Dressing the nucleon causally
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Chapter 1

Introduction

All properties of hadrons, as well as their interactions, are believed to be expressible in terms of interactions of more “elementary” particles – quarks and gluons – and thus should be part of the solution of Quantum Chromodynamics. Due to the complexity of this theory, especially at energies of a few hundred MeV, such a solution has not been obtained up to now. However, the full knowledge of the underlying quark-gluon dynamics is not always necessary to describe many reactions involving hadrons. Rather than dealing with quarks and gluons, one often uses effective degrees of freedom which, though possibly having quite a complicated quark structure, are germane to the processes in the given energy range. Such effective approaches have been applied to study various properties of hadrons and their interactions. Thus, much interest has been drawn recently by the subject of nucleon polarizabilities. These fundamental constants characterize deformation of the nucleon in the external electromagnetic field and as such bear an imprint of the composite nature of the nucleon. The polarizabilities are extracted from the cross section of Compton scattering, one of the simplest processes involving the interaction between the nucleon (N) and the photon (γ). The theoretical framework of the present thesis, which falls into the category of dynamical approaches using hadronic degrees of freedom, can be conveniently introduced by considering Compton scattering.

To study Compton scattering $\gamma N \to \gamma N$ at energies above the pion ($\pi$) production threshold, one needs to take into account effects of the rescattering of a pion in the intermediate state, $\gamma N \to \pi N \to \gamma N$, which proceeds via the pion photoproduction reaction $\gamma N \to \pi N$. Pion photoproduction is in turn affected by pion-nucleon scattering through the rescattering $\gamma N \to \pi N \to \pi N$. Since the pion mass is small, $m_\pi \approx 138$ MeV, the effects of pion rescattering are significant for understanding of Compton scattering even at
relatively low energies. A unified framework to describe these three reactions is thus necessary.

What constraints should be incorporated in such a coupled-channel approach? First of all, it should be two-body ($\pi N$) unitary, i.e. it should yield a scattering matrix $S$ which is unitary at energies below the three-body ($\pi \pi N$) threshold. Unitarity takes care that the sum of probabilities of transitions from an initial state to all final states equals unity, even at energies where pion production is possible. Another crucial constraint is gauge invariance, which ensures conservation of electromagnetic current in the reactions involving photons. The next condition is related to the existence of identical particles in the incoming and outgoing states of the reactions $\gamma N \rightarrow \gamma N$ and $\pi N \rightarrow \pi N$. This leads to certain restrictions on the scattering amplitude $T$ (the T matrix) of these processes, known as the crossing symmetry relations and the invariance with respect to the charge conjugation, space inversion and time reversal (CPT) transformations. The gauge invariance, crossing and CPT symmetries constitute the basis of the low-energy theorem for Compton scattering [1], which gives the cross section up to first order in a small energy of the photon. An important constraint on pion-nucleon interaction at low energies stems from chiral symmetry [2, 3].

Motivated by the above considerations, the approach of this thesis is based on the K-matrix formalism. Such a framework provides an easy way to obtain a T matrix in which all rescattering contributions necessary for two-body unitarity are summed up, which is called unitarization. The kernel in this approach, the K matrix, determines the physical contents of the model. The K-matrix approach is very suitable for satisfying gauge invariance and crossing symmetry constraints for the T matrix: it suffices to construct a K matrix which satisfies these conditions. Another motivation for choosing the K-matrix formalism is that it presents an appropriate framework in which we embed a dynamical model for nucleon form factors relevant for Compton scattering, pion photoproduction and pion-nucleon scattering, so-called half-off-shell form factors.

In addition to the constraints mentioned above, the scattering amplitude should possess certain analyticity properties. It has been shown [4–9] that these properties are essential for reproducing the characteristic energy-dependence of the Compton scattering amplitude near the pion production threshold. From a theoretical point of view, analyticity is related to the condition of causality [10–13, 2]. It is an obvious requirement of causality that detection of a particle cannot precede its emission. Thus, the detectable – real – particles can transport only causal signals. The energy $E$ and 3-momentum $\vec{p}$ (together forming the 4-momentum $p$) of a real particle are not independent
but rather confined to the so-called mass shell:

\[ p^2 \equiv E^2 - \vec{p}^2 = m^2, \]  

(1.1)

where \( m \) is the mass of the particle. Complementary to the notion of a real (on-shell) particle is the concept of a virtual (off-shell) particle, which is defined by requiring that its energy and 3-momentum do not obey Eq. (1.1). According to this definition, a particle cannot be detected if it is off the mass shell. In spite of being purely abstract objects, such particles do enter in the theoretical picture of scattering processes. In particular, intermediate states through which the scattering proceeds are described in terms of propagation of virtual particles. If a particle is off the mass shell, it may transport noncausal signals, which are unmeasurable (and, in this sense, unphysical). However, the presence of off-shell particles in the theoretical description of the intermediate states must not be in conflict with the observed fact that physical processes are causal and independent of the frame of reference (Lorentz invariant). This requirement is met if the functions characterizing propagation of and interactions among the intermediate off-shell particles have certain analyticity properties.

Every rescattering contribution is represented by a loop integral which can be written in terms of one or several spinor structures each of which is multiplied with a scalar function. In the loop integrals describing the rescattering \( \gamma N \rightarrow \pi N \rightarrow \gamma N \), for example, the imaginary (pole) and real (principal-value) parts of the scalar functions are associated, respectively, with the propagation of on-shell and off-shell particles in the intermediate state. Due to analyticity properties, the real and imaginary parts of these functions are related through dispersion integrals [11, 14, 12, 15, 2]. Usually, in K-matrix models one does not include contributions from rescattering with off-shell particles in the intermediate state and thus constructs the K matrix as a sum of tree Feynman diagrams in which free propagators and bare vertices are used [16–20]. Such a form of the K matrix implies that, although the pole parts of the loop integrals are included in the T matrix due to unitarization, their principal-value parts are completely ignored. Consequently, the property of analyticity is strongly violated.

In contrast to the traditional approach, we construct the K matrix using vertices and propagators which contain the principal-value parts of a wide class of loop integrals whose pole parts are generated in the T matrix through unitarization. Since the T matrix now contains both pole and principal-value parts of these loop integrals, we incorporate some of the analyticity constraints in the K-matrix framework. The inclusion of the principal-value parts in an
iteration procedure constitutes a “dressing” of free propagators and bare vertices with an infinite number of meson loops corresponding to virtual rescattering contributions. The dressed nucleon propagator and the dressed vertices ($\pi NN$ and $\gamma NN$) are written in terms of scalar functions which are called self-energy functions and form factors, respectively. It is in the calculation of these functions that the analyticity properties are incorporated in the present model. Namely, we employ dispersion relations in the dressing procedure to obtain the real parts of the form factors and self-energy functions, in terms of which the K matrix is constructed.

The implementation of the analyticity properties of the self-energy functions and form factors is a necessary, albeit not sufficient, condition for the analyticity of the full T matrix. Violation of analyticity of the T matrix is mainly due to the fact that the K matrix does not contain the principal-value parts of all loop integrals whose pole parts are generated in $T$, which is most significant for Compton scattering. This deficiency is mitigated in the physically important region of energies near the pion threshold by adding an appropriate “contact” $\gamma\gamma NN$ vertex to the K matrix.

Besides K-matrix models, other approaches have been utilized to calculate a two-body unitary S matrix for various reactions. For example, in the context of pion-nucleon scattering, a solution of the Bethe-Salpeter equation [21] has been presented recently [22]. Also, models based on other relativistic wave equations (such as three-dimensional reductions of the Bethe-Salpeter equation) have been developed [23–26]. While, compared to the present model, in these approaches one takes into account both pole and principal-value parts of all loop integrals which are included, the crossing symmetry constraint is usually not preserved. Another difference concerns the extension to channels including photons: unlike in the K-matrix approach, gauge invariance of the kernel of such a wave equation does not automatically lead to gauge invariance of the full amplitude [27–29]. In the present model, we obtain a crossing symmetric and gauge invariant amplitude by constructing a kernel, the K matrix, which possesses these properties.

To obey gauge invariance, we include appropriate $\gamma\gamma NN$ and $\gamma\pi NN$ contact terms both in the K matrix and in the dressing procedure for the $\gamma NN$ vertex. These are constructed using the prescription of minimal substitution. An inherent limitation of minimal substitution is that it allows one to determine unambiguously only that part of a contact terms which is parallel to the photon 4-momentum. We exploit the ambiguity in the transverse part in order to reproduce the behaviour of the pion photoproduction and Compton amplitudes at the pion production threshold.

One of the main objectives of this work is to study effects of the dressing on
observables in Compton scattering, pion photoproduction and pion-nucleon scattering. In particular, we will show that the dressing has a considerable effect on nucleon polarizabilities, and the use of dispersion relations in the dressing procedure is essential for reproducing the observed cusp-like behaviour of the Compton amplitude in the vicinity of pion threshold. In the present work, the effects of nucleon dressing are expressed in terms of the constructed nucleon self-energy functions and \( \pi NN \) and \( \gamma NN \) form factors. One often considers only the dependence of form factors on the 4-momentum squared of the meson or photon. This dependence is important for the description of such reactions as electron-nucleon and nucleon-nucleon scattering. By contrast, both the dressing procedure and the calculation of the observables in our approach are arranged in a way that allows us to deal only with half-off-shell form factors. These form factors depend on the 4-momentum squared of the intermediate off-shell nucleon. Off-shell form factors are a ubiquitous ingredient of many models of hadronic reactions; they have been employed in calculations of proton-proton bremsstrahlung \([30]\), pion photoproduction \([27, 28, 31, 29, 33, 20]\), pion-nucleon scattering \([23-25, 31, 18, 26]\), virtual Compton scattering \([34]\), meson production in nucleon-nucleon collisions \([35]\) and other reactions. Off-shell \( \pi NN \) and \( \gamma NN \) form factors have been studied in the past, using dispersion methods \([14, 36, 37]\), field-theoretical models \([38]\) and one-loop calculations \([39]\). The main distinguishing features of our dressing model for the half-off-shell form factors are the following. Firstly, we effectively include loop corrections up to infinite order. Secondly, in calculating the loop integrals, we implement analyticity properties of the form factors through the usage of dispersion relations. Thirdly, the form factors and the nucleon self-energy are calculated together, being obtained as a solution of a system of coupled integral equations in terms of which the dressing is formulated. Fourthly, our model for the form factors is developed as a built-in part of the \( K \)-matrix approach in which physical observables are calculated.

It is important that we use the same representation of the effective Lagrangian for both the dressing procedure and the calculation of the scattering amplitude. The off-shell 3-point functions (such as the dressed \( \pi NN \) and \( \gamma NN \) vertices considered in this work) and 2-point functions (propagators) as well as 4-point functions (for example, a \( \pi \pi NN \) contact term) may be entirely different in different representations, while the total scattering amplitude is the same \([40-45, 13]\). In other words, in transforming from one representation to another, all the \( n \)-point functions change consistently with each other in such a way that the scattering amplitude, which is built out of
these quantities, remains the same in all representations\textsuperscript{1}. The representation-
dependence of n-point functions implies, in particular, that the form factors
and self-energy functions, which are central to the present approach, are not
measurable quantities. Clearly, the scattering amplitude and all its off-shell
building blocks should be calculated in the same representation. Each rep-resentation corresponds to a particular choice of interacting field coordinates
and thus to a particular form of the effective Lagrangian. Using the fact that
the full contents of a theory can be equivalently expressed either in terms of
interacting fields or in terms of n-point functions [46, 42, 13], we define a rep-resentation by imposing constraints on n-point functions. In particular, in the
representation used throughout this work 4-point $\pi\pi NN$ (and higher-point)
vertices are excluded.

In view of the special role that the dressed nucleon propagator and the
dressed half-off-shell $\pi NN$ and $\gamma NN$ vertices play as principal building blocks
of the K matrix, the dressing procedure for these 2- and 3-point functions will
be described in detail, being in fact the central subject of this thesis. In Chap-
ters 2 and 3 we shall discuss the purely hadronic sector of the model, including
the K matrix for the pion-nucleon scattering, the dressed nucleon propagator
and the dressed $\pi NN$ vertex. To explain the dressing procedure retaining its
most essential ingredients, in Chapter 2 we shall work in a simplified model
including only the nucleon and pion degrees of freedom. This model will be
augmented in Chapter 3 by incorporating the $\Delta$ resonance and the $\rho$ and
$\sigma$ mesons, providing a satisfactory quantitative description of pion-nucleon
scattering up to pion laboratory energies of 400 MeV. The extension of the
model to include the photon is described in Chapters 4 and 5, following the
same line of presentation as laid out for the hadronic sector. In Chapter 4 the
coupled-channel K-matrix approach is outlined and the dressing procedure for
the half-off-shell $\gamma NN$ vertex is explained employing only the nucleon, pion
and photon degrees of freedom. Also, in Chapter 4 we describe the procedure
of minimal substitution and prove gauge invariance of the model. The full
development of the model is presented in Chapter 5, including calculations of
observables in pion photoproduction and Compton scattering. A good agree-
ment with experiment is achieved up to photon laboratory energies of 500
MeV. In particular, the calculated scalar and vector polarizabilities of the
proton and neutron are shown to be consistent with the measured values. In
presenting results of calculations, the main emphasis will be made on studying
effects of the developed dressing procedure.

\textsuperscript{1}For example, one can construct representations in which 3-point off-shell form factors
are completely eliminated and their effect on the amplitude is encapsulated in an appropriate
4-point vertex.