equation of the form \(k_a = f\left(P/V_{L}^a\right)\left(V_{G}^{b}\right)\) with a maximum deviation of \(\pm 30\%\). Data of Zhu et al. (2001) for DT were analyzed in the present work on the basis of \(N/N_{cd}\).

The correlation presented by Hughmark (1980) for power input in gas–liquid dispersion by a six-bladed disc turbine over a wide range of system configurations and operating parameters (Table 3) is:

\[
\frac{P_{G}}{P_{O}} = 0.1 \left(\frac{Q_{G}}{N_{V_{L}}}\right)^{-0.25} \left(\frac{N_{b}D_{T}^{4}}{gD_{V_{L}}^{2/3}}\right)^{-0.2}
\] (2)

Equation (2) is based on 391 data with standard deviation of 0.117. This correlation is considered to be reliable (Hughmark, 1980; van’t Riet and Tramper, 1991; Yawalkar et al., 2002). Therefore, \(k_a\) versus impeller speed \((N)\) data for different superficial gas velocities and power input for all the above discussed literature on DTs, based on power input criterion obtained by using Equation (2). Thereafter, \(k_a\) versus \(N/N_{cd}\) data were obtained by using Equation (1).

**Data Obtained from Tabular and Graphical Results**

Chandrasekharan and Calderbank (1981) measured \(k_a\) by dynamic method in an STR of \(T = 1.22\) m. Analyzing \(k_a\) values obtained by Figueiredo and Calderbank (1979) for \(T = 0.91\) m and their own data, Chandrasekharan and Calderbank (1981) concluded that for a given specific power input and gas superficial velocity there is an optimum size of tank with respect to \(k_a\) for a particular geometry. In other words, for a given \(P/V_{L}\) and \(V_{G}\) as tank diameter increases, \(k_a\) also increases up to certain tank diameter, but at a higher tank diameter above the critical size, \(k_a\) decreases with tank diameter. However, Linek et al. (1987) have shown that the \(k_a\) measurement method used by Figueiredo and Calderbank (1979) is erroneous and, therefore, their \(k_a\) values for \(T = 0.91\) are not correct.

Chandrasekharan and Calderbank (1981) presented their results in tabular form (Table 2 of their paper), which have been...
converted in the form of $k_L a$ versus $N/N_{cd}$ using Equation (1) in the present work.

Bakker and van den Akker (1994) studied gas dispersion characteristics and gas–liquid mass transfer in STR equipped with axial flow impellers, such as PBT, A315 impeller and Leeuwrik impeller. They have presented an excellent discussion on prevailing gas flow patterns and impeller hydrodynamics for these impellers for different operating and geometric conditions. Bakker and van den Akker (1994) reported that an A315 impeller is the most suitable axial flow impeller with respect to gas dispersion capacity and stability of the system. They observed that for a given power input and gas superficial velocity PBT yields lower gas hold-up and $k_L a$ than A315. Bakker and van den Akker (1994) have given plots of $N_{PC}$ versus $F_G$ for A315 impeller and PBT with large ring sparger (Figures 9 and 10 of their paper). From these plots minimum impeller speed for complete gas dispersion ($N_{PC}$) for both these impellers and respective impeller speed (N) for different power inputs were obtained. $N_{cd}$ values for PBT and A315 impellers were 4.99 rev/s and 6.1 rev/s, respectively.

Using the graphs of $P/V_L$ versus $k_L a$ (Figure 17 of their paper) for the A315 impeller and large-ring sparger, $k_L a$ versus $N/N_{cd}$ data for the A315 impeller were obtained. Data of Bakker and van den Akker (1994) show that for a given $P/V_L$ and $V_G$ for A315 impeller, a small-ring sparger (SRS) gives slightly higher $k_L a$ values (10% to 15%) than a large-ring sparger (LRS). They also observed a similar 10% to 15% deviation of the gas hold-up data for an A315 impeller with both these spargers. Contrary to the results of the A315 impeller, Bakker and van den Akker (1994) observed an insignificant effect of sparger types on gas hold-up for PBT. However, considering the close relationship between gas hold-up and $k_L a$, it can be concluded that the effect of sparger design on $k_L a$ for PBT in the work of Bakker and van den Akker (1994) is negligible or very small. They did not report any effect of sparger design on $k_L a$ for STR with SRS and not for LRS. Thus, $k_L a$ values obtained from their plot of $P/V_L$ versus $k_L a$ for PBT with SRS and not for LRS can be approximately considered to be the same as those for PBT with LRS for respective $P/V$ and gas flow rate. Therefore, using the graph of $P/V_L$ versus $k_L a$ (Figure 19 of their paper) for PBT and SRS, data of $k_L a$ versus $N/N_{cd}$ for PBT have been obtained in the present work.

**Results and Discussion**

In our previous work it was observed that the correlation of the form $e_C = f(N/N_{cd}, vvm, T, D/T)$ shows good fit to the gas hold-up data for larger tanks available in the literature over a wide range of system and operating conditions. Dependence of $e_C$ on $T$ and $D/T$ is rather small as compared to dependence on the terms $N/N_{cd}$ and $vvm$. Here, vvm is volume of gas sparged per unit volume of liquid per minute. Therefore, it was thought desirable to correlate the $k_L a$ data obtained from different studies discussed earlier in the form of $k_L a = f(N/N_{cd}, vvm, T, D/T)$. Dependence of $k_L a$ on $D/T$ at a given $N/N_{cd}$, vvm and $T$ was found to be negligible. The expression obtained after the regression is:

$$k_L a = 0.0558 \left(\frac{N}{N_{cd}}\right)^{1.464} (vvm)^{1.05} \quad (3)$$
Equation (3) is based on a total of 178 data points and has $R^2$ value of 0.96 with standard deviation of 0.13. The exponent of $T$ in Equation (3) is very close to 1. Therefore, Equation (3) can be written in the form:

$$k_d a = 3.35(N/N_{cd})^{1.464}(V_c)$$

(4)

Even though application of Equation (4) instead of Equation (3) to estimate $k_d a$ increases the deviation of predicted $k_d a$ from actual $k_d a$ values, the increases in deviation is very small (2% to 4%). $k_d a$ data along with the correlating line given by Equation (4) are shown in Figure 2. It can be seen that most of the data except few data points of Smith et al. (1977) lie within ±22% of the correlating line. Hughmark’s (1980) correlation was used to determine impeller speed $N$ for a given power input for some data sets. Standard deviation of this correlation given by (4%).

The data studied in this work. Smith’s (1991) dimensionless correlation proposed by Smith (1991) for the present work, Equation (4) is compared with the parameter $\alpha_{NG}$ along with the correlating line given by Equation (4) of the data points around the correlation given by Equation (4) arises from the use of Hughmark’s correlation.

Table 4 indicates that $k_d a$ data of different workers can be correlated satisfactorily by the relative dispersion parameter, $N/N_{cd}$. To study the usefulness of the proposed correlation in the present work, Equation (4) is compared with the dimensionless correlation proposed by Smith (1991) for the data studied in this work. Smith’s (1991) dimensionless correlation is based on a wide range of system configurations using DT. Table 5 shows some data based on the relative dispersion parameter $N/N_{cd}$ along with the $k_d a$ values estimated by the correlation of Smith (1991) and Equation (4). It can be concluded that the correlation of Smith (1991) shows reasonable agreement (within 33%) with most of the data for disc turbines. However, the A315 impeller and PBT data correlation of Smith (1991) predicts $k_d a$ values almost two times higher than that observed by Bakker and van den Akker (1994). The correlation proposed in Equation (4) in the present work, on the other hand, shows satisfactory agreement (within 22%) with the data for DT, A315 impeller and PBT. Even though disc turbines are high shear-low pumping impellers while A315 and PBT impellers are low shear-high pumping, the $k_d a$ data of all these impellers show good agreement with each other when correlated on the basis of $N/N_{cd}$.

It should be noted that Equation (4) is independent of the scale of the reactor, type, size and position of the impeller and sparger. It indicates that at a particular superficial gas velocity and $N/N_{cd}$, $k_d a$ is same irrespective of system configuration. It can be argued that in Equation (4) all the geometric details of the system are defined by the term $N_{cd}$, which is a characteristic of particular system configuration ($T$, $D/T$, type of the impeller, etc.). Jadhav and Pangarkar (1991) and Kushalkar and Pangarkar (1994) have shown that in solid–liquid/solid–liquid–gas STR the term $N_{cd}$ (minimum impeller speed for complete dispersion of the solid particles) defines geometric details of the system and therefore a correlation based on $N/N_{cd}$ for predicting $k_d a$ is geometric configuration independent.

The R.H.S. of Equation (4) has unit m/s while the L.H.S. has unit 1/s. This implies that, to make Equation (4) dimensionally consistent, the L.H.S. of Equation (4) should be multiplied by a term that has a unit of length. Considering the close dependence of gas–liquid mass transfer coefficient on the gas bubble size ($d_g$) which has the unit of length, the $k_d a$ data was regressed in the form of $k_d a d_g/V_c$ versus $N/N_{cd}$. The correlation of Calderbank (1958) was used to determine values of the Sauter mean bubble diameter ($d_g$) in a tank for different system and operating conditions studied in this work. The correlation given by Calderbank (1958) for pure solutions is:

$$d_g = 4.15 - \frac{\sigma^{0.6}}{(\rho/\rho_l)^{0.4} \cdot L^{0.2} \cdot \epsilon_C^{0.5} + 0.0009}$$

(5)

The gas hold-up data required in this expression were obtained from the gas hold-up correlations given by respective authors and, if not, then derived from the gas hold-up correlation proposed in our previous work (Yawalkar et al., 2002). The expression obtained is:

$$k_d a d_g = 0.0186 (N/N_{cd})^{0.82}(V_c)$$

(6)

Equation (5) has an $R^2$ value 0.88 with standard deviation of 0.14. Figure 3 shows the large tank data for the works of Smith et al. (1977), Chandrasekharan and Calderbank (1981), Hickman (1988), Smith (1991) and Whitton and Nienow (1993) in the form of Equation (6). Taking into account the deviation introduced due to the application of Hughmark’s (1980) and Calderbank’s (1958) correlations to generate impeller speed ($N$) and bubble diameter ($d_g$) data, respectively, at a given power input per unit volume ($P/V_c$), the agreement of the data points around a correlating line is satisfactory (30%).

**Significance of the Terms vvm and $T$ in Equation (4)**

Equation (4), which arises out of Equation (3), is distinctly different in its dependence on gas flow rate as compared to the correlation for gas hold-up presented in our earlier paper (Yawalkar et al., 2002). Yawalkar et al. (2002) found that the use of vvm in the correlation for $\epsilon_C$ makes the correlation independent of scale. In the present correlation for $k_d a$ the terms on the R.H.S. of Equation (3), (vvm), $T$ ultimately reduces to $V_c$ and Equation (4) is obtained. This can be viewed as an inadequacy of the concept of vvm. However, in the following an explanation is given which indicates that the direct dependence of $k_d a$ on $V_c$ arises out of complex dependence of the terms $k_d a$ on vvm and $T$ and not because of any inadequacy of the vvm concept.

Studying large amount of data of different workers Hughmark (1980) has given a correlation for gas–liquid interfacial area per unit volume term, $a$, in STR. The correlation given by Hughmark (1980) is:

$$a = 1.38 \left( \frac{\rho_l}{\sigma} \right)^{0.50} \left( \frac{Q_c}{NV_c} \right)^{0.33} \left( \frac{N^2 D^4}{g V_c^{0.66}} \right)^{0.592} \left( \frac{d_g N D^4}{\sigma V_c^{0.66}} \right)^{0.187}$$

(7)

This correlation was applied to the system configuration and operating parameters of different workers (given in Table 1) to estimate values of $a$ in their respective works. Then the values of $a$ obtained were regressed as $a = f(N/N_{cd}, \text{vvm}, T)$. The expression obtained is:

$$a = 7.47 (N/N_{cd})^{1.23}(\text{vvm})^{0.95}(T)^{0.30}$$

(8)

The $R^2$ value of Equation (8) is 0.99 with standard deviation of 0.05. Using this dependence of $a$ on $N/N_{cd}$, vvm and $T$ and that of $k_d a$ given in Equation (3) (i.e. dividing Equation 3 by Equation 8) it can be noted that $k_d a \propto (N/N_{cd})^{0.234}(\text{vvm})^{0.05}(T)^{0.73}$. Thus, $k_d a$ and $a$ have different functional relationships with $N/N_{cd}$, vvm and $T$ but the combined dependence is such that ultimately an equation Equation(4) is obtained which
directly involves $V_G$. Therefore, it can be concluded that the functional dependence Equation (4) proposes, $k_La \sim (vvm)$. ($T$) (which eventually is equal to $V_G$) arises out of the combined effect of dependence of the terms $k_L$ and $a$ on $vvm$ and tank diameter, $T$.

Conclusions

In this study, data available in the literature on volumetric gas–liquid mass transfer coefficient ($k_La$) for larger STRs ($T = 0.39$ m to 2.7 m) have been analyzed in the form of $k_La$ versus relative dispersion parameter, $N/N_{cd}$. An attempt has been made to provide a single correlation based on $N/N_{cd}$ for $k_La$ data obtained by different researchers for a coalescing (air–water) system. It was observed that, at a given superficial gas velocity ($V_G$), $k_La$ values were approximately the same irrespective of geometric configuration (size of the reactor, type and size of the impeller, sparger type, etc) for a particular $N/N_{cd}$. The correlation obtained was:

$$k_La = 3.35(N/N_{cd})^{1.464}(V_G)$$

Almost all the $k_La$ data points were found to lie within 22% of the correlating line.

Thus, results of the previous work on gas hold-up (Yawalkar et al., 2002) and the present work on $k_La$ indicate that the relative dispersion parameter is useful in reliably correlating/estimating hydrodynamic and mass transfer properties of gas–liquid STR.

**Nomenclature**

- $a$: gas–liquid interfacial area per unit volume, (m$^2$/m$^3$)
- $C$: clearance of the impeller from the bottom of STR, (m)
- $D$: impeller diameter, (m)
- $D_i$: width of the impeller blade, (m)
- $G_i$: gassed flow number, $Q_G/N_D^3$
- $Fr$: Froude number, $N_D^2/g$
- $g$: gravitational acceleration, (=9.81 m/s$^2$)
- $H$: height of clear liquid in STR, (m)
- $k_L$: volumetric gas–liquid mass transfer coefficient, (1/s)
- $k_{SL}$: solid–liquid mass transfer coefficient, (m/s)
- $m$: mass of liquid in STR, (kg)
- $N$: impeller speed, (rev/s)
- $N_{cd}$: minimum impeller speed for complete dispersion of the sparged gas, (rev/s)
- $N_s$: minimum impeller speed for complete suspension of all the solid particles in STR, (rev/s)
- $N_{PG}$: power number of the impeller in presence of gas, $P_G/\rho N_D^5$
- $N_{PA}$: power number of the impeller in absence of gas, $P_A/\rho N_D^5$
- $N_k$: impeller speed at which recirculation of the sparged gas within the liquid starts, (rev/s)
- $P_o$: power input in absence of gas, (W)
- $P_G$: power input in presence of gas, (W)
- $Q_G$: volumetric gas flow rate, (m$^3$/s)
- $Re_N$: Reynolds number based on impeller speed, $N_D^2\rho/\mu$
- $Re_i$: Reynolds number based on impeller diameter, $D_i^2\rho/\mu$
- $T$: inside diameter of STR, (m)
- $u'$: turbulence intensity, (m/s)
- $V_G$: superficial gas velocity, (m/s)
- $V_L$: volume of the liquid in the reactor, (m$^3$)
- $w$: impeller blade width, (m)

**Greek Symbols**

- $\rho$: density, (kg/m$^3$)
- $\sigma$: surface tension, (N/m)

**Subscripts**

- $cd$: complete dispersion
- $G$: gas
- $L$: liquid
- $S$: complete suspension
- $SL$: solid-liquid

**References**


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