6

Galaxy Clusters into the Future †

We explore the effects of future evolution on several properties of galaxy cluster halos in cosmological simulations. These properties include the morphology, angular momentum, virialization and scaling relations. We also explore some global properties, such as the mass function and the mass accretion history. The simulations span a wide range of cosmological parameters, representing open, flat and closed Universes. The simulations are run into the far future when the Universe is 60-70 Gyr old. This timespan is long enough for halos to essentially reach dynamical equilibrium. We find that there is no significant increase of low mass halos, while large mass objects continue to evolve until $a_f \sim 1.85$. As a consequence of the latter, the mass accretion history of cluster halos does not show any significant difference between cosmologies. We also find that in the far future, halos will become nearly spherical as a consequence of virialization. Also, the evolution of the angular momentum is constant. As a consequence, the spin parameter shows that halos will slow down. In contrast with $a_f = 1$, halos in the far future will become more virialized, with a virial state $|U| \sim 1.8 - 1.9K$. This is reflected in the scaling relations of galaxy cluster halos. The Kormendy and the Faber-Jackson are closer to the theoretical relations. The Fundamental Plane shows that in the far future, all galaxy cluster halos will be virialized, an aspect that is reflected in its width: it is similar in every cosmology. In the far future, it will become increasingly difficult to discriminate in which type of Universe we live in.

6.1 Introduction

The existence (or lack) of a cosmological constant has profound consequences in the future evolution of the Universe. Although there seems to be an agreement in the range of values that both $\Omega_m$ and $\Omega_\Lambda$ can have in order to match observations of high-redshift objects such as quasars and be in accordance with the age of globular clusters, there has not been a detailed study of how the interplay between these two cosmological parameters affects the future evolution of the large scale structure, particularly galaxy clusters. The effect of a nonzero cosmological constant drives the Universe towards unbounded exponential expansion (see Carroll et al. 1992, for a detailed review of the cosmological constant), while a zero cosmological constant, makes the expansion to decelerate. Previous studies (Nagamine & Loeb 2003; Busha et al. 2003; Dünner et al. 2006; Hoffman et al. 2007, and chapter 7 of this thesis) have shown that for a $\Lambda$CDM Universe with present day cosmological parameters $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ structure formation and halo growth come to a rapid end, at approximately $a_f \sim 3$. At this time, the cosmological constant is nearly 1. The exponential expansion causes mergers and accretion to stop and pushes halos further away from each other. From $a_f \sim 3$ and beyond, no considerable matter is left on the surrounding of the halos to accrete, and they start to relax towards equilibrium in effective isolation.

For different cosmological models as the one stated before, the time at which structure formation and halo growth stops will be different because of the different values of the cosmological constant. The values of $\Omega_m$ and $\Omega_\Lambda$ have a large effect on the time scales of the Universe. The cosmological constant either aids or resists the attraction of matter. This can lead to younger or older Universes, respectively, with the consequent effect on the dynamical evolution of galaxy clusters. In some cases, the cosmological constant already took over the expansion of the Universe and some structures are already growing in isolation. In others, it is just beginning. But the final scenario will be the same: objects grow in complete isolation. In all cases, the internal dynamics of clusters of galaxies will be affected by this gravitational expansion. However, the question that remains is how they will behave.

In order to study the role of the cosmological constant in the evolution towards the future of galaxy clusters and how this influences their final dynamical structure, we extract information of the future gravitational growth of the large scale structure of the Universe and of physical quantities such as shape, angular momentum, kinetic and potential energy and scaling relations from a set of thirteen cosmological simulations. Each of these models describe different cosmological models with different values of curvature. The physical quantities under study will tell us when and how galaxy clusters reach dynamical equilibrium, and will allow us to determine the importance of the cosmological constant in the fate of the Universe.

The chapter is organized as follows. In section 6.2 we describe the numerical simulations that we carried out. Power spectrum evolution is described in 6.3. Mass function towards the future of the different cosmogonies are studied in section 6.4. The mass accretion history of four galaxy clusters of one particular cosmology is presented in section 6.5, and then they are used as a base to compare with the remaining models. Section 6.6 study the shape evolution of this same four models, together with the evolution of the entire sample. A comparison is made between all thirteen models. Dynamical quantities are studied in the remaining sections. Section 6.7 look at the angular momentum of galaxy clusters, while in section 6.8 we present analysis on the virialization. In section 6.9 we study how the mass, velocity dispersion and radius correlate with the virialization by looking into the scaling relations of galaxy clusters. Finally, in section 6.10 we present our conclusions.

6.2 Numerical Experiments

6.2.1 The simulations

The simulations and the method to identify halos are extensively described in chapter 2. Here, we summarize this description.
We perform thirteen N-body simulations that follows the dynamics of $N = 256^3$ particles in a periodic box of size $L = 200h^{-1}$Mpc. The initial conditions are generated with identical phases for Fourier components of the Gaussian random field. In this way each cosmological model contains the same morphological structures. For all models we chose the same Hubble parameter, $h = 0.7$, and the same normalization of the power spectrum, $\sigma_8 = 0.8$. The principal differences between the simulations are the values of the matter density and vacuum energy density parameters, $\Omega_m$ and $\Omega_\Lambda$. By combining these parameters, we get models describing the three possible geometries of the Universe: open, flat and closed. The effect of having the same Hubble parameter and different cosmological constants translates into having different cosmic times.

The initial conditions are evolved from $a_f = 1$ until $a_f = 54$, assuming that at late epochs structure formation will decrease significantly, so no major changes will be seen from then on (see Nagamine & Loeb (2003)). We use the massive parallel tree code GADGET2 (Springel 2005). The Plummer-equivalent softening was set at $\epsilon_{pl} = 15h^{-1}$kpc in physical units. For each cosmological model we wrote the output of 14 snapshots, equally spaced in log($a$). The only exceptions were the SCDM and OCDM05 cosmologies. They were stopped at $a_f \sim 3.5$ because the power at small scales was increasing dramatically, with the result that the high clustering affected the simulations.

6.2.2 Halo identification

We use the HOP algorithm (Eisenstein & Hut 1998) to extract the groups present in the simulations. HOP associates a density to every particle. In a first step, a group is defined as a collection of particles linked to a local density maximum. To make a distinction between a high density region and its surroundings, HOP uses a regrouping procedure. This procedure identifies a group as an individual object on the basis of a specific density value. Important for our study is the fact that for this critical value we chose the virial density value $\Delta_c$ following from the spherical collapse model. In order to have the proper $\Delta_c$ we numerically compute its value for each of the cosmologies (see appendix 2.A). Table 6.1 lists the values of the cosmological parameters and the values of the virial density for each cosmology at $a_f = 0$.

![Figure 6.1 — Evolution of a single cluster in the ΛCDMF2 cosmoslogy in comoving (upper panels) and physical (lower panels) coordinates. Box sizes are of $14 \times 14$ Mpc$^{-1}$.](image)

Note that we only consider groups containing more than 100 particles. Because the particle mass depends on the cosmological scenario, this implies a different mass cut for the halos in each of our
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Table 6.1 — Cosmological parameters for the runs. The columns give the identification of the runs, the present matter density parameter, the density parameter associated with the cosmological constant, the age of the Universe in Gyr since the Big Bang at the present epoch, the age of the Universe in Gyr at \( a_f = 54 \), the mass per particle in units of \( 10^{10} h^{-1}M_\odot \) and the mass cut of the groups given by HOP in units of \( 10^{10} h^{-1}M_\odot \). For the SCDM and OCDM05, the simulations were stopped at \( a_f \sim 3.4 \), hence, the age shown is the correspondent to that expansion factor.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Omega_m )</th>
<th>( \Omega_\Lambda )</th>
<th>( \Omega_k )</th>
<th>Age at ( a_f = 1 )</th>
<th>Age at ( a_f = 54 )</th>
<th>( m_{dm} )</th>
<th>( m_{cut} )</th>
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<td>791.46</td>
<td>1.32</td>
<td>132</td>
</tr>
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<td>397</td>
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<td>49.571</td>
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<td>662</td>
</tr>
<tr>
<td>( \Lambda )CDM01</td>
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<td>0.5</td>
<td>0.4</td>
<td>14.65</td>
<td>90.01</td>
<td>1.32</td>
<td>132</td>
</tr>
<tr>
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<td>0.7</td>
<td>0.2</td>
<td>15.96</td>
<td>81.21</td>
<td>1.32</td>
<td>132</td>
</tr>
<tr>
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<td>0</td>
<td>17.85</td>
<td>76.36</td>
<td>1.32</td>
<td>132</td>
</tr>
<tr>
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<td>0.2</td>
<td>12.70</td>
<td>88.55</td>
<td>3.97</td>
<td>397</td>
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<tr>
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<td>79.07</td>
<td>3.97</td>
<td>397</td>
</tr>
<tr>
<td>( \Lambda )CDMC1</td>
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<td>-0.2</td>
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<td>73.21</td>
<td>3.97</td>
<td>397</td>
</tr>
<tr>
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<td>0</td>
<td>11.61</td>
<td>87.97</td>
<td>6.62</td>
<td>662</td>
</tr>
<tr>
<td>( \Lambda )CDMC2</td>
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<td>0.7</td>
<td>-0.2</td>
<td>12.17</td>
<td>78.13</td>
<td>6.62</td>
<td>662</td>
</tr>
<tr>
<td>( \Lambda )CDMC3</td>
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<td>0.9</td>
<td>-0.4</td>
<td>12.84</td>
<td>71.89</td>
<td>6.62</td>
<td>662</td>
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</tbody>
</table>

Simulations. As a result, SCDM does not have groups with masses lower than \( 10^{13} M_\odot \). We have to keep in mind this artificial constraint when considering collapse and virialization in hierarchical scenarios at high redshifts. When structure growth is still continuing vigorously at the current epoch, the collapsed halos at high redshifts will have been small. Our simulations would not be able to resolve this.

Fig. 6.1 shows the evolution at \( a_f = 1, 1.85, 6.3 \) and \( a_f = 54 \) of a single cluster in both comoving (upper panels) and physical (lower panels) coordinates. The difference between both coordinates is clear: while in physical coordinates it has nearly the same size throughout its history, in comoving coordinates it shrinks, to the point it is almost invisible. By looking the evolution in physical coordinates, we see that the cluster gains its mass via a merging process. This can be seen at \( a_f = 1 \), where it has many substructure. By \( a_f = 1.85 \) the merging process is almost finished, and then it starts to relax (\( a_f = 6.3 \), until it becomes an isolated compact object (\( a_f = 54 \)).

6.2.3 Halo properties

For each group in every output we compute the mass, the kinetic energy, the potential energy, the angular momentum and the inertia tensor. All properties are computed using only the particles within the group, i.e., no post-processing is done to any of the groups. In some situations we limit ourselves to those dark matter halos that would correspond to rich galaxy clusters. A galaxy cluster is defined to have a dark matter mass \( M \) of \( M > 10^{14} h^{-1} M_\odot \).

Given a halo of \( N \) particles, we compute the above quantities as follows:

- **Mass**: the number of halo particles multiplied by the mass per particle present in each halo:
  \[
  M = N m_{\text{part}},
  \]
  where \( m_{\text{part}} \) is the mass per particle.

- **Shape**: in order to calculate the shape of the halo, we calculate the inertia tensor using all
particles inside the region of interest:

\[ I_{ij} = \sum x_i x_j. \]  (6.2)

We chose our coordinate system with respect to the center of mass of the halo. By diagonalizing the matrix, we obtain eigenvalues \( a_1 > a_2 > a_3 \). The eigenvalues of the inertia tensor are a quantitative measure of the degree of symmetry of the distribution of particles. The axis ratios follow from the ratio of the eigenvalues

\[ \frac{b}{a} = \sqrt{\frac{a_2}{a_1}}, \quad \frac{c}{a} = \sqrt{\frac{a_3}{a_1}}, \]  (6.3)

with \( a_f > b > c \) the axes of the object.

- **Angular momentum**: defined as

\[ J = \sum_{i=0}^{N} m_i r_i \times v_i, \]  (6.4)

where \( r_i \) and \( v_i \) are the position and velocity of the \( i \)th particle with respect to the center of mass of the group.

It is often useful to define the spin parameter, a dimensionless quantity which relates the angular momentum and the energy of a group (Peebles 1971),

\[ \lambda = \frac{J}{\sqrt{|E|}} \frac{G M_5}{2}; \]  (6.5)

where \( J \) is the angular momentum of the group (see eqn. 6.4), \( E \) is the total energy, \( M \) its mass and \( G \) is the gravitational constant. Note that its dependence on the total energy of the system is rather weak. The spin parameter is essentially the ratio of the angular momentum of an object to that required for rotational support. A value of \( \lambda = 0.05 \), for example, implies very little systematic rotation and negligible rotational support.

- **Kinetic Energy**: the total sum of the particle kinetic energies (with respect to the center of the halo):

\[ K = \frac{1}{2} \sum_{i=1}^{N} m_i (v_i - v_{\text{center}})^2, \]  (6.6)

where \( v_i \) is the physical velocity of particle \( i \) and \( v_{\text{center}} \) the physical velocity of the halos’ center of mass.

- **Potential Energy**: defined as

\[ U = -\sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{G m_i m_j}{|r_i - r_j|}, \]  (6.7)

where \( r_i \) and \( r_j \) are the locations of particles \( i \) and \( j \).

- **Mean harmonic radius**: \( r_h \) is defined as the inverse of the mean distance between all pairs of particles in the halo:

\[ \frac{1}{r_h} = \frac{1}{N} \sum_{i<j} \frac{1}{|r_{ij}|}, \quad N = \frac{n_{\text{part}}(n_{\text{part}} - 1)}{2}, \]  (6.8)

where \( r_{ij} \) is the separation vector between the \( i \)th and the \( j \)th particle. The great virtue of this radius is that it is a good measure of the effective radius of the gravitational potential of the clump, certainly important when assessing the virial status of the clump. Also, it has the practical advantage of being independent of the definition of the cluster center. To some extent, it is also an indicator of the internal structure of the halo because it put extra weight to close pairs of particles.
6.3 Power Spectrum Evolution

Fig. 6.2 shows the evolution of the power spectrum for three cosmological models: ΛCDM02, ΛCDMF2 and ΛCDMC2. We also plot the linear power spectrum for comparison. The measured power spectrum at high-k modes (small length scale) have become strongly nonlinear, while the low-k modes (large length scale) are still following the linear power spectrum. As explained in chapter 2, the shape of the (linear) power spectrum depends on the value of Ωm, while the amplitude depends on ΩΛ. The power spectrum is plotted in comoving space, making that the expansion of the Universe transfers power to larger scales with no real change in the shape.

For the open model case, we see that there is hardly any growth from $a_f = 1$ to $a_f = 54$, caused by the freezing of structure in an expanding Universe (it can be seen on both the linear and the measured spectrum). On the other hand, ΛCDMC2 shows an increase in the growth from $a_f = 1$ to $a_f = 1.85$, and freezes at late times. ΛCDMF2 is the case in between. The growth from $a_f = 1$ to $a_f = 1.85$ is less than in the ΛCDMC2 but more than in the ΛCDM02. These results are in agreement with the ΛCDM simulations studied by Busha et al. (2007).

The “break” from linearity occurs at different $k$ modes depending on the cosmology. For the ΛCDM02 case, it happens at $k \sim 0.1 \, h \text{Mpc}^{-1}$, while for the ΛCDMF2 and the ΛCDMC2, at $k \sim 0.2$

![Image of power spectrum evolution for different cosmological models](image)

Figure 6.2 — The evolution of the power spectrum for three different cosmological models. Each panels shows four different times, from bottom up: $a_f = 1$, $a_f = 1.85$, $a_f = 6.3$ and $a_f = 54$. 
$h\text{Mpc}^{-1}$ and $k \sim 0.3~h\text{Mpc}^{-1}$, respectively. This means that the non linear evolution of the power spectrum increases the power on small scales more rapidly in the low $\Omega_m$ Universes than in the high density cosmologies.

### 6.4 Mass Functions

The mass function is the number density of objects of a given mass. Fig. 6.3 shows the evolution of the cumulative number of dark matter halos for the ΛCDMO2, ΛCDMF2 and ΛCDMC2 cosmologies. The mass functions are shown at expansion factors $a_f = 1$, $a_f = 1.85$, $a_f = 6.3$ and $a_f = 53$.

The first notable difference is the halt of structure formation after $a_f = 1.85$: the mass function freezes in every cosmology. In the ΛCDMO2 cosmology we see a small increase of large mass objects from $a_f = 1$ to $a_f = 1.85$, but for the rest, there is no increase of the mass function. This is tied in with the fact that in low $\Omega_m$, structure growth comes to a halt at high redshift. The increase of large mass objects between $a_f = 1$ and $a_f = 1.85$ is moderate in the ΛCDMF2 model, while in the ΛCDMC2 it is higher.

![Graph of mass function](image)

**Figure 6.3** — Evolution of the mass function for the ΛCDMO2, ΛCDMF2 and the ΛCDMC2 cosmology at four different epochs: $a_f = 1$, $a_f = 1.85$, $a_f = 6.3$ and $a_f = 54$.

The Jenkins mass function is derived from the Press-Schechter formalism (see chapter 2). Its form is given by

$$
\frac{dn_f}{dM} = A \frac{\bar{\rho}}{M^2} (\ln \sigma(M)) \exp(-\ln \sigma^{-1} + B) C.
$$

(6.9)

where $a_f = 0.315$, $B = 0.61$ and $C = 3.8$ are the original values given by Jenkins et al. (2001). $a_f$ sets the overall mass fraction in collapsed objects, $e^B$ plays the role of a (linearly evolved) collapse perturbation threshold (similar to the parameter $\delta_c$ in the Press-Schechter model) and $C$ is a stretch parameter that provides the correct shape of the mass function (Evrard et al. 2002).

Fig. 6.4 shows the Jenkins mass function (JMF) (Jenkins et al. 2001) as dashed lines at two expansion factors, $a_f = 1$ and $a_f = 54$, for the ΛCDMO2, ΛCDMF2 and the ΛCDMC2 model.

At $a_f = 1$, we use the original parameters of the JMF. At $a_f = 54$ we use the parameters given by Evrard et al. (2002): $a_f = 0.199$, $B = 0.76$ and $C = 3.90$. This parameters were derived for models with $\Omega_\Lambda = 1$ and/or $\Omega_m = 0$. We find that the JMF is consistent at both epochs, although there is a slight overestimation of large mass halos in the ΛCDMO2 model. At $a_f = 54$, the original values of the JMF do not manage to reproduce the simulated mass functions.
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Figure 6.4 — Mass functions for the $\Lambda$CDMO2, $\Lambda$CDMF2 and the $\Lambda$CDMC2 cosmology at $a_f = 1$ (left panel) and $a_f = 54$ (right panel). Dashed lines represent the fit of the Jenkins mass function.

Although not shown, we also fitted the Press-Schechter mass function (PSMF) (Press & Schechter 1974) and the Sheth & Tormen mass function (STMF) with published values (Sheth & Tormen 1999). At $a_f = 1$, both the PSMF and STMF agree well with the mass functions. At $a_f = 54$, PSMF shows a slight mass excess for every cosmology. This may be corrected by a precise value of $\delta_c$. STMF shows a significant mass excess at this expansion factor.

6.5 Mass Accretion Histories

The freezing of growth that structures in some Universes will suffer in the far future will have an effect in their mass accretion history (MAH). Structures which are bound at the present cosmic epoch $a_f = 1$

Figure 6.5 — Average mass accretion histories for galaxy cluster halos in three cosmological models: $\Lambda$CDMO2 (top frame), $\Lambda$CDMF2 (bottom left frame) and $\Lambda$CDMC2 (bottom right frame). The shaded area denotes the corresponding standard deviations.
will merge, while others will separate from each other and will grow in isolation. To investigate the MAH of cluster sized halos we construct their merging tree history. We do this by identifying the particles that belong to cluster halos at $a_f = 54$ and we trace them back in history, checking for the most massive progenitor.

We determine the general mass accretion history by averaging all individual mass accretion histories of the halos in each cosmological model. The average MAH of these galaxy cluster halos is shown in Fig. 6.5. A few facts can be immediately inferred. Galaxy clusters in high density Universes ($\Lambda$CDMC2) did absorb some amount of matter during their evolution from $a_f = 1$ to $a_f = 54$. The opposite effect can be observed for the low $\Omega_m$ $\Lambda$CDMO2 model: galaxy cluster halos had at $a_f = 1$ $\sim 80\%$ of the mass at $a_f = 54$. We find this behavior in every high density simulated cosmology. The spread of the MAHs are similar in the depicted cosmologies. Nevertheless, there are hardly any significant differences on the mass accretion history between the depicted cosmologies. The impression is that in the far future it will become increasingly difficult to infer in what cosmology we live on.

Figure 6.6 — Evolution of the shapes of objects present in the $\Lambda$CDMO2 (top-left panel), $\Lambda$CDMF2 (top-right panel) and $\Lambda$CDMC2 model (bottom panel).
6.6 Shapes in the far future

In chapter 4 we studied the shape of dark matter halos as a function of the expansion factor. We found that, although the halos tend to retain a rather prolate shape, there was a shift towards a more spherical shape.

In Fig. 6.6 we have plotted the halos in a scatter diagram of axis ratio $c/a$ versus $b/a$. We have indicated where one can find prolate, oblate and spherical halos. Instead of plotting individual halo shapes, we have plotted the contour of the 32% percentile around the average shape of the halos in a simulation. In each of the panels we plotted these contours for a sequence of timesteps, with the lightest contour level corresponding to $a_f = 1$ and the darkest one to $a_f = 54$.

The trend found in chapter 4 is confirmed when appreciating Fig. 6.6: halos become more and more spherical when evolving into the far future. They have had enough time to virialize and relax.

6.7 Angular Momentum and Spin Parameter into the far future

In chapter 4 we studied the angular momentum of dark matter halos as a function of the expansion factor for a variety of cosmologies. We found that the angular momentum of halos increases erratically but steadily, and that it is intimately related to the increasing mass of the halos. It is interesting, then, to investigate the behavior of the angular momentum in the far future, where halos stop increasing in mass.

Fig. 6.7 shows the evolution of the median of the angular momentum $J$ as a function of expansion time. We look at the time evolution in two different ways: as a function of cosmic time $t$ (top panel) and as a function of expansion factor $a_f$. Unlike the evolution of the angular momentum $J$ from early times to the present epoch, where we saw a steady increase, we now see that it remains constant from
Figure 6.8 — Evolution of the spin parameter as a function of the expansion factor of the cluster size halos for the ΛCDMO2 (solid line), ΛCDMF2 (dotted line) and the ΛCDMC2 (dashed line) cosmologies.

\( a_f = 1 \) to \( a_f = 54 \).

The evolution of the spin parameter \( \lambda \) is perhaps more revealing (see Fig. 6.8). In chapter 4 we saw that it showed a decreasing trend, from \( \lambda \sim 0.06 \) at early times to \( \lambda \sim 0.04 \) at present epoch. We now see that towards the far future it continues to decrease, from \( \lambda \sim 0.04 \) to \( \lambda \sim 0.03 \) at \( a_f = 54 \).

In order to see possible differences between different cosmological models, we fit the lognormal distribution

\[
p(\lambda) d\lambda = \frac{1}{\sigma_\lambda \sqrt{2\pi}} \exp\left[-\frac{\ln^2(\lambda/\lambda_0)}{2\sigma_\lambda^2}\right] d\lambda,
\]

(6.10)
to the sample of cluster halos. In Fig. 6.9 we have plotted the spin parameter distribution of the cluster halos in the ΛCDMO2, ΛCDMF2 and ΛCDMC2 cosmologies at \( a_f = 54 \). Superimposed are the fits following the lognormal distribution in Eqn. 6.10. Clearly, it provides a more than satisfactory description. This ties in with the findings of chapter 4, where we saw that the lognormal distribution is independent of cosmology (see also Peirani et al. (2004)). On the basis of the fits of the lognormal distribution to the \( \lambda \) distribution of the halos in our simulations, we have inferred the values of the parameters \( \lambda_0 \) and \( \sigma_\lambda \). These are shown in Table 6.2. In contrast with the values at \( a_f = 1 \) (Table 4.4 of chapter 4), we see a slight decrease on the value of \( \lambda_0 \). This is due to the slow down on the spin rotation of cluster halos in the far future.

<table>
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<th>Model</th>
<th>( \lambda_0 )</th>
<th>( \sigma_\lambda )</th>
<th>Model</th>
<th>( \lambda_0 )</th>
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</tr>
<tr>
<td>ΛCDMO2</td>
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<td>0.711</td>
<td>ΛCDMC3</td>
<td>0.023</td>
<td>0.721</td>
</tr>
<tr>
<td>ΛCDMO3</td>
<td>0.021</td>
<td>0.735</td>
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</tbody>
</table>

Table 6.2 — Inferred parameters of the lognormal distribution for \( \lambda \).
After collapsing, any given halo will virialize. During this process the internal energy of the dark matter halo is distributed such that it reaches a perfect equilibrium configuration.

In chapter 3 we investigated the virialization as a function of expansion factor of dark matter halos until the present epoch $a_f = 1$. We found that there was a considerable amount of unvirialized halos and also a large number of unbound halos.

Dark matter halos have had sufficient time to evolve from $a_f = 1$ to $a_f = 54$. We wish to see, then, what would be their virialization stage in the far future.
6.8. VIRIALIZATION TOWARDS THE FUTURE

Table 6.3 — Virial ratio $\mu$ of the halo sample in each cosmology.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mu$</th>
<th>Model</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
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<td>$\Lambda$CDMO3</td>
<td>1.86</td>
</tr>
<tr>
<td>OCDM01</td>
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<td>$\Lambda$CDMF2</td>
<td>1.83</td>
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<tr>
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<td>$\Lambda$CDMC1</td>
<td>1.81</td>
</tr>
<tr>
<td>OCDM05</td>
<td>1.80</td>
<td>$\Lambda$CDMF3</td>
<td>1.86</td>
</tr>
<tr>
<td>$\Lambda$CDMO1</td>
<td>1.87</td>
<td>$\Lambda$CDMC2</td>
<td>1.83</td>
</tr>
<tr>
<td>$\Lambda$CDMO2</td>
<td>1.84</td>
<td>$\Lambda$CDMC3</td>
<td>1.82</td>
</tr>
<tr>
<td>$\Lambda$CDMF1</td>
<td>1.80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.8.1 Virial Theorem

Any self-gravitating system has a kinetic energy given by:

$$K = \frac{1}{2} \sum_{i=1}^{N} m_i (v_i - v_{\text{center}})^2,$$

where the sum is over all particle velocities within any given region, and $m_i$ is the mass of the $i$ particle. The gravitational potential energy of the system is given by

$$U = -\sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{G m_i m_j}{|r_i - r_j|},$$

where the sum of the distances is over all particle pairs. The system is virialized if it fulfills the condition

$$2K + U = 0.$$

This is known as the Virial Theorem for a perfect isolated self-gravitating system.

A good measure for the virial state of an object may be obtained from the virial ratio

$$\mathcal{V} = \frac{2K}{|U|}.$$ 

For virialized systems, $\mathcal{V} \to 1$. The system is bound if $1 < \mathcal{V} < 2$, while it is unbound if $\mathcal{V} > 2$.

We first investigate how the entire sample of dark matter halos fits the relation between potential and kinetic energy predicted by the virial theorem. To do so, we model the virial relation in the form of the linear “virial line” $|U| - K$,

$$|U| = \mu K + \lambda,$$

where a perfect, isolated virialized object, $\mu = 2$ and $\lambda = 0$ (see also chapter 3). In order to find $\mu$, we fitted a line through the point cloud. We fitted the best virial line to the complete set of dark matter halos present in every simulation. Table 6.3 shows the inferred virial ratios for the complete set of cosmological simulations. We see that in all the cosmologies the halo sample are quite close to the virial state. The inferred virial ratio $\mu$ is higher than the one at $a = 1$. However, we do not see any significant difference between the cosmologies.

In Fig. 6.10 we plot the virial ratio $|U|/K$ as a function of the mass of the halos. The plots are scatter plots with each point representing a dark matter halo in the simulation. We have superimposed a density grey scale plot in order to identify the density of the points. The horizontal lines represent the perfect virial state $|U|/K = 2$ and the criterion for boundness $|U|/K = 1$. In each of the three cosmologies we see that over the entire mass range the majority of halos lie between the virial ratio of $|U|/K \sim 1.8 - 1.9$, higher than the virial ratio of $|U|/K \sim 1.5 - 1.6$ found at $a_f = 1$ (see chapter 3).
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Figure 6.10 — The virial ratio $|U|/K$ as a function of the mass of the halos in our simulated cosmologies. The three cosmological models represented are: ΛCDM02 (top left panel), ΛCDMF2 (top right panel) and ΛCDMC2 (bottom panel). The plot are scatter plots, with each point representing a halo in the simulation. The density of the points in the scatter plot can be inferred from the superimposed density grey scale plot. In each panel we indicate by means of horizontal lines the perfect virial state $|U|/K = 2$, and the criterion for a gravitationally bound configuration $|U|/K = 1$.

The spread in the case of low mass halos is narrower than the spread seen at $a_f = 1$. It appears that the amount of highly virialized halos has increased slightly, while those that were not even bound at $a_f = 1$ became bound and achieve a virialization. An interesting question would be that of following the history of the halos that were not bound at $a_f = 1$ and see if they merge to some larger halos or grew in isolation, with the result of relaxing and virializing. This aspect is subject of an ongoing study.

6.8.2 Virialization of Galaxy Clusters in the Far Future

By studying the virialization of of dark matter halos that we brand as cluster sized halos with a mass larger than $M > 10^{14} h^{-1} M_{\odot}$, we do not find any significant difference with respect to the complete sample of dark matter halos. Galaxy cluster halo also center around the virial ratio $|U|/K \sim 1.8 - 1.9$ (see Fig. 6.11). We do not find any significant difference between cosmologies. There is also no significant spread around the virial line. This is a consequence of the high virial state galaxy cluster halos attain in the far future.
6.9. SCALING RELATIONS IN THE FAR FUTURE

A complete description of scaling relations can be found in chapter 5. We will derive here the scaling relations for galaxy cluster halos.

Basing ourselves on the mass \( M \) of a given cluster halo, and assuming that the selected objects have the same average density, we expect an equivalent Kormendy relation given by

\[
M \propto R^3, \tag{6.16}
\]

where \( R \) is the size of the object. Any deviation in the slope may be understood as a dependence of the mean density \( \langle \rho(R_e) \rangle \) on the size \( R \) of the object.

Every selected object have a kinetic and potential energy given by

\[
K = \frac{M \sigma^2}{2}, \quad U = -\frac{GM^2}{R}, \tag{6.17}
\]

where \( \sigma = \langle v^2 \rangle^{1/2} \) is the velocity dispersion. If halos are in virial equilibrium, i.e., \( 2K + U = 0 \), then we can express the former equations as

\[
\sigma^2 = \frac{GM}{R}. \tag{6.18}
\]

Figure 6.11 — The virial ratio \( |U|/K \) as a function of the mass of the cluster halos in three cosmologies: \( \Lambda \)CDMO2 (top left panel), \( \Lambda \)CDMF2 (top right panel) and \( \Lambda \)CDMC2 (bottom panel). The plots are scatter plots, with each point representing a halo in the simulation. In each panel we indicate by means of horizontal lines the perfect virial state \( |U|/K = 2 \), and the criterion for a gravitationally bound configuration \( |U|/K = 1 \).
This would imply the following scaling relation between $M$, $R$ and $\sigma$

$$\log M = 2 \log \sigma + \log R + C_{fp}, \quad (6.19)$$

where $C_{fp}$ is a constant. This equation is known as the Fundamental Plane (FP). From Eqns. 6.17 and 6.19 we find the Faber-Jackson relation

$$M \propto \sigma^3, \quad (6.20)$$

### 6.9.1 Kormendy Relation

Fig. 6.12 shows the relation between the mass $M$ of each cluster halo and their mean harmonic radius $r_h$. Each of the three panels depicts the relation for the halos in one particular cosmology: $\Lambda$CDM02 (top left panel), $\Lambda$CDM03 (top right panel) and $\Lambda$CDM05 (bottom panel).

A visual comparison shows that halos of comparable masses have more or less the same mean harmonic radius in every cosmology. In other words, in the far future ($a_f = 54$) cluster halos have the same compactness independent of cosmology.

The Kormendy relation appears to be a good description for the $M - r_h$ relation. Also interesting to note is that the $M - r_h$ relation is much tighter at $a_f = 54$ than at $a_f = 1$. The spread of the relation is similar in every model, thus looking for a systematic trend as a function of either $\Omega_m$ or $\Omega_\Lambda$ is difficult.

The linear fits for three cosmologies are also plotted in Fig. 6.12. The inferred parameters are listed in Table 6.4. Unlike the situation at $a_f = 1$ (see chapter 5), we do not see any clear differences between the cosmologies. High density Universes ($\Lambda$CDMF2 and $\Lambda$CDMC2) have slopes in the order of $\sim 2.6$, while the $\Lambda$CDMO2 has a slope in the order of $\sim 2.4$. This seems to imply that the dependence of halo concentration in halos with $\Omega_m = 0.1$ is less dependent of its mass, as it was the case at $a_f = 1$.

We also see that the inferred parameters are far from the perfect theoretical relation of $M \propto r_h^3$. In order to investigate this, we checked the Kormendy relation with the virial radius (the size of the entire virialized halo), and found that the Kormendy parameter $a$ is higher than the inferred parameter using the mean harmonic radius, in contrast with the findings of chapter 5. This tells us that, in the far future,
Figure 6.12 — Kormendy relation. Each frame plots the relation between mass $M$ and mean harmonic radius $r_h$ of the cluster halos in the simulations corresponding to one particular cosmology. From top left to bottom these are: $\Lambda$CDM02, $\Lambda$CDM2, $\Lambda$CDM2. In each of these frames we have superimposed the fitted Kormendy relation. The linestyle of each fit is given in the top left frame.

HOP is identifying halos that are more spherical than in the present epoch due to their isolation and relaxation. The fact that the Kormendy parameter is lower when using the mean harmonic radius is just a reflection of the weak homology cluster halos present. If they were perfect homologous, the Kormendy relation will be the same independent of the radius.

We do not find any systematic trend on the influence of $\Omega_m$, $\Omega_\Lambda$, or $\Omega_{\text{total}}$ on $a$.

### 6.9.2 Faber-Jackson Relation

Fig. 6.13 shows the Faber-Jackson relation between the mass $M$ and the velocity dispersion $\sigma$ of the cluster halos. Each panel correspond to one particular cosmology: $\Lambda$CDMO2 cosmology (top left panel), $\Lambda$CDM2 cosmology (top right panel) and $\Lambda$CDM2 cosmology (bottom panel).

For comparison, in each of the panels we show the three lines corresponding to the linear fits of this relation in each of the three depicted cosmologies. The $M - \sigma$ relation is well fitted by the Faber-Jackson relation and it is tighter than in the cosmic epoch $a_f = 1$ (see Fig. 5.4 of chapter 5). It is also close to the theoretical relation $M \propto \sigma^3$ and somewhat slightly tighter than the Kormendy relation.

The inferred FJ relation does not vary significantly as a function of the underlying cosmology. However, there is an evolution from $a_f = 1$ to $a_f = 54$. While at $a_f = 1$ the slope $b$ was in all cases in the order of $\sim 2.75$, at $a_f = 54$ it is of the order of $\sim 3.10$ (with the exception of the SCDM cosmology,
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Figure 6.13 — Faber-Jackson relation. Each frame plots the relation between mass $M$ and velocity dispersion $\sigma$ of the cluster halos in the simulations corresponding to one particular cosmology. From top left to bottom these are: $\Lambda$CDMO2, $\Lambda$CDMF2, $\Lambda$CDMC2. In each of these frames we have superimposed the fitted Kormendy relation. The linestyle of each fit is given in the top left frame.

see Table 6.4). There is no clear dependence on $\Omega_m$, neither on $\Omega_\Lambda$ nor on $\Omega_{total}$. In other words, galaxy cluster halos have evolved from $a_f = 1$ to $a_f = 54$ achieving a high degree of virialization.

As with the Kormendy relation, we do not find any influence of the cosmological parameters on the Faber-Jackson parameter $b$ at $a = 54$.

6.9.3 Fundamental Plane

Fig. 6.14 shows the Fundamental Plane relation for the same three cosmologies: $\Lambda$CDMO2 (top left panel), $\Lambda$CDMF2 (top right panel) and $\Lambda$CDMC2 (bottom panel). In each of the panels we plotted the mass $M$ of the halos against the quantity $r^c_0 \sigma^d$.

As with the Kormendy and Faber-Jackson relation, the Fundamental Plane relation at $a_f = 54$ is tighter than at $a_f = 1$. The inferred parameters $c$ and $d$ are close to the ones theoretically expected for perfect virialized halos, $c = 1$ and $d = 2$. This just confirms what it was found in chapter 5: the Fundamental Plane is almost independent of cosmology and of expansion factor.

The notable difference is with respect to the width of the Fundamental Plane. While in chapter 5 we found that the width was dependent of cosmology, i.e., high density Universes having larger width, at $a_f = 54$ we see that every cosmology have a more or less similar width, with the exception of the SCDM and OCDM05 models which were stopped at $a \sim 3.4$. If at $a_f = 1$ we found that clusters in
6.9. SCALING RELATIONS IN THE FAR FUTURE

Figure 6.14 — Fundamental Plane. Each frame plots the relation between mass $M$ and the quantity $r_h^c \sigma^d$, the combination of mean harmonic radius $r_h$ and velocity dispersion $\sigma$, with $r_h$ to the power of $c$ and $\sigma$ to the power of $d$ of the FP linear fit. The dots in each frame represent the cluster halos in the $\Lambda$CDMO2 cosmology (top left frame), in the $\Lambda$CDMF2 cosmology (top right cosmology) and in the $\Lambda$CDMC2 cosmology. The superimposed lines in each frame represent the fitted Fundamental Plane for the four different cosmologies (see insert in the top left frame).

High density Universes still undergo substantial levels of merging and accretion, they have had the necessary time to relax and virialized in the far future.

Figure 6.15 — Inferred parameters for the FJ relation according to three different parameters: $\Omega_m$ (top left panel), $\Omega_\Lambda$ (top right panel) and $\Omega_m + \Omega_\Lambda$ (bottom panel).
**6.10 Conclusions**

We have evolved thirteen different cosmological simulations into the far future \((a_f = 54)\). From these set of simulations, we have identified the dark matter cluster halos and studied several individual properties in order the investigate how they compare to the ones at the present cosmic epoch. The main conclusion are:

- The mass function of every cosmological models freezes after \(a_f = 1.85\). This effect is observed before for low \(\Omega_m\) Universes due to their early formation epoch.

- There is hardly any differences between the mass accretion history of halos in any given cosmology. They all show an slight increase of mass between \(a_f = 1\) and \(a_f = 3\), and after, halos do not accrete significant mass clumps.

  Interestingly, the spread of the MAHs are similar in the depicted cosmologies. The impression is that in the far future it will become increasingly difficult to infer in what cosmology we live on.

- In all cosmologies, halos evolve towards a nearly spherical shape.

- The angular momentum remains constant between the present epoch and \(a_f = 54\). This is a reflection of the mass accretion history: halos do not gain angular momentum due to the absence of mass gain.

- On average, the spin parameter decreases from \(\lambda \sim 0.04\) at \(a_f = 1\) to \(\lambda \sim 0.03\) at \(a_f = 54\). Dark matter halos slow down due to their isolation.

- Halos show a high degree of virialization, with a relation \(|U| \sim 1.8 - 1.9K\), higher than the relation found at \(a_f = 1\) \((|U| \sim 1.5 - 1.6K)\), but still not a perfect virial state \(|U| = 2K\).

- In each cosmological model we recover the Kormendy, Faber-Jackson and Fundamental Plane relation in the far future.

- These relations are much tighter than the ones at \(a_f = 1\), showing that in the far future halos are highly virialized.

- The width of the Fundamental Plane at \(a_f = 54\) is thinner than at \(a_f = 1\) and nearly the same for every cosmological model: galaxy cluster halos in every cosmology have had enough time to virialized and reach dynamical equilibrium.

- Due to the above finding, it will become increasingly difficult in the far future to identify in which Universe we live in.