High-precision measurements of proton-proton bremsstrahlung at 190 MeV

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This chapter deals with the theory relevant for this work. First, the kinematics is discussed in two different choices of coordinates, whereby the advantages of one choice over the other are delineated. Then, the physical observables measured in this experiment are defined. Finally, some theories capable of predicting these observables are reviewed.

2.1 Kinematics

In this section the kinematics of the $pp\gamma$ reaction is discussed. The procedure used in the data analysis for solving the kinematic equations in spherical coordinates is outlined. The definitions used here are similar to the ones used by Drechsel and Maximon [29], but differ in details. These adjustments are introduced, because this experiment is performed with a polarized beam, which complicates the picture to some extent.

2.1.1 Solving the kinematic equations

A $pp\gamma$ event is described by the momentum vectors of the three outgoing particles. Each momentum vector has three components, and the final state has thus nine parameters, which describe it completely. These nine parameters are, however, not independent, because of the laws of energy and momentum conservation. These four conservation laws reduce the number of independent parameters to five. In the laboratory system, the $z$-axis is chosen along the direction of the incoming proton, which has momentum $p_0$. The $y$-
axis is chosen along the direction of polarization of the beam, orthogonal to \( \vec{z} \). The \( x \)-axis completes the right-handed set of orthogonal directions. The target proton has zero momentum. The momenta of the two outgoing protons are \( \vec{p}_A \) and \( \vec{p}_B \). The momentum of the photon is \( \vec{k} \). The energy and momentum conservation laws now yield (natural units, \( \hbar = c = 1 \) are used):

\[
\sqrt{\left(p_A^2 + m_p^2\right)} + \sqrt{\left(p_B^2 + m_p^2\right)} + k = \sqrt{p_b^2 + m_p^2 + m_p}
\]

(2.1)

Here, \( m_p \) is the mass of the proton. The conventional choice of the five parameters to fix the kinematics are the spherical angles of the momenta: \( \theta_A, \phi_A, \theta_B, \phi_B \) and \( \theta_\gamma \). The conservation laws for momentum in spherical coordinates are:

\[
p_A \sin \theta_A \cos \phi_A + p_B \sin \theta_B \cos \phi_B + k \sin \theta_\gamma \cos \phi_\gamma = 0
\]

(2.3)

\[
p_A \sin \theta_A \sin \phi_A + p_B \sin \theta_B \sin \phi_B + k \sin \theta_\gamma \sin \phi_\gamma = 0
\]

(2.4)

\[
p_A \cos \theta_A + p_B \cos \theta_B + k \cos \phi_\gamma = p_b
\]

(2.5)

From the law of energy conservation (2.2) the photon momentum can be expressed as:

\[
k = \sqrt{\left(p_A^2 + m_p^2\right)} + \sqrt{\left(p_B^2 + m_p^2\right)} - \sqrt{\left(p_A^2 + m_p^2\right)} - \sqrt{\left(p_B^2 + m_p^2\right)}
\]

(2.6)

If one substitutes this expression for \( k \) in Eq. (2.5), it can be solved for \( p_B \):

\[
p_B = \frac{q \cos \theta_B \pm \sqrt{q^2 \cos^2 \theta_\gamma - m_p^2 \cos^2 \theta_A \left(\cos^2 \theta_\gamma - \cos^2 \theta_B\right)}}{\cos^2 \theta_\gamma - \cos^2 \theta_B}
\]

(2.7)

where \( q \) is defined as:

\[
q = p_A \cos \theta_A + \left(\sqrt{\left(p_A^2 + m_p^2\right)} + m_p - \sqrt{\left(p_B^2 + m_p^2\right)}\right) \cos \theta_\gamma - p_b
\]

(2.8)

With some algebra, one can eliminate \( \phi_\gamma \) from Eq. (2.3) and Eq. (2.4):

\[
k^2 \sin^2 \theta_\gamma = (p_A \sin \theta_A - p_B \sin \theta_B)^2 + 4 p_A p_B \sin \theta_A \sin \theta_B \sin^2 \Phi
\]

(2.9)

Here, the new variable \( \Phi \) is defined as:

\[
\Phi = \frac{\pi}{2} + \frac{\phi_A - \phi_B}{2}
\]

(2.10)

In Eq. (2.9) one can substitute \( k \) from Eq. (2.6) and \( p_B \) from Eq. (2.7). The resulting expression has only \( p_A \) as an unknown, and can be solved by numerical means.
2.1. KINEMATICS

2.1.2 Conventions for the kinematics

From Eq. (2.7) it can be seen that there are in general two possible choices for \( p_B \) given a certain \( p_A \). Furthermore, it turns out that Eq. (2.9) has in general two solutions, resulting in a total of four different mathematical solutions. Two of these solutions are non-physical in the sense that they produce negative values for either \( p_A \), \( p_B \) or \( k \). The other two solutions are valid solutions of the kinematic equations, given \( \theta_A, \phi_A, \theta_B, \phi_B \). The labels \( A \) and \( B \) have been assigned arbitrarily to the two protons. By assigning labels 1 and 2 to the two protons in a unique way, one can discriminate between the two different solutions. For the sake of the argument, the solution with the lowest value of \( p_A \) is called solution I and the other solution II, i.e. \( p_{I, A} < p_{II, A} \). One can now adopt the convention to label \( p_A \) as \( p_1 \) and \( p_B \) as \( p_2 \) in case of solution I. In the case of solution II, it is the other way around: \( A=2 \) and \( B=1 \). This results in a unique labelling of the two protons, independent of the initial \( A, B \) labelling.

For the case of coplanar kinematics, the difference between the two solutions is depicted in Fig. 2.1. In the first possibility (solution I), the photon is emitted to the same side of the beam axis as proton \( A \). For the other possibility (solution II), the photon is emitted to the same side as proton \( B \). With the above definition, this means that proton 1 lies on the same side of the beam axis as the photon. Note, that this interpretation holds only for coplanar scattering. In the case of non-coplanar scattering, the definition of proton 1 and 2 is still unique, but a trivial interpretation is not always available.

With protons 1 and 2 so defined, the three vectors \( p_1, p_2 \) and \( k \) can either form a right-handed, where \( (p_1 \times p_2) \cdot k > 0 \), or a left-handed system, where \( (p_1 \times p_2) \cdot k < 0 \). These two situations are depicted in Fig. 2.2. The parity \( \Pi \) of a \( pp\gamma \) event is now defined.
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Figure 2.2: Geometrical interpretation of $\Phi'$ and $\phi_{\text{event}}$. In a right-handed (left handed) system $\Phi'$ is defined as positive (negative).

as the sign of this vector product:

$$\Pi = \text{sign}[(p_1 \times p_2) \cdot k]$$  \hspace{1cm} (2.11)

From Eq. (2.10) one can see that in the case of coplanar kinematics $\Phi$ is either 0 or $\pi$. In this special case the parity $\Pi$ is not defined, since the vector product in Eq. (2.11) yields 0. One can define a second coordinate system for each event in such a manner that it is rotated around the beam direction with respect to the external coordinates. The $x'$-axis is chosen such that proton 1 makes an angle $\Phi'$ with the positive $x'$-axis and proton 2 makes an angle $\Phi'$ with its negative part. The $y'$-axis is orthogonal to the $x'$-$z'$ plane. Since the angle $\Phi'$ in this new coordinate system is 0 for coplanar events, it is referred to as the “non-coplanarity angle”. One can define it as:

$$\Phi' = \Pi \text{asin}(\text{asin}(\Phi))$$  \hspace{1cm} (2.12)

where $\Pi$ is the parity of the event. Using this definition, the non-coplanarity angle contains the information on the handedness of the vectors $p_A$, $p_B$ and $k$ and the ambiguity of $\Pi$ in the coplanar case is solved in a natural way. The angle $\Phi'$ lies between $-\pi/2$ and $\pi/2$. The external coordinate system and the coordinate system of the event make an angle which is called $\phi_{\text{event}}$. The geometrical interpretation of the non-coplanarity angle and $\phi_{\text{event}}$ is depicted in Fig. 2.2.
2.2 Harvard Coordinates

There are many ways of choosing a set of five independent variables to fix the kinematics. One alternative way used in the literature is the so-called “Harvard coordinate” system, see e.g. [21, 30, 31, 32]. The calculations involved in dealing with this coordinate system are more tedious than the standard spherical coordinates. The reason these coordinates are preferred in certain cases is the fact that the differential cross section does not show singular behaviour at kinematic extremes (see section 2.4).

2.2.1 Definition of the Harvard angles

The definitions of the cartesian coordinate system is similar to the one in the previous subsection. The beam comes in along the $z$-axis. The beam proton has three-momentum $p_b$ and the target proton is at rest. The two outgoing protons have momenta $p_1$ and $p_2$, respectively. The proton mass is $m_p$ and the photon has momentum $k$. When the 3 particles in the final state are in one plane, this scattering plane defines the $x$-axis and the $y$-axis: the $x$-axis and the $z$-axis are in the plane, the $y$-axis is normal to the plane. When the 3 particles are not in one plane, one defines a reference plane by choosing an $x$-axis. The $y$-axis is again normal to this reference plane. In this system one projects the final-state momenta on the reference plane (defined by $x$- and $z$-axis); the angle between the momentum and its projection is called $\phi$ and the angle between the projection and the $z$-axis is $\theta$. It is clear that the reference plane is not uniquely defined. In the Harvard geometry, one takes the non-coplanarity angles $\phi$ for the 2 protons equal, so $\phi_1 = \phi_2 \equiv \phi$. This uniquely defines an $x$- and a $z$-axis.

The transformation from Harvard coordinates to cartesian coordinates is given by:

\begin{align}
    p_1 &= p_1 \cos \phi_1 \sin \theta_1 \hat{x} + p_1 \cos \phi_1 \cos \theta_1 \hat{z} + p_1 \sin \phi_1 \hat{y} \\
    p_2 &= -p_2 \cos \phi_2 \sin \theta_2 \hat{x} + p_2 \cos \phi_2 \cos \theta_2 \hat{z} + p_2 \sin \phi_2 \hat{y} \\
    k &= k \cos \phi_\gamma \sin \theta_\gamma \hat{x} + k \cos \phi_\gamma \cos \theta_\gamma \hat{z} - k \sin \phi_\gamma \hat{y}
\end{align}

In the special coplanar case where $\phi_1 = 0, \phi_2 = 0$, all momenta are in one plane. The Harvard coordinates and spherical coordinate are then identical.

\begin{align}
    \phi_1 = 0 \quad , \quad \theta_1 = \theta_1 \quad ; \quad \phi_2 = \pi \quad , \quad \theta_2 = \theta_2 \quad ; \quad \phi_\gamma = 0 \quad , \quad \theta_\gamma = \theta_\gamma.
\end{align}

2.2.2 The limiting gamma ray

The Jacobian for non-coplanar case was first solved by Liou and Sobel [30]. The non-coplanarity angle of the protons as defined in Harvard geometry, $\phi$, has a kinematically allowed maximum, which is called $\phi_{\max}$. The corresponding polar and azimuthal angles for the photon, $\phi_0$ and $\theta_0$, define the so-called “limiting $\gamma$ ray” [21]. How to obtain these angles is explained in the next section. One defines a new coplanar photon momentum in the $x$-$z$ plane by

\begin{align}
    k' \equiv k - \alpha k_0
\end{align}
where \( k_0 \) is the momentum of the limiting \( \gamma \) ray. The parameter \( \alpha \) is chosen such that the vector \( k' \) has no \( y \)-component. The polar angle of this new photon momentum is called \( \psi_{\gamma} \), so that

\[ k' = k' \sin \psi_{\gamma} \hat{x} + k' \cos \psi_{\gamma} \hat{z} \quad (2.18) \]

One finds

\[ k' \sin \psi_{\gamma} = k \cos \theta_{\gamma} \sin \theta_{\gamma} - \alpha k_0 \cos \theta_0 \sin \theta_0 \quad (2.19) \]
\[ 0 = -k \sin \theta_{\gamma} + \alpha k_0 \sin \theta_0 \quad (2.20) \]
\[ k' \cos \psi_{\gamma} = k \cos \theta_{\gamma} \cos \theta_{\gamma} - \alpha k_0 \cos \theta_0 \cos \theta_0 \quad (2.21) \]

From these equations, one obtains

\[ \alpha k_0 = k \sin \theta_{\gamma} / \sin \theta_0 \quad (2.22) \]

and

\[ \tan \psi_{\gamma} = \frac{\sin \theta_{\gamma} - \cot \theta_0 \tan \theta_{\gamma} \sin \theta_0}{\cos \theta_{\gamma} - \cot \theta_0 \tan \theta_{\gamma} \cos \theta_0} \quad (2.23) \]

### 2.2.3 Solution of the Harvard kinematics

Suppose one is given \( \theta_1 \) and \( \theta_2 \). The limiting \( \gamma \) ray is defined by two angles, \( \theta_0 \) and \( \theta_0 \). The 4 equations for energy and momentum conservation read:

\[ \sqrt{p_b^2 + m_p^2 + m_p} = \sqrt{p_1^2 + m_p^2} + \sqrt{p_2^2 + m_p^2} + k \quad (2.24) \]
\[ 0 = p_1 \cos \phi \sin \theta_1 - p_2 \cos \phi \sin \theta_2 + k \cos \theta_{\gamma} \sin \theta_{\gamma} \quad (2.25) \]
\[ 0 = p_1 \sin \phi + p_2 \sin \phi - k \sin \theta_{\gamma} \quad (2.26) \]
\[ p_b = p_1 \cos \phi \cos \theta_1 + p_2 \cos \phi \cos \theta_2 + k \cos \theta_{\gamma} \cos \theta_{\gamma} \quad (2.27) \]

First \( \phi_{\gamma} \) and \( \theta_{\gamma} \) are eliminated from the 3 equations for momentum conservation. This gives a quadratic equation in \( \cos \phi \) with the solution:

\[ \cos \phi = \frac{-p_1 (p_1 c_1 + p_2 c_2) + \sqrt{p_1^2 (p_1 c_1 + p_2 c_2)^2 + 4 p_1 p_2 s^2 (p_1^2 + (p_1 + p_2)^2 - k^2)}}{4 p_1 p_2 s^2} \quad (2.28) \]

where \( c_1 = \cos \theta_1, c_2 = \cos \theta_2, s = \sin \theta \) and \( \theta = (\theta_1 + \theta_2) / 2 \). \( k \) is given by the equation for energy conservation in terms of \( p_1 \) and \( p_2 \). The maximum value for \( \phi \) is obtained by minimizing \( \cos \phi \). One thus has to solve:

\[ \partial \cos \phi / \partial p_1 = \partial \cos \phi / \partial p_2 = 0 \quad (2.29) \]
This gives 2 coupled equations for \( p_1 \) and \( p_2 \) that have to be solved numerically:

\[
p_1 c_2 \sqrt{p_1^2 (p_1 c_1 + p_2 c_2)^2 + 4p_1 p_2 s_2 (p_1^2 + (p_1 + p_2)^2 - k^2)} - \\
p_2 c_2 (p_1 c_1 + p_2 c_2) + 4 s^2 k p_1^2 / E_1 + 2 s^2 p_1 (p_1^2 - p_1^2 - p_2^2 + k^2) = 0
\]  
(2.30)

and a second equation, where \( p_1 \leftrightarrow p_2 \) and \( c_1 \leftrightarrow c_2 \). Furthermore, the energies are used:

\[
E_1 = \sqrt{p_1^2 + m_e^2} \quad \text{and} \quad E_2 = \sqrt{p_2^2 + m_e^2}.
\]

After having obtained in this manner \( p_1 \) and \( p_2 \) for the kinematic maximum, one gets \( k \) from energy conservation, and \( \theta_{\text{max}} \) from Eq. (2.28). Finally, the azimuthal and polar angles of the limiting \( \gamma \)-ray are given by

\[
\tan \theta_0 = \frac{(p_2 \sin \theta_2 - p_1 \sin \theta_1) \cos \theta_{\text{max}}}{p_b - (p_1 \cos \theta_1 + p_2 \cos \theta_2) \cos \theta_{\text{max}}}
\]

(2.31)

and

\[
\sin \theta_0 = \sin \theta_{\text{max}} (p_1 + p_2)/k
\]

(2.32)

Having found the limiting \( \gamma \)-ray, one has still to solve the kinematics. Given are \( \theta_1 \), \( \theta_2 \), \( \phi \), and \( \psi \). The goal is to obtain \( p_1 \), \( p_2 \), \( k \), \( \theta_\gamma \), and \( \phi_\gamma \). The calculation is straightforward, so only the algorithm is outlined here. First the following determinants are defined:

\[
\Delta_1 = p_b \begin{vmatrix}
-\cos \phi \sin \theta_2 & \sin \theta_2 \\
\sin \phi & -\tan \phi_\gamma 
\end{vmatrix}
\]

(2.33)

\[
\Delta_2 = -p_b \begin{vmatrix}
\cos \phi \sin \theta_1 & \sin \theta_1 \\
\sin \phi & -\tan \phi_\gamma 
\end{vmatrix}
\]

(2.34)

\[
\Delta_\gamma = p_b \sqrt{1 + \tan^2 \phi_\gamma} \begin{vmatrix}
\cos \phi \sin \theta_1 & -\cos \phi \sin \theta_2 \\
\sin \phi & \sin \phi 
\end{vmatrix}
\]

(2.35)

and

\[
\Delta = \begin{vmatrix}
\cos \phi \sin \theta_1 & -\cos \phi \sin \theta_2 & \sin \theta_\gamma \\
\sin \phi & \sin \phi & -\tan \phi_\gamma \\
\cos \phi \cos \theta_1 & \cos \phi \cos \theta_2 & \cos \theta_\gamma 
\end{vmatrix}
\]

(2.36)

Then \( p_1 \), \( p_2 \), and \( k \) are given by

\[
p_1 = \Delta_1 / \Delta \quad \text{,} \quad p_2 = \Delta_2 / \Delta \quad \text{and} \quad k = \Delta_\gamma / \Delta .
\]

(2.37)

Inserting these equations in the equation for energy conservation, and eliminating \( \phi_\gamma \) by using:

\[
\tan \phi_\gamma = \tan \phi_0 \frac{\cos \theta_\gamma - \cot \psi_\gamma \sin \phi_\gamma}{\cos \phi_\gamma - \cot \psi_\gamma \sin \phi_\gamma}
\]

(2.38)

obtained from Eq. (2.23), an equation is obtained that can be solved numerically for \( \phi_\gamma \). Then one solves Eq. (2.38) for \( \phi_\gamma \) and finally Eq. (2.37) for \( p_1 \), \( p_2 \), and \( k \).
2.3 The bremsstrahlung observables

In this section the definition of the proton-proton bremsstrahlung observables relevant for this work are defined, i.e. the cross section and the analyzing powers. The cross section is a measure of the chance an interaction takes place. The analyzing power reflects the spin dependence of the interaction.

2.3.1 The cross section

The infinitesimal cross section $d\sigma$ can be expressed as the product of the Lorentz-invariant square of the amplitude $|M|^2$ and phase-space and kinematical factors [33]. In the case of the reaction $a + b \rightarrow n$-particles, where $a$ and $b$ are fermions, the expression for the cross section is:

$$d\sigma = \frac{1}{|v_a - v_b|} \left( \frac{m_a}{E_a} \right) \left( \frac{m_b}{E_b} \right) \sum_{\lambda_i, \lambda_f} |M_{fi}|^2 \times \left( \prod_{i=1}^{n} \frac{d^3 p_i}{2E_i(2\pi)^3} \right) (2\pi)^4 \delta(p_a + p_b - \sum_{i=1}^{n} p_i)S$$ (2.39)

Here, $v_a$ and $v_b$ are the velocities of the incident co-linear particles with masses $m_a$ and $m_b$ and energies $E_a$ and $E_b$. The final-state momenta $p_i$ have to be used to integrate over the four $\delta$-functions, which insure energy and momentum conservation. The statistical factor $S$ is obtained by placing a factor $\frac{1}{m!}$ for each collection of $m$ identical particles in the final state. The first kinematical factor in Eq.(2.39) contains only information on the incident particles and is thus a constant for a mono-energetic scattering experiment. The co-variant amplitude $M_{fi}$ is the factor containing all the dynamics of the interaction and can be calculated with a model of the interaction. One has to average the amplitude over all spins in the initial state and sum of the final states, thus making the cross section a spin-independent quantity. The third term, the phase-space density, is solely dependent upon the kinematics of the outgoing particles.

In the case of proton-proton bremsstrahlung, the number of independent variables, defining a scattering event, is five. This means that the fully exclusive differential cross section is a five-fold differential cross section. The most common convention for the $pp\gamma$ differential cross section is $\frac{d\sigma}{d\Omega_1 d\Omega_2 d\psi_2}$. An alternative for this form is the differential cross section in Harvard coordinates: $\frac{d\sigma}{d\Omega_1 d\Omega_2 d\psi_2}$.

2.3.2 The analyzing powers

If a beam of particles, moving along the z-axis, impinges on a target, there exists a cylindrical symmetry. This means that the differential cross section will not depend on the azimuthal angle. In the case of proton-proton bremsstrahlung the azimuthal angle of an event is defined as $\phi_{\text{event}}$ in section 2.1.2. However, the complete state of a beam particle is in general not uniquely defined by its momentum, but can in addition contain spin. This
means that the particle carries an angular momentum along with it, which can break the cylindrical symmetry. In the case of spin $\frac{1}{2}$ particles, the two eigenstates of each particle are spin up and spin down. The polarization vector, $\mathbf{P}$, of a beam consisting of spin $\frac{1}{2}$ particles, is defined as:

$$
\mathbf{P} = \frac{1}{N_{\text{tot}}} \sum_{i=1}^{N_{\text{tot}}} \langle \chi_i | \hat{\sigma} | \chi_i \rangle
$$

(2.40)

Here, $N_{\text{tot}}$ is the total number of beam particles, $|\chi_i>$ is the spinor describing the spin-state of particle $i$ and is normalized to unity. $\hat{\sigma}$ represents the three pauli spin matrices. From Eq. (2.40) one can see that the magnitude of $\mathbf{P}$ will lie between $-1$ and $1$.

If the polarization vector has a non-zero component perpendicular to the beam, the cylindrical symmetry is broken. The cross section will then in general be a function of $\phi_{\text{event}}$. Let us first consider the simple coplanar case. Since parity and time-reversal invariance are not violated by the strong and electromagnetic interaction, one can show that the cross section is only a function of $\mathbf{P} \cdot \hat{n}$, where $\hat{n}$ is the unit vector normal to the scattering plane. If the polarization vector is perpendicular to the beam, one can choose the $y$-direction along the polarization direction. The magnitude of the polarization is then defined as $p_y$. If one now defines $\sigma_{\text{pol}}$ as the cross section for polarized particles and $\sigma_{\text{ Aust}}$ as the cross section for unpolarized particles, which is independent of $\phi_{\text{event}}$, then the most general form the cross section can take for coplanar scattering is:

$$
\frac{d\sigma_{\text{pol}}}{d\mathcal{F}} = \frac{d\sigma_{\text{ Aust}}}{d\mathcal{F}}(1 + p_y A_y \cos \phi_{\text{event}})
$$

(2.41)

Here, $\mathcal{F}$ stands for any choice of coordinates and $A$ is the analyzing power. In the “Madison convention” [34], this analyzing power is denoted as $A_y$.

In the case of non-coplanar scattering, the scattering plane is not uniquely defined anymore, and consequently there is an ambiguity in the definition of the vector $\hat{n}$. In a three-body final state the number of independent axial vectors one can make is three. In the case of $p p \gamma$ one has, for example, $\mathbf{p}_b \times \mathbf{p}_1$, $\mathbf{p}_b \times \mathbf{p}_2$ and $\mathbf{p}_1 \times \mathbf{k}$ as independent axial vectors. Consequently, the number of analyzing powers is three in the case of non-coplanar scattering. If one defines a scattering plane for a three-body final state, as has for example been done in section 2.1.2, one can define the axial vector normal to this plane as the one corresponding to the analyzing power $A_y$. This vector is then denoted as $\hat{n}_y$. One can define a second plane as the one going through the beam-axis and perpendicular to the scattering plane. This would be the $y' - z$ plane in Fig. 2.2. The axial vector corresponding to this plane is denoted as $\hat{n}_z$ and corresponds to the observable $A_z$. The third axial vector $\hat{n}_c$ can be chosen such that it completes the set of right-handed orthonormal vectors. The vector $\hat{n}_c$ is then in the direction of the incoming beam. The “polarized” cross section for a $\mathbf{P}$ perpendicular to the beam then becomes:
Here, \( \Pi \) is the parity of the event as defined in section 2.1.2. The observable \( A_z \) drops out, since it can only be measured with a polarization component in the direction of the beam. The analyzing powers are constrained by \( A_x^2 + A_y^2 + A_z^2 \leq 1 \) since the cross section can never be negative. Note that the definition of the scattering plane and consequently of \( \phi_{\text{event}} \) is different in spherical and Harvard coordinates. Therefore, the definition of \( A_y \) and \( A_z \) is also different.

Another convention which is commonly used, is to define the scattering plane by the incoming beam and the photon momentum vector. In this convention the variable \( \phi_{\text{event}} \) should be replaced by \( \phi_\gamma \) in Eq. (2.42). This convention will be used in Chapter 6 for the presentation of non-coplanar data. In order to distinguish the different conventions, Eq. (2.42) is rewritten to:

\[
\frac{d\sigma^{\text{pol}}}{d\Omega} = \frac{d\sigma'}{d\Omega}(1 + p_\gamma A_x \sin \phi_{\text{event}} + \Pi p_\gamma A_y \cos \phi_{\text{event}}) \tag{2.43}
\]

Here two new analyzing powers, \( A_x^\gamma \) and \( A_y^\gamma \), are introduced. In the coplanar case, the analyzing power \( A_x^\gamma \) reduces to \( A_y \) and \( A_x^\gamma \) reduces to zero due to parity conservation. The general relation between these analyzing powers is:

\[
A_x^\gamma = \sin(\phi_\gamma - \phi_{\text{event}})A_x + \cos(\phi_\gamma - \phi_{\text{event}})A_y \tag{2.44}
\]

\[
A_y^\gamma = \cos(\phi_\gamma - \phi_{\text{event}})A_x + \sin(\phi_\gamma - \phi_{\text{event}})A_y \tag{2.45}
\]

The reason to use these analyzing powers, instead of \( A_x \) and \( A_y \), in the presentation of the data, resides in the fact that for one experiment the azimuthal range of photon detection lies around 0° and 180°. Therefore, only \( A_x^\gamma \) is obtained, since the contribution of \( \sin \phi_\gamma \) is not measurable at 0° and 180°.

### 2.4 Phase-space considerations

In this section the evaluation of the invariant phase-space density is performed in spherical coordinates along the lines outlined by Drechsel and Maximon [29]. Furthermore, the advantages and disadvantages of the Harvard geometry are discussed. In the case of proton-proton bremsstrahlung the phase-space density is given by:

\[
dI = \delta(E_b + m - E_1 - E_2 - k)\delta^3(p_b - p_1 - p_2 - k) \frac{d^3p_1 d^3p_2 d^3k}{E_1 E_2 k} \tag{2.46}
\]

The goal is now to obtain an expression for the differentiated phase-space density, the Jacobian:

\[
J = \frac{dI}{d\Omega_1 d\Omega_2 d\phi_\gamma}
\]
\[ \delta^3(p_b - p_1 - p_2 - k) \]

\[ \times \frac{p_1^2 dp_1 p_2^2 dp_2 k dk d\theta_1 d\phi_1}{E_1 E_2} \]

To obtain a useful expression for \( J \), the integral over \( p_1, p_2, k \) and \( \phi_1 \) has to be carried out. One can write:

\[ \delta^3(p_b - p_1 - p_2 - k) = \frac{1}{k \sin \theta_1} \delta(\phi_1 - \phi_{p_1 + p_2}) \delta(f(p_1, p_2, k)) \times \delta(k \cos \theta_1 + p_1 \cos \theta_1 + p_2 \cos \theta_2 - p_b) \]

where

\[ f(p_1, p_2, k) = \sqrt{(p_1 \sin \theta_1 - p_2 \sin \theta_2)^2 + 4p_1 p_2 \sin \theta_1 \sin \theta_2 \sin^2 \Phi} + k \sin \theta_1 \]

With this expression the integral over \( \phi_1 \) is trivial. Since \( \delta(k \cos \theta_1 + p_1 \cos \theta_1 + p_2 \cos \theta_2 - p_b) \) is linear in \( k \), one simply obtains a factor \((\cos \theta_1)^{-1}\) from integration over \( k \). Since after this integration \( k \) is fixed in terms of \( p_1 \) and \( p_2 \), the function \( f \) is now no longer explicitly dependent on \( k \). One is then left with:

\[ J = \int \delta(f(p_1, p_2)) \delta(g(p_1, p_2)) \frac{p_1^2 p_2^2 dp_1 dp_2}{E_1 E_2 \cos \theta_1} \]

where \( g(p_1, p_2) \) is given by the law of energy conservation. Integrating this expression over \( p_1 \) and \( p_2 \) yields:

\[ J = \frac{p_1^2 p_2^2}{E_1 E_2 \cos \theta_1} \left| \frac{\partial f}{\partial p_1} \frac{\partial g}{\partial p_2} - \frac{\partial f}{\partial p_2} \frac{\partial g}{\partial p_1} \right| \]

\[ = \frac{p_1^2 p_2^2 \sin \theta_1}{E_1 E_2 |N|} \]

where \( p_1, p_2 \) and \( k \) are functions of \( \theta_1, \theta_2, \theta_3 \) and \( \Phi \). The quantity \( N \) is defined as:

\[ N = (p_2 \sin \theta_2 - p_1 \sin \theta_1)[\sin(\theta_1 + \theta_2) - (\frac{p_1 \sin \theta_2}{E_1} + \frac{p_2 \sin \theta_1}{E_2}) \cos \theta_1] - k(\frac{p_1 \cos \theta_2}{E_1} - \frac{p_2 \cos \theta_1}{E_2}) \sin^2 \theta_2 + 2 \sin \theta_1 \sin \theta_2 \sin^2 \Phi \]

\[ \times [p_1 \cos \theta_1 - p_2 \cos \theta_2 - (\frac{p_1}{E_1} - \frac{p_2}{E_2}) \cos \theta_1] \]

One can then use this phase-space factor to obtain an expression for the differential cross section \( \frac{d\sigma}{d\Omega_1 d\Omega_2 d\Phi_1} \) from Eq. (2.39).

Since the cross section is proportional to the Jacobian \( J \), any singularity in it will produce a singularity in the cross section. An investigation of the behaviour of the Jacobian as a function of non-coplanarity angle \( \Phi \) reveals that it will go to infinity when
\( \Phi^\prime \) reaches its maximum value. This means that the assumption of constant cross section within a finite bin will not be valid anymore close to the kinematical maximum. Therefore, a direct comparison of experiment and theory is not possible. There are two ways to resolve this problem. The first way is to integrate the theoretical cross section over the experimental bin. This integration is rather difficult due to the asymptotic behaviour, but can be performed as will be discussed below. Another way to resolve the problem is to choose another set of independent variables with a Jacobian without singularities in the measured kinematical region.

The Harvard coordinates have specifically been constructed to circumvent this problem of asymptotic behaviour. An expression for the Jacobian of the Harvard coordinates can be derived in a similar fashion as has been done for spherical coordinates. This much more tedious calculation is described in detail in Ref. [30]. The behaviour of the Jacobian for both sets of coordinates is depicted in Fig. 2.3. Both Jacobians are plotted for identical kinematics. One can see that the Jacobian of the Harvard geometry does not diverge at the kinematic extreme. For this reason Harvard coordinates have been advocated [31]. However, Harvard coordinates introduce another problem. This is illustrated in Fig. 2.4, where for four fixed proton geometries the relation between \( \theta^\gamma \) and \( \psi^\gamma \) is plotted. The polar scattering angles of both protons are fixed at 16°. The non-coplanarity angle, as defined in Eq. (2.12), is kept constant at 0° (coplanar), 10°, 20° and 30°, respectively. For the coplanar case, \( \psi^\gamma \) simply reduces to \( \theta^\gamma \). At non-coplanar kinematics, the allowed range of \( \theta^\gamma \) is stretched to the complete 0°–180° range. The experimental problem resides in the determination of \( \psi^\gamma \). The experimentally-determined \( \theta^\gamma \) is always associated with a certain uncertainty. In the bremsstrahlung measurements which are subject of the thesis for example, the resolution in \( \theta^\gamma \) is typically 4°. At \( \Phi = 20^\circ \) and \( \theta^\gamma = 110^\circ \) this translates to a 20° uncertainty in \( \psi^\gamma \). Here, the resolutions of the proton angles are not yet folded in. This will make the resolution in \( \psi^\gamma \) even worse. Unfortunately, it is in general exactly there where the standard representation of the cross section shows singular behaviour, where \( \psi^\gamma \) is experimentally ill-defined. In order to make small bins in Harvard-coordinates, one would need an infinite resolution for the photon kinematics. Large bins in \( \psi^\gamma \) are experimentally feasible, but this would hamper a direct comparison between theory and experiments.

The only way out of this situation is to perform the integration of the theoretical cross section over the experimental bin. Theoretically, the cross section is a product of the invariant matrix element squared (\( |\mathcal{M}|^2 \)) and a phase-space factor (\( J \)). An explicit expression for \( J \) in spherical coordinates is given in Eq. (2.51). The variations within a small experimental bin are due to the phase-space factor, and not due to the matrix element, which reflects the reaction dynamics. Under the assumption of a constant matrix element within the experimental bin, one can make the following approximation:

\[
\left( \frac{d\sigma}{d\Omega_1 d\Omega_2 d\theta^\gamma} \right)_{\text{exp}} = \frac{1}{\text{binsize}} \int_{\text{bin}} J |\mathcal{M}|^2 d\Omega_1 d\Omega_2 d\theta^\gamma
\]

\[
\approx |\mathcal{M}|^2 \frac{1}{\text{binsize}} \int_{\text{bin}} J d\Omega_1 d\Omega_2 d\theta^\gamma \quad \text{(2.53)}
\]
2.4. PHASE-SPACE CONSIDERATIONS

Figure 2.3: Typical behaviour of the Jacobians of spherical and Harvard coordinates for identical kinematics. The spherical coordinates $\theta_1$, $\theta_2$ and $\theta_\gamma$ have been fixed, while the non-coplanarity $\Phi'$ is varied between 0 and its kinematically-allowed maximum.

Figure 2.4: The relation between $\theta_\gamma$ and $\psi_\gamma$, for fixed proton kinematics. Both proton polar angles are fixed at 16°. Observe that for large non-coplanarity angles, a small step in $\theta_\gamma$ results in a large step in $\psi_\gamma$. 
The integral of the phase-space factor $J$ over the bin is model independent and can be performed by the experimentalists. Even if the singularity is contained within the experimental bin and the integral over $J$ is thus an improper integral, it can be integrated numerically. It is stressed that the approximation made in Eq. (2.53) is only valid for small bins. The thus obtained matrix element squared is a Lorentz-invariant object and, therefore, independent of the initial choice of the coordinate system. Hence, along with the cross sections also the invariant squared matrix elements $|\mathcal{M}|^2$ will be presented, whenever a direct comparison with theory is hampered due to phase-space variations within the experimental bin.

2.5 Theoretical description of $pp\gamma$

In this section an overview is given of the models and the ingredients used for the calculation of bremsstrahlung observables. First, the classical description of bremsstrahlung is discussed. Then a derivation of the electromagnetic current is given in a quantum-mechanical description. Subsequently, the low-energy theorem and the soft-photon approximation are outlined. This theorem makes the connection between elastic scattering and the bremsstrahlung process. Finally, the state-of-the-art nuclear models are discussed.

2.5.1 Classical description of bremsstrahlung

According to Maxwell’s theory of electromagnetism, acceleration of charged particles results in the emission of electromagnetic radiation. In a scattering process involving charged particles, the charged particles change their momentum and are thus accelerated. This explains the origin of the word bremsstrahlung, which is German for “braking radiation”. The general classical expression for the intensity of radiation in cgs units is given by [35]:

$$\frac{dI}{d\omega d\Omega} = \frac{e^2}{4\pi^2c} \left| \int \frac{dt}{d\tau} \left( \frac{n \times (n \times \mathbf{\beta})}{1 - n \cdot \mathbf{\beta}} \right) e^{i\omega(t-n/c)d\tau} \right|^2$$  \hspace{1cm} (2.54)

The quantity $\frac{dI}{d\omega d\Omega}$ is the energy radiated per solid angle ($\Omega$) per frequency ($\omega$). Furthermore, $\mathbf{\beta}$ is the velocity vector of the particle in units of light-speed $c$, $\mathbf{r}$ is the position vector of the particle and $\mathbf{n}$ is the unit vector in the direction of $d\Omega$. In the case of a collision between two particles, all accelerations take place in both a short time and a small region of space. The case where the observer of the radiation is far away, the vector $\mathbf{n}$ is a constant, and the velocity changes from $\mathbf{\beta}_i$ to $\mathbf{\beta}_f$. It is clear from Eq. (2.54) that the distribution of radiation is dependent on the microscopic details of the collision. However, in the low energy part of the spectrum, where $\omega \approx 0$, the exponent $e^{i\omega(t-n/c)}$ reduces to unity. The integral is then a trivial one:

$$\frac{dI}{d\omega d\Omega} = \frac{e^2}{4\pi^2c} \left| e^{i\mathbf{\epsilon} \cdot \left( \frac{\mathbf{\beta}_f}{1 - n \cdot \mathbf{\beta}_f} - \frac{\mathbf{\beta}_i}{1 - n \cdot \mathbf{\beta}_i} \right)} \right|^2$$  \hspace{1cm} (2.55)
Here, $e^*$ is the photon polarization. Dividing the intensity by $E_\gamma = \hbar \omega_\gamma$ results thus in the number of photons produced in the collision, i.e. $dI = dN E_\gamma$. If the cross section $d\sigma/d\Omega_1$ for the process transforming a particle with velocity $\vec{\beta}_i$ to a particle with velocity $\vec{\beta}_f$ is known, one can express the bremsstrahlung cross section for “soft photons” as:

$$d\sigma = \frac{\alpha}{4\pi^2 E_\gamma} \left| e^* \left( \frac{\vec{\beta}_f}{1 - \vec{n} \cdot \vec{\beta}_f} - \frac{\vec{\beta}_i}{1 - \vec{n} \cdot \vec{\beta}_i} \right) \right|^2 d\sigma/d\Omega_1$$

(2.56)

where $\alpha = e^2/\hbar c$ is the fine-structure constant. This expression is known as the classical soft-photon approximation. The assumption is made that the bremsstrahlung photon does not alter the primary interaction.

This classical soft-photon approximation is valid for scattering of a light projectile on an infinitely heavy target. In the case of proton-proton bremsstrahlung, the target and projectile have the same mass. Henceforth, the acceleration of the target proton cannot be neglected and one has to make a coherent sum of the bremsstrahlung amplitudes of protons 1 and 2. This results in:

$$d\sigma = \frac{\alpha}{4\pi^2 E_\gamma} \left| e^* \left( \frac{\vec{\beta}_{f,1}}{1 - \vec{n} \cdot \vec{\beta}_{f,1}} - \frac{\vec{\beta}_{i,1}}{1 - \vec{n} \cdot \vec{\beta}_{i,1}} \right) + \frac{\vec{\beta}_{f,2}}{1 - \vec{n} \cdot \vec{\beta}_{f,2}} - \frac{\vec{\beta}_{i,2}}{1 - \vec{n} \cdot \vec{\beta}_{i,2}} \right|^2 d\sigma/d\Omega_1$$

(2.57)

If one expands this expression in $\vec{n} \cdot \vec{\beta}$, the leading order term is the dipole contribution. The dipole amplitude is proportional to $e^* \cdot (\vec{\beta}_{f,1} - \vec{\beta}_{i,1})$. In the center-of-mass system, the sum of velocities is by definition zero, i.e. $\vec{\beta}_i + \vec{\beta}_f = 0$. Therefore, one can conclude that the dipole contribution is absent in proton-proton bremsstrahlung.

From this expression one can make two other observations. Eq. (2.56) is only valid if the wavelength of the photons is large compared to the region of space where all accelerations take place. Therefore, to extract more information about the system of interest than can be extracted from elastic scattering, one has to go beyond the validity of this soft-photon approximation. Secondly, the bremsstrahlung cross section is proportional to $\alpha d\sigma/d\Omega_1$, where $\alpha \approx 1/137$. The bremsstrahlung cross section is thus roughly two orders of magnitude lower than the elastic-scattering cross section of the same system.

### 2.5.2 Derivation of the electromagnetic current

From the expression for the infinitesimal cross section, Eq. (2.39), one can see that all dynamics is stored in the invariant amplitude $M$. In the case of bremsstrahlung, this amplitude is given by:

$$M = e^* \langle \psi_f | J_\mu | \psi_i \rangle$$

(2.58)
Here, $\varepsilon^\mu$ is the polarization vector of the photon and $J^\mu$ is the nuclear current. The final and initial states, $\psi_f$ and $\psi_i$, are the strongly interacting $NN$ wavefunctions. In order to find the nuclear current, first the nuclear Hamiltonian is considered:

$$ H = \frac{-\nabla_1^2 - \nabla_2^2}{2m_p} + V_{NN} \quad (2.59) $$

Here, $V_{NN}$ is the nuclear potential. The operators $\nabla_1^2$ and $\nabla_2^2$ are the kinetic-energy operators acting on the wave functions of particles one and two, respectively. The one-body electromagnetic potential can be obtained by minimal substitution:

$$ \nabla \rightarrow \nabla - i e A $$

$$ H \rightarrow H + 2eA_0 \quad (2.60) $$

where $e$ is the charge of the proton and the factor 2 in Eq. (2.61) comes from the fact that the system contains two protons. Since the electromagnetic force can be treated perturbatively to first order with sufficient accuracy, only terms linear in $A^\mu$ are kept.

If one assumes a local potential, where $V_{NN}$ is independent of momenta, one then finds for the electromagnetic potential in the nuclear Hamiltonian:

$$ V_{em} = \frac{ie}{2m_p} \{\nabla, A\} + 2eA_0 \quad (2.62) $$

where the symbol $\{\, , \}$ denotes the anti-commutator. The electromagnetic current is defined as a functional derivative:

$$ j^\mu = \frac{\delta H}{\delta A^\mu} \bigg|_{A^\mu=0} \quad (2.63) $$

$$ \rho_i = e\delta^3(x - r_1) + e\delta^3(x - r_2) \quad (2.64) $$

$$ j_1^{con} = \frac{e}{2m_p} (\{\nabla_1, \delta^3(x - r_1)\} + \{\nabla_2, \delta^3(x - r_2)\}) \quad (2.65) $$

Here, $\rho_i$ is the one-body charge density and $j_1^{con}$ is the one-body convection-current density. From minimal substitution, only the electric current is obtained, and not the magnetic current, which acts on the spin. The spin magnetic current is given by:

$$ j_1^{magn} = \nabla \times \left( \frac{e\mu_p}{2m_p} \sigma_1 \delta^3(x - r_1) + \frac{e\mu_p}{2m_p} \sigma_2 \delta^3(x - r_2) \right) \quad (2.66) $$

Here, $e\mu_p/2m_p$ is the magnetic moment of the proton and $\sigma_1$ and $\sigma_2$ are spin operators on protons 1 and 2, respectively. The total one-body current is given by the sum of the convection and the magnetic currents.

An important requirement of any reasonable theory is current conservation. This condition is expressed by the continuity equation:

$$ \partial^\mu J_\mu = -i[H, \rho] \quad (2.67) $$
With a little algebra one can show that:

\[ \nabla \cdot (j_1^{\text{ion}} + j_4^{\text{magn}}) = -[T, \rho_i] \]  

(2.68)

where \( T \) is the kinetic-energy operator. In the case that the potential \( V_{NN} \) is non-local, it does not commute with the one-body charge density. Therefore, in that case an additional two-body current is necessary in order to ensure current conservation:

\[ \nabla \cdot j_2 = -[V_{NN}, \rho_i] \]  

(2.69)

A problem that arises in constructing a two-body current is that it is not uniquely defined by the requirement of current conservation. One needs knowledge of an underlying dynamical model, like QCD, to construct the complete two-body current. The most notable examples of two-body currents are the so called “meson-exchange currents” and the \( \Delta \)-isobar (see section 2.5.4).

### 2.5.3 The low-energy theorem and soft-photon approximations

The interest in proton-proton bremsstrahlung is guided by the fact, that one is able to probe the \( NN \)-interaction in a region not probed by elastic scattering. In 1958, Low proved that to the two lowest orders in the photon energy, the bremsstrahlung amplitude is governed by “on-shell information” only [7]. This low-energy theorem states that if the co-variant amplitude \( M \) is expanded in a series of the photon momentum \( k_0 \), the first two terms \( A \) and \( B \) are uniquely determined by on-shell \( NN \) phase shifts and the on-shell electromagnetic vertex:

\[ M = \frac{A}{k_0} + B + O(k_0) \]  

(2.70)

The phase shifts and thus the parameters \( A \) and \( B \) are known with high accuracy from elastic \( NN \)-scattering experiments. This theorem is a consequence of the law of current conservation and its detailed derivation may be found in Ref. [8]. According to this theorem, any reasonable theory capable of calculating \( M \), should reproduce the correct \( A \) and \( B \) coefficients in Eq. (2.70).

One can exploit this theorem to obtain an expression for the bremsstrahlung amplitude, without any knowledge of the details of the underlying interaction. Such amplitudes are commonly referred to as Soft-Photon Approximations (SPA). The SPA relies on the following lemma, due to Adler and Dothan [36]. The four-vectors \( M_\mu(x, k_\nu) \) and \( M_\mu^0(x, k_\nu) \) are functions of \( x \) and \( k_\nu \) and \( M_\mu^0(x) \) is a function of \( x \) only, where \( x \) is an arbitrary set of parameters. If they are chosen such that:

\[ M_\mu(x, k_\nu) = M_\mu^0(x, k_\nu) + M_\mu^0(x) + O(k) \]  

(2.71)

\[ k^\mu M_\mu(x, k_\nu) = k^\mu M_\mu^0(x, k_\nu) \]  

(2.72)

then one can conclude that:

\[ M_\mu^0(x) = 0 \]  

(2.73)
An alternative way of expressing the law of current conservation, Eq. (2.67), is:

$$k^\mu M_\mu = 0$$  

(2.74)

One can now apply the above lemma to the $pp\gamma$ amplitude. If one calculates an amplitude in SPA, one only has to calculate the zeroth order amplitude $M_{SPA}^{(0)}(x,k_\nu)$. If, in addition, imposes the law of current conservation to this amplitude, such that

$$k^\mu M^{(0)}(x,k_\nu) = 0,$$

the resulting amplitude is by the lemma guaranteed to be correct to first order in $k$, since $k^\mu M^n(x,k_\nu) = k^\mu M^{SPA}(x,k_\nu)$.

The first framework for SPA-calculations for the nucleon-nucleon system was developed by Nyman [8]. At about the same time an extension of the Low-Energy Theorem for particles with spin was developed by Burnett and Kroll [37], which was applied to the case of $NN\gamma$ by Fearing [38]. More recently, SPAs have been developed by Liou et al. [9,10] and by Korchin et al. [39], which respects both current conservation and in addition the Pauli principle. All the approaches mentioned above provide phenomenological amplitudes, which work for low-energy photon production. When applied to high-energy photon production, these amplitudes are “ad-hoc” extrapolations, containing no information on the underlying dynamics of the interaction.

### 2.5.4 Microscopic nuclear models

In order to investigate the reaction mechanism in proton-proton bremsstrahlung, one needs a model of the nucleon-nucleon interaction. In the 1970’s calculations were performed with one-pion exchange models, see for example Ref. [40] and references therein. In the 1980’s the first calculations for bremsstrahlung were carried out with meson-exchange potentials [41, 42]. Although many refinements have been made since, the meson-exchange models of the $NN$-interaction are still considered state-of-the-art. Examples of such models are those developed by the Bonn group [1], the Paris group [2], the Nijmegen group [3] and the Utrecht group [4]. The bosons describing the forces in these $NN$-models are mesons, like the three pions ($\pi^+$, $\pi^0$, $\pi^-$), the eta-meson ($\eta$), the rho-meson ($\rho$) and the omega-meson ($\omega$). Since the meson-exchange models could not describe the data taken at TRIUMF [23] with satisfactory accuracy, several attempts have been made to investigate the sensitivity of the bremsstrahlung amplitude to higher order effects [12, 43, 44].

Since the coupling constants of the $NN$-force are not small compared to 1, one cannot treat this force in a perturbative framework. Instead, one has to find the $NN$-transition amplitude, the T-matrix, as the solution of the integral equation:

$$T = V_{NN} + V_{NN}G T$$  

(2.75)

Where $V_{NN}$ is the nucleon-nucleon potential and $G$ is the two-nucleon propagator. This integral equation is in a non-relativistic framework the Lippman-Schwinger equation, and in a relativistic framework the Bethe-Salpeter equation.

The T-matrix is a function of three independent variables: $T(p_i,p_f,E)$. Here $p_i$ and $p_f$ are the initial and final relative momenta of the $NN$-pair, and $E$ is the total energy of
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Figure 2.5: The single-scattering diagrams.

Figure 2.6: The double-scattering or re-scattering diagrams.

The $NN$-system. In the case of elastic scattering, all particles in the initial and final states are on their mass shell. This implies that $p_i = p_f$ and $E = E(p_i)$. In the case of off-shell nucleons participate in the interaction, one can define the half-off-shell $T$-matrix, where $p_i \neq p_f$ and $E = E(p_i)$ and the full-off-shell $T$-matrix, where $p_i \neq p_f$ and $E \neq E(p_i)$. In the bremsstrahlung reaction half-off-shell $T$-matrices enter only.

There are different components which contribute to $M$, which can be depicted as Feynman-like diagrams. They are not “true” Feynman diagrams, because they contain the $T$-matrix, which is calculated in a non-perturbative framework. The first part of the amplitude is the single-scattering contribution, depicted in Fig. 2.5. Here, the photon is coupled to the external legs of the $NN$-interaction. The contribution of these diagrams to the total amplitude is roughly 75 percent for the $pp$ case.

The second part of the amplitude is the re-scattering contributions, depicted in Fig. 2.6. Here, the two nucleons scatter both before and after the photon emission. These diagrams contribute roughly 20 percent to the total amplitude.

In addition to the simple single- and double-scattering contributions, there are higher-order contributions to the co-variant amplitude $M$, which are due to the two-body currents. The most important higher-order terms are the virtual $\Delta$-isobar and the magnetic meson-exchange currents [13, 14, 15, 16, 17]. The $\Delta$-isobar is an excited state of the
three-quark system composing the nucleon. Since this intermediate state can only be understood in the context of the underlying QCD-structure, it has to be put into the models by hand. This is related to the fact that only the divergence of the two-body current is determined by the $NN$-potential in Eq. (2.69) and not the curl. In Fig. 2.7 the $Δ$-isobar contribution is depicted schematically in the left most picture. The $Δ$ has to be taken into account in both the single-scattering and the re-scattering contributions. The magnetic meson-exchange contribution consists of an intermediate vector-meson with quantum numbers $J^P = 1^- (\rho \text{ or } ω)$ being transformed into a pseudo-scalar meson with quantum numbers $J^P = 0^- (\pi^0)$. This change of quantum numbers classifies the transition as M1. The magnetic meson-exchange contribution is depicted in the middle diagram of Fig. 2.7. This diagram is only one out of four diagrams, which are usually taken into account for consistency. The MEC-line can occur without a T-matrix, but it can also occur before and after the $NN$-pair has interacted via a T-matrix. Furthermore, it can occur in a rescattering, where both before and after the MEC interaction, a T-matrix couples to the $NN$-pair. The $Δ$-isobar and magnetic meson-exchange currents have to be implemented together in a consistent way, since they can be transformed into each other at the quark level. The effect the meson-exchange current and the $Δ$-isobar have on the bremsstrahlung observables is in general small at energies well below the pion-production threshold, but they can be probed with high-quality data. In Fig. 2.8 the predictions of the same model are plotted twice, once without the two-body contribution, and once including them. An experiment aiming to probe the higher-order effects, should have an accuracy high enough to disentangle the two calculations.

Another higher-order contribution to the amplitude is that of the negative-energy states [18]. In these states, a real photon and a proton anti-proton pair are generated out of vacuum. The virtual anti-proton then annihilates with another proton. The coherent sum of all contribution containing this process, is negligible at 190 MeV beam energy, but can become substantial beyond to the pion-production threshold. It is depicted in the most right diagram of Fig. 2.7. The negative-energy states are often denoted as $NN$-states. Strictly speaking, the negative energy states contribute in first order to the amplitude. The
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reason that the contribution is quite small at intermediate energies resides in the fact that it is a relativistic effect.

Finally, another effect which can be taken into account is the off-shell behaviour of the electromagnetic vertex [19, 20]. Usually, this vertex is assumed to be a function of the on-shell invariant mass of the nucleon. However, this does not necessarily have to be the case. The extent to which this effect contributes is still a subject of discussion.