We contrast the large scale evolution of the population of marginally bound supercluster-like objects in an accelerating Universe with their continuing internal development while they collapse gravitationally. We identify these objects in a large ΛCDM cosmological simulation of 512^3 particles in a cube of 500h^{-1}Mpc side length, on the basis of the binding density criterion introduced by Dünner et al. (2006). We construct the “supercluster” mass function at $a = 1$ and at $a = 100$ in order to measure the accuracy of the criterion. The recovered mass functions are in good agreement with the theoretical predictions of the Press-Schechter formalism, and the Sheth-Tormen and the Jenkins approximation. According to these mass functions, we may expect to find up to two Shapley-like superclusters in a volume comparable to that of the Local Universe ($z < 0.1$). Our simulations do recover one massive supercluster with a mass of $\sim 8 \times 10^{15}h^{-1}M_{\odot}$ containing 15 cluster members, slightly larger than the Shapley supercluster. The three manifestations of the internal evolution which we investigated in some detail are the shape of the bound objects, their compactness and density profile, and their substructure in terms of the supercluster multiplicity function. While most superclusters are prolate at $a = 1$, we find them to evolve towards a spherical shape at $a = 100$. We also find that they become highly concentrated in the far future while their substructure merges and produces one compact massive object.

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CHAPTER 7: Future Evolution of Superclusters in an Accelerating Universe

7.1 Introduction

The evidence for an accelerated expansion of the Universe has established the possibility of a ‘dark energy’ component. In this scenario, the universe is entering into an accelerating phase, which is increasingly dominated by the dark energy. It is assumed that this dark energy, in its simplest form, behaves like Einstein’s cosmological constant. The possibility of an exponential expansion has motivated the concept of ‘island universes’: the largest bound structures will grow isolated as the expansion accelerates (Chiueh & He 2002; Nagamine & Loeb 2003; Busha et al. 2003; Dünner et al. 2006). In periods when the matter density was larger than the dark energy, structures formed by gravitational instability. Nowadays, the universe is already starting to accelerate, meaning that structure formation is virtually finished. At this stage, those structures that are much denser than the dark energy are not affected by the latter and remain bound, separating from each other at an accelerating rate which prevents them from joining larger structures. Therefore, at the present cosmological time, the largest bound structures are just forming.

Dünner et al. (2006, hereafter Paper I) presented a criterion to determine the limits of bound structures, defining superclusters as the biggest gravitationally bound structures that will be able to form. The criterion defined a density contrast over which a spherical shell will remain bound to a spherically distributed overdensity. Here we use this criterion to identify superclusters in a large cosmological box and study their abundance and shape at the present time and in the future. The abundance or mass function serves as a good indicator of the growth of structure of a cosmological model, while the shape allows us to study their internal evolution and compare with real data.

In this study we aim at contrasting the large scale evolution of structure in an accelerated Universe with that of the internal evolution of bound objects. While clusters of galaxies are the most massive and most recently fully collapsed and virialized structures, superclusters should be seen as the largest bound but not yet fully evolved objects in our Universe. In the future they may develop into genuine “island Universes”. We study the mass function of these objects in order to see the largest “island Universes” that will form. On the other hand, we study three aspects of the continuing internal evolution of the superclusters. Their shape will be one of the most sensitive probes of their continuing contraction and collapse. In addition, their collapse will produce a continuous sharpening of their internal mass distribution and density profile. Finally, we also study their evolving and diminishing substructure, in terms of the supercluster multiplicity function, as its subclumps merge during the collapse process.

The study of future evolution has already been addressed by different authors Chiueh & He (2002) solved the spherical collapse equations numerically, obtaining a theoretical criterion for the mean density enclosed in the last gravitationally bound shell. Busha et al. (2003) followed the internal evolution of the density and velocity structures of bound objects, while Nagamine & Loeb (2003) focused on the evolution of the Local Universe. Dünner et al. (2007) used the criterion of Paper I to study the limits of bound structures in redshift-space. Hoffman et al. (2007) repeated the study of Nagamine & Loeb (2003), adding the dependence on dark matter and dark energy to the fate of the Local Universe. Later, Busha et al. (2007) studied the effects of small-scale structure on the formation of dark matter halos in two different cosmologies. Of the previous works, Nagamine & Loeb (2003), Hoffman et al. (2007) and Busha et al. (2007) studied the mass functions of objects present in their simulation. They all found that after the present time, the mass function hardly changes.

The definition of superclusters have presented a problem because they are not virialized and their overdensity is very low, making it difficult to impose a limit. Using different definitions, shape of superclusters have been studied both using real data (e.g. Plionis et al. 1992; Basilakos et al. 2001; Einasto et al. 2007) and in N-Body simulations (e.g. Basilakos et al. 2006; Wray et al. 2006; Einasto et al. 2007). The different studies have concluded that superclusters have a prolate shape.

This chapter is organized as follows. In section 7.2, we present a review of the spherical collapse model and a prescription for deriving an analytical solution for the spherical collapse equations. Section 7.3 describes the simulation and the group finder algorithm that we employ when determining the mass functions. Section 7.4 presents the mass functions of the bound structures at $a = 1$ and $a = 100$ and a comparison with the ones obtained by the Press-Schechter formalism and its variants. Shapes of
7.2 Spherical Collapse Model

The spherical collapse model (Gunn & Gott 1972; Lilje & Lahav 1991) describes the evolution of a spherically symmetric mass density perturbation in an expanding Universe. It is still a powerful tool for understanding how a spherical patch of homogeneous overdensity forms a bound system through gravitational instability. Since we assume sphericity, the non-linear dynamics of a collapsing shell is determined by the mass interior to it.

We present here a short description of the spherical collapse model following Dünner et al. (2006). Consider a flat Universe with a cosmological constant ($\Lambda$). In this model, a given mass shell with radius $r(t)$ contains a fixed mass $M$ and satisfies the energy equation (Peebles 1984):

$$E = \frac{1}{2} \left( \frac{dr}{dt} \right)^2 - \frac{GM}{r} - \frac{\Lambda r^2}{6} = \text{constant},$$

where $\Lambda = 3H^2\Omega_\Lambda$, $r$ is the shell’s radius, $M$ is the total mass enclosed and the constant $E$ is the total energy per unit mass of the shell. Introducing the following dimensionless variables:

$$\tilde{r} = \left( \frac{\Lambda}{3GM} \right)^{1/2} r,$$

$$\tilde{t} = \left( \frac{\Lambda}{3} \right)^{1/2} t,$$

we can rewrite 7.1 as

$$\tilde{E} = \frac{1}{2} \left( \frac{d\tilde{r}}{dt} \right)^2 - \frac{1}{\tilde{r}} - \frac{\tilde{r}^2}{2},$$

where

$$\tilde{E} = \left( \frac{G^2M^2\Lambda}{3} \right)^{1/2} E.$$

7.2.1 Criterion for Bound Structures

In this section, we briefly describe the steps required in order to obtain an analytical solution for the spherical collapse equations for a bound structure. A full description can be found in Paper I.

We are looking for a critical shell that will asymptotically stay at the limit between expanding forever or recollapsing into the structure. In order to find the energy of such a shell, we maximize the (dimensionless) potential energy defined as:

$$\tilde{V} = -\frac{1}{\tilde{r}} - \frac{\tilde{r}^2}{2}.$$  

The maximum of this potential occurs at $\tilde{r}^* = 1$, so $\tilde{E}^* = \tilde{V}(\tilde{r}^* = 1) = -\frac{3}{2}$ is the maximum possible energy for a shell to remain attached to the structure.

Integrating Eqn. 7.4 for a critical shell (with $\tilde{E} = -\frac{3}{2}$), we get

$$\tilde{t} = \int_0^{\tilde{r}} \frac{\sqrt{\tilde{r}} dr}{(1 - r) \sqrt{r + 2}}.$$
The solution to this integral is given by
\[
\tilde{t} = \frac{1}{2} \sqrt{3} \ln \left( \frac{1 + 2\tilde{r} + \sqrt{3\tilde{r}(\tilde{r} + 2)}}{1 + 2\tilde{r} - \sqrt{3\tilde{r}(\tilde{r} + 2)}} \right) - \ln(1 + \tilde{r} + \sqrt{\tilde{r}(\tilde{r} + 2)}).
\] (7.8)

Knowing that in a flat Universe with cosmological constant the vacuum energy density parameter varies with time as (Peebles 1980)
\[
\Omega_\Lambda \equiv \frac{\Lambda}{3H^2} = \tanh^2 \left( \frac{3\tilde{t}}{2} \right),
\] (7.9)
we can replace Eqn. 7.8 in the latter, obtaining
\[
\Omega_\Lambda(\tilde{r}) = \left[ \chi(\tilde{r}) - 1 \right]^2 \left[ \chi(\tilde{r}) + 1 \right]^{\frac{\chi}{2}},
\] (7.10)

where
\[
\chi(\tilde{r}) = \left[ \frac{1 + 2\tilde{r} + \sqrt{3\tilde{r}(\tilde{r} + 2)}}{1 + 2\tilde{r} - \sqrt{3\tilde{r}(\tilde{r} + 2)}} \right]^{\frac{1}{2}} \times \left( 1 + \tilde{r} + \sqrt{\tilde{r}(\tilde{r} + 2)} \right)^{-3}.
\] (7.11)

Eqns. 7.10 and 7.11 allow us to determine the radius of a critical shell to remain bound in a $\Lambda$CDM Universe. If we evaluate Eqn. 7.10 for our preferred cosmology ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$), we get $\tilde{r}_0 = 0.84$, meaning that a critical shell has a present radius that is 84% of the maximum radius it will reach as $t \to \infty$.

It is convenient to express the critical condition as the minimum enclosed mean density needed by a shell to stay bound. The critical density of the Universe is given by
\[
\rho_c = \frac{3H^2}{8\pi G},
\] (7.12)
and the average mass density enclosed by any given shell is
\[
\bar{\rho}_s = \frac{3M}{4\pi \tilde{r}^3}.
\] (7.13)

The ratio between the enclosed density and the critical density is then given by:
\[
\frac{\bar{\rho}_s}{\rho_c} = \frac{2\Omega_\Lambda}{\tilde{r}_{cs}^3}.
\] (7.14)

The condition for a shell to be bound is then
\[
\frac{\bar{\rho}_s}{\rho_c} \geq \frac{\rho_{cs}}{\rho_c} = \frac{2\Omega_\Lambda}{\tilde{r}_{cs}^3}.
\] (7.15)

where the subscript ‘cs’ stands for critical shell. For $\Omega_\Lambda = 0.7$, we get that this ratio is $\rho_{cs}/\rho_c = 2.36$, where we have used the fact that $\tilde{r}_0 = 0.84$. Evaluating at $\Omega_\Lambda = 1$ ($t \to \infty$), we get $\rho_{cs}/\rho_c = 2$.

This criterion has the property that it has an analytical relation with $\Omega_\Lambda$, allowing us to evaluate it easily at any cosmological time. Verifying this criterion through simulations in Paper I it was found that, on average, 72% of the mass enclosed by the estimated radius is really bound to the structure, while 0.3%, although bound to the structure, is not enclosed by the radius.

7.2.2 Determining the linear overdensity for a marginally bound object

In view of our intention to identify the bound superclusters at any redshift, and their mass spectrum, we also need a proper estimate for the linear critical density value for bound and collapsing mass
7.2. SPHERICAL COLLAPSE MODEL

halos. Among others, this is crucial for a comparison with the theoretical mass functions predicted by Press-Schechter theory (see 7.A). Here we derive the corresponding value.

We know that \( \delta(a) = \delta_0 D(a) \), where \( \delta \) is the density contrast given by Eqn. A-1 (see 7.A) and \( D(a) \) is the amplitude of the growing mode, given by (Heath 1977; Peebles 1980):

\[
D(a) = \frac{5\Omega_m H_0^2}{2} \int_0^a \frac{da'}{a'^3 H(a')} \rho \equiv ag(a), \quad (7.16)
\]

where \( g(a) \) is the linear growth factor. An accurate approximation formula for \( g(a) \) is (Carroll et al. 1992):

\[
g(a) \approx \frac{5}{2} \Omega_m(a)^{4/7} - \Omega_\Lambda(a) + \left( 1 + \frac{\Omega_m(a)}{2} \right) \left( 1 + \frac{\Omega_\Lambda(a)}{70} \right)^{-1}. \quad (7.17)
\]

We need to find \( \delta_0 \) for the critical shell. In order to achieve that, we consider its initial evolution \( (a \ll 1) \).

The density of a spherical region of mass \( M \) and radius \( r \) is given by Eqn. 7.13 and the background density can be computed at any time simply by doing \( \rho_b = \rho_{c,0} \Omega_{m,0}(a^3) \), where \( \rho_{c,0} \) and \( \Omega_{m,0} \) are the critical density and the matter density parameter, respectively, at the present time, and \( a \) is the expansion factor. Using Eqn. 7.2, we can rewrite Eqn. A-1 as:

\[
1 + \delta = \frac{\rho}{\rho_b} = \frac{2\Omega_{\Lambda,0}}{\Omega_{m,0}} \left( \frac{a}{r} \right)^3, \quad (7.18)
\]

where \( \Omega_{\Lambda,0} \) denotes the present value of the cosmological density parameter.

For the problem at hand we make the approximation that the expansion of the Universe is still reasonably approximated by an EdS expansion

\[
a(t) = \left( \frac{t}{t_*} \right)^{2/3}, \quad (7.19)
\]

where \( t \) is some generic time and \( t_* \) is a characteristic constant time. Replacing this equation in Eqn. 7.18 we get

\[
t(\tilde{r}) = t_* \left( 1 + \frac{\delta}{2} \left( \frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}} \right)^{1/2} \right)^{3/2}, \quad (7.20)
\]

were we have kept terms of order lower than \( \delta^2 \). Since we are interested in objects that will remain bound, we can take Eqn. 7.7 and replace the integral by a derivative by doing

\[
\frac{d\tilde{r}}{d\sqrt{\tilde{r}}} = \frac{\sqrt{\tilde{r}}}{(1 - \tilde{r}) \sqrt{r} + 2 \sqrt{r}} \cdot 2 \sqrt{\tilde{r}}. \quad (7.21)
\]

We can expand Eqn. 7.21 (keeping low order terms) and integrate. This gives:

\[
t(\tilde{r}) \approx \sqrt{\frac{2}{3\Lambda}} \tilde{r}^{3/2} + \frac{3}{10} \sqrt{\frac{3}{2\Lambda}} \tilde{r}^{5/2}, \quad (7.22)
\]

Comparing with Eqn. 7.20 we get

\[
t_* = \left[ \frac{4\Omega_{\Lambda,0}}{3\Lambda\Omega_{m,0}} \right]^{1/2} = \left( \frac{1}{6\pi G \rho_{m,0}} \right)^{1/2}, \quad (7.23)
\]

which is the well known result for the time-density relation in a matter dominated Universe, and, in the limit \( a \to 0 \),

\[
\delta_0 g(a \to 0) = \frac{\delta}{a} = \frac{9}{10} \left( \frac{2\Omega_{\Lambda,0}}{\Omega_{m,0}} \right)^{1/3} = 1.504. \quad (7.24)
\]

Evaluating Eqn. 7.24 for our preferred cosmology \( (\Omega_m = 0.3, \Omega_\Lambda = 0.7) \) and using Eqn. 7.17, we get \( \delta(a = 1) = 1.17 \). This is the present linear density contrast for marginally bound structures that will collapse when \( a \to \infty \).
7.3 The Numerical Simulation

We simulate one cosmological model, assuming a standard flat Lambda Cold Dark Matter ($\Lambda$CDM) Universe. The cosmological parameters in the simulation are $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$, and $h = 0.7$, where the Hubble parameter is given by $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, and the normalization of the power spectrum is $\sigma_8 = 1$. In order to have a large sample of bound objects, the simulation box has a side length of $500h^{-1}$ Mpc and contains $512^3$ dark matter particles of mass $m_{dm} = 7.75 \times 10^{10} h^{-1} M_\odot$. The initial conditions were generated at expansion factor $a = 0.02$ (redshift $z = 49$), and were evolved until $a = 100$ using the massive parallel tree N-Body/SPH code GADGET-2 (Springel 2005). The Plummer-equivalent softening was set at $\epsilon_{Pl} = 20 h^{-1}$ kpc in physical units from $a = 1/3$ to $a = 100$, while it was taken to be fixed in comoving units at higher redshift. Given the mass resolution and the size of the box, our simulation allows us to reliably identify massive superclusters with $\sim 80000$ particles. The simulation was performed on the Beowulf Cluster at the University of Groningen.

We took snapshots at the present time ($a = 1$) and in the far future ($a = 100$). We do not expect major large scale evolution anymore beyond $a = 1$, while $a = 100$ is representative for epochs at which the internal evolution of all bound objects has also been completed.

The top two panels of Fig. 7.1 show a slice of $30h^{-1}$ Mpc width of the particle distribution projected along the $z$ axis, both at $a = 1$ and $a = 100$. At $a = 1$, the large-scale structures of the cosmic web are well established, and, as was pointed out by Nagamine & Loeb (2003), they hardly change thereafter. The lower left panel depicts the region shown in the box of the top-left panel in physical coordinates at $a = 1$, centered on a massive structure following the criterion of Eqn. 7.15. The radius of the circle is that of the bound region. We see how it is well connected with the surrounding structures. The lower right panel has the same physical size, and shows the same object, but now at $a = 100$. We see that the size of it (the circle shown) is nearly similar at both expansion factors. Although the accelerated expansion of the Universe freezes in comoving coordinates, in physical coordinates the separation of structures grows exponentially in time. Objects grow in complete isolation, as can be seen from the figure.

7.3.1 HOP Halos

In order to find the objects present in our simulation, we use HOP (Eisenstein & Hut 1998). This algorithm first assigns a density estimate at every particle position by smoothing the density field with an SPH-like kernel using the $n_{dens}$ nearest neighbors of a given particle. In our case, we use $n_{dens} = 64$. Subsequently, particles are linked by associating each particle to the densest particle from the list of its $n_{hop}$ closest neighbors. We use $n_{hop} = 16$. The process is repeated until it reaches the particle that is its own densest neighbor. The algorithm associates all particles to their local maxima. It often happens that the local maxima causes groups to fragment. To correct this, groups are merged if the bridge between them exceeds some chosen density thresholds. Three density thresholds are defined as follows (Cohn et al. 2001):

- $\delta_{outer}$ is the required density for a particle to be in a group.
- $\delta_{saddle}$ is the minimum boundary density between two groups so that they can be merged.
- $\delta_{peak}$ is the minimum central density for a group to be independently viable.

Eisenstein & Hut (1998) claim that the method works best is the values are in the ratio $\delta_{outer}:\delta_{saddle}:\delta_{peak}=1:2.5:3$. We follow their recommendation and use this relation.

7.3.2 Virialized Groups

To identify the bound groups present at each output time, we will first identify virialized groups. For this purpose, we associate $\delta_{peak}$ with the corresponding virial density, $\Delta_{vir}(z)$. 
7.3. THE NUMERICAL SIMULATION

Figure 7.1 — Top panels: $30h^{-1}\text{Mpc}$ slice through the $z$ axis in comoving coordinates, at $a = 1$ (left) and $a = 100$ (right). We see that structures are well defined at $a = 1$ and change little thereafter. The panel on the lower left depicts the region enclosed in the upper panel in physical coordinates, centered on a massive bound structure. The panel on the right has the same physical coordinates, but now at $a = 100$. It is also centered on the same massive bound structure. The radius of the circles are of the size of the structure. We see how the structures surrounding the object at $a = 1$ have detached from it, making it grow in complete isolation.

The value of $\Delta_{\text{vir}}(z)$ is taken from the solution to the collapse of a spherical top-hat perturbation under the assumption that the object has just virialized. Its value is $18\pi^2$ for a critical Universe, and it has to be obtained numerically for other cases. A good approximation for the case of a flat Universe ($\Omega_m + \Omega_\Lambda = 1$) was found by Bryan & Norman (1998):

$$\Delta_{\text{vir}}(z) \approx \frac{18\pi^2 + 82x - 39x^2}{1 + x}$$  \hspace{1cm} (7.25)

where $x = \Omega_m(z) - 1$. This relation is accurate in the range $\Omega_m(z) = 0.1 - 1$. For the cosmology described here, $\Delta_{\text{vir}} \approx 337$. Note that this value is with respect to the background density of the Universe, not the critical density.

At $a = 100$, $\Omega_m = 4.3 \times 10^{-7}$, so we can not apply Eqn. 7.25. We need to find the characteristic radius that will correspond to the virial radius. The energy at that time is $E_{\text{vir}} = K_{\text{vir}} + V_{\text{vir}}$. From the
virial theorem we know that \( K = V/2 \). Then, the (dimensionless) energy at virialization is given by

\[
\tilde{E}_{\text{vir}} = \frac{1}{2} \tilde{V}_{G,\text{vir}} + 2\tilde{V}_{\Lambda,\text{vir}},
\]

where \( \tilde{V}_G \) is the potential energy due to gravity and \( \tilde{V}_\Lambda \) is the potential energy due to the cosmological constant. By energy conservation, the energy at maximum expansion is equal to the energy at virialization. We can then write

\[
\tilde{r}^2 + \frac{1}{2\tilde{r}} = \frac{3}{2}. \tag{7.27}
\]

This is a cubic equation with solutions \( \tilde{r} = 1 \) (maximum radius of the shell), \( \tilde{r} \approx -1.366 \) (unphysical solution) and \( \tilde{r} \approx 0.366 \), which corresponds to the virial radius. Note that Eqn. 7.27 is Eqn. 26 of Lahav et al. (1991) but using our dimensionless variables. We use the corresponding value of \( \tilde{r} \) in Eqn. 7.14, and get \( \tilde{r}/\rho_c \approx 40.8 \). Then, in order to get physically virialized objects with HOP at \( a = 100 \), we use this value divided by \( \Omega_m \) at that time.

### 7.3.3 Halo identification

To extract the groups present in the simulation, we take a subsample of \( 256^3 \) particles (1/8 of the total particle number), both at \( a = 1 \) and \( a = 100 \). We do this because of computer limitation. HOP found \( \sim 20600 \) independently virialized groups at \( a = 1 \) and \( \sim 18000 \) at \( a = 100 \) with more than 50 particles, i.e., total masses \( m \geq 3.1 \times 10^{13} h^{-1} M_\odot \).

Having found the groups, we apply the density criterion of Eqn. 7.15 at \( a = 1 \) and \( a = 100 \). We do this in the following way. We first find the center of mass of the groups given by HOP using the subsampled particles. We first take the densest particle of the group as a guess for the center of mass. We grow a sphere around this center, with the radius being increased until the mean overdensity (density of the sphere with respect to the critical density of the Universe) reaches a value of 300. This value is chosen in order to find the densest core of the structure. We then calculate the center of mass of this sphere and repeat the process, iterating until the shift in the center between successive iterations is less than 1% of the previous value. Once we have the center of mass, we proceed to define the objects present in the simulation. Since we used the subsampled particles to find the center of mass, we proceed to find again the center of mass, but this time considering all the particles in the simulation. We follow the same procedure as described before. Having found the center of mass of the groups, we apply the criterion for a structure to remain bound as given by Eqn. 7.15, i.e., we calculate the density ratio of concentrical spheres with increasing radius until the desired condition is reached.

Now, objects defined by applying Eqn. 7.15 will have larger radii than the groups found by HOP. Since we are constructing bound objects from virialized structures, it is to be expected that we may be counting the same object twice since two or more virialized objects might be bound to each other. We take care of this issue by checking if there is another center of mass inside the bound radius, starting from the most massive group. Any lower mass group whose center of mass is inside a more massive one is taken out of the sample. Doing this gives \( \sim 15000 \) bound groups. Repeating the procedure at \( a = 100 \) does not change the number of bound groups. This indicates that at \( a = 1 \) bound groups are well identified.

### 7.3.4 Sample completeness

Identifying structures at \( a = 1 \) in this way poses another problem. Since we are selecting bound structures that have a virial group in its center, we are forcing our structures to have one virial group with a mass equal or higher to the mass cut we gave HOP (50 particles). This means that we will not detect bound groups with masses lower than \( 3.1 \times 10^{13} h^{-1} M_\odot \). This is not a problem for the most massive groups, but we may have an incomplete sample for lower mass groups. Fig. 7.2 (left panel) shows a scatter plot of the virial groups found by HOP and the bound structure constructed from it. As expected, the lower right region is empty: HOP groups will always be less massive than the bound
groups. There is a correlation between both masses, but with a high scatter. By cutting at 50 particles, we are eliminating many bound structures with masses that reach $2 \times 10^{14} h^{-1} M_\odot$. It is fair to say, then, that our sample of bound structures at $a = 1$ is complete for masses greater than $2 \times 10^{14} h^{-1} M_\odot$. We do the same analysis for objects at $a = 100$. We see from the right panel of the same figure that the correlation is better and that our sample will be complete for masses greater than $6 \times 10^{13} h^{-1} M_\odot$.

Figs. 7.3 and 7.4 show four structures, two at each epoch ($a = 1$ and $a = 100$). At both epochs we show a massive object and a less massive one. At $a = 1$ (Fig. 7.3), the presence of substructure in the massive object is evident. The massive central structure is the virial object from which we constructed the final object. On the other hand, the less massive object does not show substructure, just the central region and a few particles within the bound radius. At $a = 100$, the less massive object is quite similar to that at $a = 1$ in terms of mass and appearance. The massive object is highly concentrated: all
Figure 7.4 — Bound objects at $a = 100$. The left panel shows a massive bound group, on the right, a less massive structure. We see that both structures are highly concentrated.

substructure within its radius will have collapsed, and will form one big mass concentration.

7.4 Supercluster Mass Functions

Perhaps the most outstanding repercussion of the slowdown of large scale structure formation in hierarchical cosmological scenarios is the fact that new objects will no longer condense out of the density field. This should be reflected in the mass spectrum of the objects that were just on the verge of formation around the time of the cosmological transition. Hence, here we investigate the mass distribution of bound but not yet fully virialized structures (we also investigate a sample of virialized halos). In our Universe these are the superclusters.

7.4.1 Press-Schechter mass functions

Fig. 7.5 shows the evolution of the mass function predicted by the Press-Schechter and the Sheth-Tormen formalism for our cosmology. The plot on the left follows the evolution at $z = 4, 2, 1, 0.5$ and 0. It is clear that, as the Universe evolves, structures grow in mass and in number. The panel on the right shows evolution into the future for $a = 1, 2, 4, 10$ and 100. We see that after $a = 1$ the number of low mass objects does not change substantially, and after $a = 4$ the evolution stops. In fact, the curves for $a = 10$ and $a = 100$ overlap. We also see that the Sheth-Tormen approximation predicts a lower number of less massive objects than Press-Schechter.

7.4.2 Mass functions in Simulations

Considering two epochs ($a = 1$ and $a = 100$), and three different identification criteria (HOP, virial, and bound), we obtain six different mass functions from our simulation data. Two of them are the ones given by HOP. Two others correspond to virialized objects, this is, objects that in their radius enclose 100 times the critical density at $a = 1$ and 40.8 times the critical density at $a = 100$. The other two mass functions are the ones defined by the bound objects as given by the criterion described in section 4.2. Given the way we identified objects (see section 7.3), both the HOP and the virial mass functions are similar. For convenience, we will use the HOP mass functions in our analysis.

Fig. 7.6 shows the mass functions of the objects found by HOP (left panel) and of the groups that satisfy the criterion given by Eqn. 7.15 (right panel), both at $a = 1$ and at $a = 100$. We see that objects
at $a = 100$ are more massive than those at $a = 1$, but not substantially, confirming the freezing of the mass functions shown by Nagamine & Loeb (2003) and the theoretical results shown in Fig. 7.5. Since some of the groups at $a = 1$ will end up together at $a = 100$, at this expansion factor there are more massive groups than at $a = 1$, and fewer low mass groups.

Although the binding criterion found in Paper I predicts that the mass for bound objects should be the same at both expansion factors, we can see from the right panel of Fig. 7.6 that there is a slight difference. This was also noted in Paper I, where the authors found that, on average, 72% of the mass enclosed today by the critical radius will end up in a bound structure at $a = 100$.

As done in Paper I we also marked the particles that belong to a group at $a = 1$ and follow them to $a = 100$. In order to check the amount of mass that remains bound. We identified which groups at $a = 1$ correspond to the ones at $a = 100$. Having done this, we get that the fraction of mass that will remain bound is 72%, with a standard deviation of 13%. Note that meanwhile accretion did not enrich the halo with more than 1% of its final mass. Both are in agreement with the findings in Paper I.

Fig. 7.7 shows the same as the right panel of the previous figure, but now we plotted the “reduced” mass function at $a = 1$, this is, we reduced all the masses at $a = 1$ by a factor of 0.72. We see there is a good overlap with the mass function at $a = 100$, as expected.

If we consider the entire sample of bound groups at $a = 100$, we get a total mass of $2.83 \times 10^{18} h^{-1} M_{\odot}$. Since the mass of our simulated Universe is $1.04 \times 10^{19} h^{-1} M_{\odot}$, 27% of the total mass ends up in bound structures.

Using the criterion given by Eqn. 7.15 as a physical definition of superclusters, we estimate the most massive supercluster present in the local Universe ($z < 0.1$) to have a mass of $\sim 8 \times 10^{15} h^{-1} M_{\odot}$. This mass is slightly larger than the one of Shapley, which we found to be $\sim 7 \times 10^{15} h^{-1} M_{\odot}$ (Dürrer et al. 2008). According to the mass functions, we may find up to two Shapley-like superclusters in our Local Universe.

### 7.4.3 Comparison of simulated and theoretical mass functions

Fig. 7.8 shows the cumulative mass function of the objects found by HOP at $a = 1$ (left panel) and at $a = 100$ (right panel), together with the three theoretical mass functions. At $a = 1$, we can see that the Jenkins (Jenkins et al. 2001) mass function is the one that fits best (right). The value of $\delta_c$ was
CHAPTER 7: Future Evolution of Superclusters in an Accelerating Universe

Figure 7.6 — Mass functions for HOP objects (left panel) and for bound groups (right panel), both at $a = 1$ and $a = 100$.

Figure 7.7 — Mass functions of bound objects, now showing the “reduced” mass function at $a = 1$.

calculated according to Eqn. A-9, where for our cosmology we get $\delta_c = 1.675$. Governato et al. (1999) argues that, for their simulation (they also have a box of $500h^{-1}\text{Mpc}$ in a side and $\Omega_m = 0.3$) a better value for $\delta_c$ is $1.775$ (at $z = 0$). We find that this value does make a small difference, specially with the Sheth-Tormen approximation, where the fit improves. For the Press-Schechter mass function, we see a good fit for lower masses, but it underestimates higher masses. The Jenkins approximation does not change, since it is independent of $\delta_c$.

For the $a = 100$ mass function, we used the fitting parameters for $\Omega_m = 0$ listed in Evrard et al. (2002) for the Jenkins mass function. With these parameters, it agrees very well with the HOP mass function, although it slightly overestimates the number of lower mass objects. However, for Jenkins’ original parameter values, it would lead to a significant overabundance of objects with respect to the ones found in the simulations. Also note that neither the pure Press-Schechter nor the Sheth-Tormen
functions manage to fit the mass spectrum over the entire mass range. PS does agree at the high mass end while ST results in a better agreement at lower masses. This is an indication for the more substantial role of external tidal forces on the evolution of the low mass halos.

Fig. 7.9 shows the mass functions for bound objects at $a = 100$. We did not plot the objects at $a = 1$ since it gives a similar plot. We use the value $\delta_c = 1.17$ derived in section 7.2.2. Since the Jenkins approximation is independent of $\delta_c$, it does not fit the simulated mass function. We see that the best fit is provided by the PS mass function. This is may be due to the fact that the original implementation of PS was based on spherical objects, which are the type of objects that we find at $a = 100$. 

Figure 7.9 — The mass function of bound objects at $a = 100$ and the three theoretical mass functions. With our calculated value of $\delta_c$, the PS approximation is the best fitting.
7.5 Shapes of Bound Structures

Arguing that the large scale formation and evolution of structure comes to a halt once the Universe starts to accelerate, while the internal evolution of overdense patches continues, one particular manifestation of this will be the changing shape of these collapsing objects.

The issue of shapes of superclusters of galaxies has already been addressed, both using real data (e.g., Plionis et al. 1992; Basilakos et al. 2001) and in N-body simulations (e.g., Basilakos et al. 2006; Wray et al. 2006; Einasto et al. 2007). In every study, the authors state that the dominant shape of superclusters at the present time is prolate. Their definition of superclusters is done using percolation analysis. This makes it into a somewhat arbitrary definition since a supercluster will depend on the chosen percolation radius. By contrast, our definition of a supercluster – including its size, mass and shape – is based upon a strict physical criterion.

7.5.1 Definitions

In order to determine the shape, we calculate the inertia tensor using all particles inside the region of interest:

$$I_{ij} = \sum x_i x_j m_i.$$  \hfill (7.28)

Since the matrix is symmetric, it is possible to find a coordinate system such that the matrix representing the tensor has elements only along the diagonal. Our coordinate system will be chosen with respect to the center of mass of our bound objects. It is therefore possible to diagonalize the matrix, obtaining the eigenvalues $a_1$, $a_2$ and $a_3$. The eigenvalues of the inertia tensor give a quantitative measure of the degree of symmetry of the distribution. For example, if $a_1$ is a far larger number than $a_2$ and $a_3$, the distribution is not spherically symmetric. The two axis ratios are given by

$$\frac{b}{a} = \sqrt{\frac{a_2}{a_1}}, \quad \frac{c}{a} = \sqrt{\frac{a_3}{a_1}},$$  \hfill (7.29)

where $a_1 > a_2 > a_3$. This means that, if both ratios $b/a$ and $c/a$ are close to one, the object is almost spherical.

7.5.2 Shape evolution

Fig. 7.10 shows the distribution of axis ratios of the groups that satisfy the binding criterion (Eqn. 7.15) at present time. Objects must lie in the lower portion of the plot. Spherical groups should be at $(1,1)$, oblate groups should tend to $b/a = 1$ and prolate groups should lie near the diagonal $b/a = c/a$. There is a big empty region for lower values of $c/a$, meaning that there are no pancake-like structures (a pancake shape is defined such that $c/a \to 0$). We also see that there are almost no nearly-spherical objects. This is to be expected given the fact that in reality there are no spherical peaks in the initial density field (Bardeen et al. 1986).

The distribution of supercluster shapes is a combination of at least two factors. One is the shape of the proto supercluster in the initial density field. The second factor is the evolutionary state of the supercluster. We know that objects collapse anisotropically and proceed from a pancake-like shape via filamentary configuration towards a fully collapsed triaxial object. While it is still in the process of full collapse one may still recognize the internal substructure of its constituent clusters.

While the Universe is entering into an accelerating phase, the bound structures become increasingly isolated. No major mergers between structures happen after $a = 1$. The substructures that are within the bound radius will merge with each other, giving rise to spherical concentrated structures. Nagamine & Loeb (2003) ran an N-Body simulation that resembles the Local Universe, and found that the Local Group will get detached from the rest of the Universe, and the physical distance from it to the other systems that are not bound will increase exponentially.

We see this process reflected in Fig. 7.11. The left hand frame shows the shape distribution at $a = 1$, the one on the right hand side the resulting distribution at $a = 100$. At $a = 1$ the mean value of
7.5. SHAPES OF BOUND STRUCTURES

Figure 7.10 — Distribution of axis ratios for bound objects at $a = 1$ (left panel) and at $a = 100$ (right panel). The shape evolution towards less elongated objects is clearly visible.

the axis ratios is $(\langle b/a \rangle, \langle c/a \rangle) = (0.69, 0.48)$, with a standard deviation of $(\sigma_{b/a}, \sigma_{c/a}) = (0.13, 0.11)$. The figure shows that there is a wide spread of shapes with a tendency of the majority of groups to lie near the diagonal: bound groups have a prolate shape.

If we now turn our attention to the groups at $a = 100$, the distribution of group axis ratios in Fig. 7.10 is clearly different from the one at $a = 1$. The mean values of the axis ratios for all bound groups are $(\langle b/a \rangle, \langle c/a \rangle) = (0.94, 0.85)$, with standard deviations of $(\sigma_{b/a}, \sigma_{c/a}) = (0.03, 0.05)$. We see that for the majority of objects there is a predominance of nearly spherical shape. This demonstrates that, although there is virtually no large scale structure evolution after $a = 1$, there is an internal evolution of each individual group. It manifests itself in the object’s shape to evolve from prolate to more or less spherical.

7.5.3 Mass dependence

One potentially relevant issue concerns the possible dependence of shape on the mass shape of bound structures. In order to investigate this, we divide our sample into three mass ranges. We will consider massive groups (superclusters-like structures) with $M > 10^{15} h^{-1} M_\odot$, massive clusters with $5 \times 10^{14} h^{-1} M_\odot < M \leq 10^{15} h^{-1} M_\odot$, and less massive clusters with $M \leq 5 \times 10^{14} h^{-1} M_\odot$.

Figure 7.11 shows a contour map of the distribution of axis ratios for the three mass ranges described. The first mass ranges has $\sim 530$ objects (upper left panel); the second, $\sim 1020$ (upper right) and the third, $\sim 3330$ (lower panel). We see a predominance for prolate shape in every mass range.

Keeping in mind that our groups are bound but perhaps not virialized, it is instructive to contrast our objects to galaxy clusters, which are virialized by definition. Usually, superclusters are defined using different percolation radii, but clusters are defined using Eqn. 7.25, which, for our cosmology, has a value of $\sim 101$. By definition, these clusters are virialized. In our case, although the masses are comparable, the radius are not, since our bound groups have lower densities than clusters. Hence, they have larger radius, which will make them to have more substructure. Shapes of galaxy cluster halos in N-Body simulations have already been addressed by many authors (e.g., Dubinski & Carlberg 1991; Katz 1991; van Haarlem & van de Weygaert 1993; Jing & Suto 2002; Kasun & Evrard 2005; Paz et al. 2006; Allgood et al. 2006), and all of them agree that they tend to be more prolate as the halo mass increases. Dubinski & Carlberg (1991) found that halos are “strongly triaxial and very flat”, with mean axis ratios of $\langle b/a \rangle = 0.71$ and $\langle c/a \rangle = 0.50$, Katz (1991) found, for simulations of isolated halos
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with different power spectra indices, values ranging from 0.84 to 0.93 for $b/a$ and 0.43 to 0.71 for $c/a$, while Kasun & Evrard (2005) found for massive clusters peak values of $(b/a, c/a) = (0.76, 0.64)$.

The bound superclusters in our study also tend to be more prolate in shape, although their radii are different (larger) than the virial radius. We found that the three mass ranges have similar mean axis ratios (see Table 7.1). The shapes are quite close to the ones mentioned in the previous paragraph. This is indeed very interesting given that cluster mass virialized objects are quite different from the less dense supercluster objects we have studied. This immediately raises the following question: if the axis ratios for bound objects are similar to the ones of virialized objects, what would be the values for our virial objects?

To this end, we calculated the moment of inertia and its eigenvalues for the virial objects in

<table>
<thead>
<tr>
<th>Mass Range</th>
<th># Objects</th>
<th>$\langle b/a \rangle$</th>
<th>$\langle c/a \rangle$</th>
<th>$\sigma_{b/a}$</th>
<th>$\sigma_{c/a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M &gt; 10^{15} h^{-1} M_\odot$</td>
<td>533</td>
<td>0.70</td>
<td>0.50</td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td>$5 \times 10^{14} h^{-1} M_\odot &lt; M \leq 10^{15} h^{-1} M_\odot$</td>
<td>1015</td>
<td>0.70</td>
<td>0.49</td>
<td>0.13</td>
<td>0.10</td>
</tr>
<tr>
<td>$2 \times 10^{14} h^{-1} M_\odot &lt; M \leq 5 \times 10^{14} h^{-1} M_\odot$</td>
<td>3333</td>
<td>0.69</td>
<td>0.48</td>
<td>0.13</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 7.1 — Number of objects and average values of the axis ratios with their standard deviation according to mass range for bound objects in $a = 1$. 

Figure 7.11 — Contour maps of the distribution of axis ratios at $a = 1$ according to the three mass ranges described in the text. We see a predominance of prolate shapes.
our simulation at $a = 1$. The obtained values are $((b/a),(c/a)) = (0.83,0.71)$ with standard deviation $(\sigma_{b/a},\sigma_{c/a}) = (0.09,0.09)$. This means that also the virial halos are prolate, somewhat more pronounced than those quoted in other studies.

Figure 7.12 — Same as in Fig. 7.11 but for bound objects at $a = 100$. For clarity, the third and fourth mass ranges are plotted with thinner dots due to the amount of points. We see that in all four mass ranges there is a predominance of spherical shapes.

When we turn towards the situation in the future, we find that, in all mass ranges, objects attain a predominantly spherical morphology. Fig. 7.12 clearly shows this (see also Table 7.2). It is good to realize that bound objects grow in complete isolation and that all substructures within the binding radius merge into one, single, compact spherical object.

<table>
<thead>
<tr>
<th>Mass Range</th>
<th># Objects</th>
<th>$\langle b/a \rangle$</th>
<th>$\langle c/a \rangle$</th>
<th>$\sigma_{b/a}$</th>
<th>$\sigma_{c/a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M \geq 10^{15} h^{-1} M_\odot$</td>
<td>347</td>
<td>0.93</td>
<td>0.84</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>$5 \times 10^{14} h^{-1} M_\odot \leq M &lt; 10^{15} h^{-1} M_\odot$</td>
<td>727</td>
<td>0.94</td>
<td>0.85</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>$10^{14} h^{-1} M_\odot \leq M &lt; 5 \times 10^{15} h^{-1} M_\odot$</td>
<td>6436</td>
<td>0.94</td>
<td>0.86</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>$M &lt; 10^{14} h^{-1} M_\odot$</td>
<td>5048</td>
<td>0.93</td>
<td>0.84</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 7.2 — Number of objects and average values of the axis ratios with their standard deviation according to mass range for bound objects in $a = 100$. 
7.6 Density Profiles of Bound Structures

Along with the evolution of shape of objects, studied in the last section, the contracting superclusters will develop into a much more compact object. In order to assess this aspect of their internal evolution we study the evolution of the density profile and mass distribution of the halos in our sample.

![Figure 7.13 — Density profiles of two objects, both at $a = 1$ and $a = 100$.](image)

Fig. 7.13 shows the density profile of two objects at two expansion factors, $a = 1$ and $a = 100$. We choose one massive object (Object 8) and a less massive one (Object 98) at $a = 1$, and track them into the far future. We check the density profile of the other structures, and they show the same behavior. These two objects represents a fair example. The profiles were constructed using equal-size logarithmic bins.

We see from the figure that the profiles show essentially the same form: a high density at the center, due to the virial structure it has, and then it becomes less dense. The only difference is at the outer boundary, stretched in order to match the density of the background Universe (see also Busha et al. 2003). We see that in Object 98 the outer boundary starts to stretch before in comparison with object 8. This is because the virial structure of object 8 is similar in mass to the virial mass it will end up having at $a = 100$, which is not the case of object 98, whose virial structure is less massive than the final structure at $a = 100$.

Figs. 7.14 and 7.15 show a clearer picture of the internal changes objects go through. Both figures show the particle distribution at $a = 1$ and at $a = 100$ (top panels). Object 8 (Fig. 7.14) is one of the most massive objects identified at $a = 1$, and it ends up as one of the most massive at $a = 100$. We see that the virial structure from which it was constructed, depicted by the dashed-dot circle, has almost half of the mass of the object, depicted by the solid circle. At $a = 100$, the object has become more compact and denser at the center, as can be deduce by looking at the position of the half-mass radius and the virial radius. Along with the substantial rearrangement of its internal mass distribution into a more compact and spherical concentration, we also note a radical change of its cosmic surroundings. At $a = 1$ this is still marked by outstanding neighboring inhomogeneities as the objects are embedded and connected with the Cosmic Web (see Figs. 7.14 and 7.15, left frames). At $a = 100$ these supercluster concentrations have turned into isolated islands.

This evolution is directly reflected in the radial mass distribution. The bottom panel of these figures show the cumulative mass distribution as a function of radius. For Object 8, we see that, at $a = 1$, there is a high concentration of mass at small radii, indicating that the virial structure at the center has almost half of the mass of the object. There is then a slight increment in mass, reflecting that there are not many particles. At $a = 100$, half of the mass is within a small radius, comparable to that at $a = 1$. However, we see that at large radii, beyond $r = r_{\text{vir}}$, the object does not gain more mass. This is reflected in the flattening of the curve.

On the other hand, the cumulative mass distribution of Object 98 (Fig. 7.15) shows the same trend, but a different behavior at $a = 1$. Half of the mass of the object is enclosed at large radii (see the particle distribution on the top-left panel). The particle distribution shows that the virial object at the center is not as massive as that of Object 3. The mass increases almost uniformly until the half-mass radius,
7.6. DENSITY PROFILES OF BOUND STRUCTURES

Figure 7.14 — Upper panels: particle distribution at $a = 1$ (left) and $a = 100$ (right) of object 8 in physical coordinates. The half-mass radius (solid circle) and the virial radius (dashed-dotted circle) are also drawn for comparison. Bottom panel: Cumulative mass distribution of the object at $a = 1$ (solid line) and at $a = 100$ (dotted line). Both quantities have been normalized to their corresponding final values. The solid lines departing from the axis show the values of the half-mass radius.

and then there is an abrupt rise until the final radius, which is due to the object at the border. As in the previous object, if it goes further of $r = r_b$, its mass will continue growing.

The cumulative mass distribution at $a = 100$ shows the same behavior as that of Object 8: half of the mass is enclosed at small radii, and at large radii will stop gaining mass. These cumulative mass distributions are a fair representation of the relaxation of the objects.

We see that objects are highly concentrated at $a = 100$, as was suggested also by their almost spherical shape (section 7.5). We define a concentration parameter as

$$c = \frac{r_{hm}}{r_b},$$

where $r_{hm}$ is the radius that encloses half of the mass and $r_b$ is the radius of the object. Fig. 7.16 shows the distribution of $c$ as a function of mass at $a = 1$ (left panel) and $a = 100$ (right panel).

As expected, objects at $a = 100$ are highly concentrated, with $\bar{c} = 0.16$ and $\sigma = 0.02$, while objects at $a = 100$, the concentration parameter has a wider distribution, with $\bar{c} = 0.35$ and $\sigma = 0.14$. To some extent, this is due to the presence of substructure.
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Figure 7.15 — Same as Fig. 7.14 but now for object 98.

Figure 7.16 — Concentration parameter as a function of mass for bound objects at $a = 1$ and at $a = 100$. 
7.7 Supercluster Multiplicity Function

As a final aspect of the internal evolution of the superclusters, we turn towards their substructure. In particular, we wish to investigate their multiplicity, i.e., the number of clusters they contain. The hierarchical development of the supercluster involves the gradual merging of its constituent subclumps into one condensed object. Therefore, we expect superclusters containing several to dozens of clusters ultimately to end up as an object of unit multiplicity.

We have to realize that the method of supercluster identification has some influence on the definition of the multiplicity function. For example, while superclusters are often identified on the basis of a percolation criterion, the multiplicity will depend on the used percolation radius. Our supercluster identification involves a more physical criterion which automatically defines the radius and, therefore, we hope yields a more natural multiplicity function.

At the current epoch, we found ∼ 4,900 structures that satisfy Eqn. 7.15, but not all of these are superclusters. We assume a lower mass limit of $5 \times 10^{15} h^{-1} M_{\odot}$ to define a supercluster. Seventeen objects satisfy the criterion. The low mass threshold of clusters members was taken to be $4 \times 10^{13} h^{-1} M_{\odot}$, which correspond to the lowest mass of virial groups found at $a = 1$.

Fig. 7.17 shows the multiplicity function for the 17 objects. We see that half of the superclusters have 10 or more members. The mean mass of clusters in the total sample is $M = 9.4 \times 10^{13} h^{-1} M_{\odot}$, which is substantially lower than the average mass of clusters in superclusters, $M = 3.6 \times 10^{14} h^{-1} M_{\odot}$. This indicates that superclusters contain more massive clusters than the mean.

According to Fig. 7.17, the largest supercluster in the Local Universe would have 15 members, and a Shapley-like supercluster would have a radius of ∼ $14 h^{-1}$ Mpc and host between 10 to 15 members, almost half of that estimated by Quintana et al. (2000). In the observational reality of our Local Universe ($z < 0.1$), we find 5 superclusters with 10 or more members, the largest one containing 12 members.

If we compare our results with the work of Wray et al. (2006), we notice there is a difference in the number of clusters members. In some cases, they obtain superclusters with more than 30 members (see their Fig. 3). This is due to the choice of the linking length when defining a supercluster. The maximum size of superclusters they find ranges from ∼ $150 h^{-1}$ Mpc to ∼ $30 h^{-1}$ Mpc for different linking lengths, which are much larger than the superclusters we find according to our physical definition.

Fig. 7.18 shows the multiplicity as a function of mass (left panel) and as a function of radius (right panel).
As expected, larger and more massive superclusters have more members. Again, if we compare these results with those of Wray et al. (2006), we get that our superclusters are smaller in size (see their Fig. 5), but the correlation between the number of members to size is the same.

Finally, we may point out that all superclusters in our sample by $a = 100$ have evolved into single, compact objects. By definition, they all have multiplicity one.

7.8 Conclusions

In this work, we studied several properties of bound structures, such as their mass function, shape and density profile. These bound structures were defined by the density criterion given in Paper I (Eqn. 7.15), and identified from a $500h^{-1}$ Mpc cosmological box with $512^3$ dark matter particles in a $\Lambda$CDM ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $h = 0.7$) Universe. We ran the simulation up to $a = 100$, which is a time where structures have stopped forming. We use HOP in order to identify independently virialized structures, both at $a = 1$ and $a = 100$. Our main results can be summarized as follows:

- The marginally bound objects that we study resemble the superclusters in the observed Universe. While clusters of galaxies are the most massive and most recently fully collapsed and virialized objects in the Universe, superclusters are the largest bound—but not yet collapsed—structures in the Universe.

- The superclusters are true island Universes: as a result of the accelerating expansion of the Universe, no other, more massive and larger, structures will be able to form.

- While the superclusters collapse between $a = 1$ and $a = 100$, their surroundings radically change. At the present epoch solidly embedded within the Cosmic Web, at $a = 100$ they have turn into isolated cosmic islands.

- The mass functions in the simulations are generally in good agreement with the theoretical predictions of Press-Schechter, Sheth-Tormen and Jenkins mass functions. At $a = 1$, the Sheth-Tormen prescription provides a better fit. At $a = 100$, the pure Press-Schechter function seems to be marginally better. This may tie in with the more anisotropic shape of superclusters at $a = 1$ in comparison to their peers at $a = 100$. 

![Figure 7.18 — Multiplicity vs. mass (right panel) and multiplicity vs. bound radius (right panel).](image-url)
While the large scale evolution of superclusters comes to a halt as a result of the cosmic acceleration, their internal evolution continues at least up to \( a = 100 \).

As a result of their collapse, the shape of the bound objects appears to change from prolate at \( a = 1 \) into almost spherical at \( a = 100 \). We find that at \( a = 1 \) their mean axis ratio are \((b/a),(c/a)) = (0.69,0.48)\). At \( a = 100 \), they have mean axis ratios of \((b/a),(c/a)) = (0.94,0.85)\).

The change in the internal mass distribution and that in the surroundings is directly reflected by the radial density profile. The inner density profile steepens substantially when the inner region of the supercluster is also still contracting. On the other hand, when at \( a = 1 \) it has already developed a substantial virialized core, the inner density profile hardly changes.

The mass profile in the outer realms of the supercluster always changes radically from \( a = 1 \) to \( a = 100 \). At \( a = 1 \) it shows an irregular increase as a function of radius, reflecting the surrounding inhomogeneous mass distribution of the Cosmic Web. By \( a = 100 \) the superclusters have developed a smooth, regular and steadily increasing mass profile.

At the current epoch the superclusters still contain a substantial amount of substructure. Particularly interesting is the amount of cluster mass objects within its realm, expressed in the so called multiplicity function. Restricting ourselves to superclusters with a mass larger than \( 5 \times 10^{15}h^{-1}M_\odot \), of which we have 17 in our simulation sample, we find a multiplicity of 5 to 15 at the current epoch. By contrast, all these have evolved into concentrated singular mass clumps of unit multiplicity.

In a region of a volume comparable to the Local Universe \((z < 0.1)\) we find that the most massive supercluster would have a mass of \( \sim 8 \times 10^{15}h^{-1}M_\odot \). This is slightly bigger than the mass of the Shapley Supercluster given in Dünner et al. (2008). Also, we find 2 Shapley-like superclusters. These host between 10 to 15 members, almost half of those estimated by Quintana et al. (2000).
7.A Press-Schechter formalism and its variants

Mass functions given by numerical simulations are good approximations. Yet they cover a limited volume, given by the size of the simulation box. There is an excellent analytic description, the Press-Schechter formalism (Press & Schechter 1974). It provides a simple but powerful way to calculate the number density of objects of a given mass and at any redshift. The Press & Schechter (PS) formalism and its extensions has been extensively studied and compared to numerical simulations (see the works by, e.g., Bond et al. (1991),Lacey & Cole (1993), Sheth & Tormen (1999), Jenkins et al. (2001)).

It considers the emergence of collapsed objects from a primordial Gaussian random density field. Such primordial circumstances have been confirmed by the mapping of the temperature fluctuations in the Cosmic Microwave Background (e.g., Spergel et al. 2003). Moreover, inflationary theories of cosmology do predict such primordial fluctuations.

Let us consider spheres of radius \( r = (3M/4\pi\rho_b)^{1/3} \), where \( \rho_b \) is the mean density of the Universe and \( M \) is some mass scale of interest. We also define the overdensity as

\[
\delta = \frac{\rho - \rho_b}{\rho_b}.
\]

The fractional rms mass fluctuation is

\[
\sigma(M) = \frac{\sqrt{\langle \delta M^2 \rangle}}{M}.
\]

Recall that the primordial density perturbations are assumed to be Gaussian fluctuations. Thus, the phases of the waves that make up the density distribution are random, and the distribution of the overdensities \( \delta \) in spheres of radius \( r \) can be described by a Gaussian function

\[
p(\delta) = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left[-\frac{\delta^2}{2\sigma^2(M)}\right].
\]

At a given time, the fraction of points which are surrounded by a sphere of radius \( r \) within which the mean overdensity exceeds some density threshold \( \delta_c \) is given by

\[
F(\delta_c, M) = \frac{1}{\sqrt{2\pi}\sigma(M)} \int_{\delta_c}^{\infty} \exp\left[-\frac{\delta^2}{2\sigma^2(M)}\right] d\delta.
\]

Changing variables, \( u = \delta/\sqrt{2\sigma} \), we can express the latter equation in the form

\[
F(\delta_c, M) = \frac{1}{\sqrt{\pi}} \int_{\frac{\delta_c}{\sqrt{2}\sigma(M)}}^{\infty} e^{-u^2} du = \frac{1}{2} \text{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma(M)}\right),
\]

From Eqn. A-5, we can obtain the comoving number density of halos of mass \( M \), the Press-Schechter mass function:

\[
\frac{dn}{dM} = 3\rho_b \left[ \frac{\partial F}{\partial M} \right] = \sqrt{\frac{2}{\pi}} \rho_b \frac{\delta_c}{M^2 \sigma(M,z)} \frac{d\ln \sigma(M,z)}{d\ln M} e^{-\frac{\delta_c^2}{2\sigma^2(M,z)}}.
\]

Note that the (notorious) factor 2 was inserted in order to account for the matter in underdense regions, which also eventually falls into overdense ones. Later, the factor has been physically explained within the extended Press-Schechter formalism proposed by Bond et al. (1991). Since, most theoretical work along these lines has followed their excursion set formalism.

The quantity \( \delta_c/\sigma \) represents how many standard deviations away from the mean amplitude a positive density perturbation must lie in order to collapse, \( \rho_b \) is the mean background density, and \( \delta_c \) is the effective linear overdensity required for the collapse.
In order to estimate the mass function from the Press-Schechter formalism, we need to specify \( \sigma(M) \) and \( \delta_c \). We can express the variance of the density fluctuations in terms of the power spectrum \( P(k) \) of the linear density field,

\[
\sigma^2(M) = 4\pi \int_0^\infty P(k)\omega(kr)k^2dk,
\]

where \( \omega(kr) \) is the Fourier space representation of a real-space top-hat filter enclosing a mass \( M \) in a radius \( r \) at the mean density of the Universe, which is given by

\[
\omega(kr) = 3 \left[ \sin(kr) \frac{(kr)^3}{(kr)^2} - \cos(kr) \right].
\]

The value of \( \delta_c \) has a weak dependence on \( \Omega_m \). A good numerical approximation is given by Navarro et al. (1997):

\[
\delta_c(\Omega_m) = \begin{cases} 
0.15(12\pi)^{2/3}/\Omega_m^{0.0185} & \text{if } \Omega_m < 1 \text{ and } \Omega_\Lambda = 0, \\
0.15(12\pi)^{2/3}/\Omega_m^{0.0055} & \text{if } \Omega_m + \Omega_\Lambda = 1.
\end{cases}
\]

For a similar expression for the critical linear virial density see Bryan & Norman (1998).

The Press-Schechter mass function has been extensively tested against N-body simulations and was shown to be in reasonable agreement with them (e.g., Efstathiou et al. 1988; Lacey & Cole 1994; Governato et al. 1999).

The standard (extended) PS formalism assumes perfectly spherical collapse. However, we know even on purely theoretical grounds that there are no spherical primordial density peaks (Bardeen et al. 1986). Sheth & Tormen (1999) improved the PS formalism by taking into account the implied anisotropic collapse of density peaks on the basis of the ellipsoidal collapse model (e.g., Icke 1973). Implicitly, this also involves the anisotropic tidal stresses imparted by external mass concentrations. They showed that this implies a more fuzzy “moving collapsed barrier”. The resulting mass function,

\[
\frac{dn_{ST}}{dM} = A \sqrt{\frac{2a}{\pi}} \left[ 1 + \left( \frac{\sigma(M)^2}{a\delta_c^2} \right)^p \right]^{p/2} \rho_b \frac{\delta_c}{M^2 \sigma(M)} \frac{d\ln \sigma(M)}{d\ln M} e^{-\frac{a\sigma^2}{2\sigma(M)}},
\]

with \( a = 0.707 \), \( p = 0.3 \) and \( A \approx 0.322 \), gives a substantially better fit to the mass functions obtained in N-body simulations. In comparison with the standard PS mass function, ST predicts a higher abundance of massive objects and a smaller number of less massive ones. Later, Jenkins et al. (2001) reported a small disagreement with respect to N-body simulations: underpredictions for the massive halos and overpredictions for the less massive halos. They suggested the alternative expression:

\[
\frac{dn_J}{dM} = A \rho_b \frac{d\ln \sigma(M)}{d\ln M} e^{-[\ln \sigma^{-1} + B]^{3/4}}.
\]

with \( A = 0.315 \), \( B = 0.61 \) and \( \epsilon = 3.8 \). Note, however, that their expression does not depend explicitly on \( \delta_c \). They showed that “for a range of CDM cosmologies and for a suitable halo definition, the simulated mass function is almost independent of epoch, of cosmological parameters, and of initial power spectrum”.