Cluster Halo Scaling Relations †

We explore the effects of dark matter and dark energy on the dynamical scaling properties of galaxy clusters. We investigate the cluster Faber-Jackson, Kormendy and Fundamental Plane relations between the mass, radius and velocity dispersion of cluster size halos in cosmological $N$-body simulations. These relations are a profound expression of the virial state of these objects. The simulations span a wide range of cosmological parameters, representing open, flat and closed Universes. Independently of the cosmology, we find that the simulated clusters do indeed lie on a Fundamental Plane, and are close to a perfect virial state. The one outstanding influence is that of $\Omega_m$. While the Fundamental Plane parameters are hardly dependent on cosmology, we do find a noticeable trend when looking at its width. The FP in low $\Omega_m$ Universes tend to converge early on to a tight and well defined plane. The redshift evolution of the scaling parameters do reveal some interesting facts. The Fundamental Plane turns out to be almost universal in nature, its only change concerning a gradual decrease of its width as the halos become more virialized. The Kormendy and Faber-Jackson parameters do change radically. The Faber-Jackson parameter increases continuously, while the Kormendy one initially increases rapidly after which it stabilizes or slightly decreases its value. On the basis of this, we conclude that the cluster population’s evolution, constrained to the Fundamental Plane, involves a weak structural and dynamical homology. Our suspicion of a possible effect of the cosmology in terms of their different power spectrum slope at cluster scale is not confirmed. Related to this we also investigated the differences between clusters that quietly accreted their mass and those that underwent massive mergers. The latter have a less well defined plane as their virial state is severely disrupted in the merging process.

Chapter 5: Cluster Halo Scaling Relations

5.1 Introduction

Recent observations of distant supernovae (Riess et al. 1998; Perlmutter et al. 1999) suggest that we are living in a flat, accelerated Universe with a low matter density. This accelerated expansion has established the possibility of a dark energy component which behaves like Einstein’s cosmological constant $\Lambda$. A positive cosmological constant solves the apparent conflict suggested by the old age of globular clusters stars and the estimated amount of it (Spergel et al. 2003, 2007) appears sufficient to yield a flat geometry of our Universe.

The role of $\Lambda$ in the process of structure formation is not yet fully understood. Although its influence can be noticed when looking at the global evolution of the Universe its role in the dynamics of the structures is not clear. Its most direct impact will be that via its influence on the amplitude of the primordial perturbation power spectrum and, perhaps most direct, via its influence on the cosmic and dynamical time scales. Its direct dynamical influence is probably minor: we do know that in the linear regime it accounts for a mere $\sim 1/70$th of the influence of matter perturbation (Lahav et al. 1991).

Most viable theories of cosmic structure formation involve hierarchical clustering. Small structures form first and they merge to give birth to bigger ones. The rate and history of this process is highly dependent on the amount of (dark) matter present in the Universe. In Universes with a low $\Omega_m$, structure formation ceases at much early times than that in cosmologies with high density values.

Within this hierarchical process, clusters of galaxies are the most massive and most recently formed structures in the Universe. Their collapse time is comparable to the age of the Universe. This makes them important probes for the study of cosmic structure formation and evolution. The hierarchical clustering history from which galaxy clusters emerge involves a highly complex process of merging, accretion and virialization. In this chapter we investigate in how far we can get insight into this history on the basis of the internal properties of the clusters. This involves characteristics like their mass and mass distribution, their size and their kinetic and gravitational potential energy. In particular, we are keen to learn whether these do show any possible trace of a cosmological constant.

One particular profound manifestation of the virial state of cosmic objects is via scaling relations that connect various structural properties. Scaling relations of collapsed and virialized objects relate two or three fundamental characteristics. The first involves a quantity measuring the amount of mass $M$, often in terms of the amount of light $L$ emitted by the object. The second quantity involves the size of the object, while the third one quantifies its dynamical state. Since the mid 70s, we know that elliptical galaxies do follow such relations. The Faber-Jackson relation (Faber & Jackson 1976) relates the luminosity $L$ and the velocity dispersion $\sigma$ of an elliptical galaxy. The Tully-Fisher relation (Tully & Fisher 1977) is the equivalent for spiral galaxies. A different, though related, scaling is that between the effective radius $r_e$ and the luminosity of the galaxy. This is known as the Kormendy relation (Kormendy 1977). These two relations turned out to be manifestations of a deeper scaling relation between three fundamental characteristics, which became known as the Fundamental Plane (Djorgovski & Davis 1987; Dressler et al. 1987). Not only do these scaling relations inform us about the dynamical state of the objects, they also function as important steps in the extragalactic distance ladder.

Similar scaling relations are also found for clusters of galaxies. This suggests that stellar systems and galaxy clusters have similar formation process, favoring the hierarchical scenario. These clusters scaling relations were first studied by Schaeffer et al. (1993). They studied a sample of 16 galaxy clusters, concluding that these systems also populate a Fundamental Plane. Adami et al. (1998) used the ESO Nearby Abell Cluster Survey to study the existence of a Fundamental Plane for rich galaxy clusters, finding that it is significantly different from that of elliptical galaxies. Later, Lanzoni et al. (2004) addressed the question of for high mass halos, which are thought to host clusters of galaxies. On the basis of 13 simulated massive dark matter halos in a $\Lambda$CDM cosmology they found that the dark matter halos follow the FJ, Kormendy and FP relations.

In this chapter we specifically address the question whether we can trace an influence of a positive cosmological constant in the scaling relations for simulated clusters. We use a set of dissipationless $N$-body simulations involving open, flat and closed Universes. All the simulations are variants of the cold
dark matter (CDM) scenario, representing different cosmological models with \( \Omega_m < 1 \) and \( \Omega_\Lambda \neq 0 \).

The organization of this chapter is as follows. In the next section we present a general description of scaling relations, addressing the relation between the Fundamental Plane and the Virial Theorem. In section 5.3 we describe the simulations and the definitions of the various parameters we use. We investigate the scaling relations of galaxy clusters in different cosmologies at \( z = 0 \) in section 5.4. In section 5.5 we study the evolution of the scaling relations as a function of redshift and cosmic time. We also investigate the dependence of merging and accretion on the scaling relations in section 5.6. Conclusions are presented in section 5.7.

5.2 Scaling Relations

From observations of elliptical galaxies we have learned that there are tight relations between a few of their fundamental structural properties (see e.g. Binney & Merrifield (1998)). These properties are the total luminosity \( L \) of a galaxy, its characteristic size \( R_e \) and its velocity dispersion \( \sigma_v \).

The first of this relations is the Faber-Jackson relation (Faber & Jackson 1976) between the luminosity of the galaxy and its velocity dispersion,

\[
L \propto \sigma_v^\gamma,
\]

where the index \( \gamma \sim 4 \). A similar relation, known as the Tully-Fisher relation (Tully & Fisher 1977), holds for HI disks of spiral galaxies. According to this relation, the galaxies’ rotation velocity is tightly correlated with the absolute magnitude of the galaxy.

Another relation was established by Kormendy (Kormendy 1977). He found that there is a strong, not entirely unexpected, correlation between the luminosity and effective radius of the elliptical galaxies:

\[
L \propto R_e^\alpha,
\]

where the index \( \alpha \sim 1.5 \).

It turns out that these relations between two structural characteristics should be seen as projections of a more fundamental and tight relations between all three structural quantities, the Fundamental Plane (FP). The Fundamental Plane of elliptical galaxies was first formulated by Djorgovski & Davis (1987) and Dressler et al. (1987). When we take the three-dimensional space defined by the radius \( R_e \) of the galaxy, its surface brightness \( I_e \) (with total luminosity \( L \propto I_e R_e^2 \)) and velocity dispersion \( \sigma_v \), we find that they do not fill space homogeneously but instead define a thin plane. In logarithmic quantities, this plane may be parametrized by

\[
\log R_e = \alpha \log \sigma_v + \beta \log I_e + \gamma.
\]

For example, Jørgensen et al. (1996) found that a reasonable fit to the Fundamental Plane is given by

\[
\log R_e = 1.24 \log \sigma - 0.82 \log I_e + \gamma.
\]

While nearly all galaxies, ranging from giant ellipticals to compact dwarf ellipticals, appear to lie on the FP (also see e.g. Jørgensen et al. 1995; Bernardi et al. 2003; Cappellari et al. 2006; Bolton et al. 2007) it is interesting to note that diffuse dwarf ellipticals do not (Kormendy 1987): they seem to be fundamentally different objects.

The galaxy scaling relations are of great importance for a variety of reasons. First of all, they must be a profound reflection of the galaxy formation process (Robertson et al. 2006). Also, they have turned out to be of substantial practical importance. Because they relate a intrinsic distance independent quantity like velocity dispersion to a distance dependent one like \( L_e \), they can be used as cosmological distance indicators.
5.2.1 Fundamental Plane and Virial Theorem

In its pure form, the Fundamental Plane is a direct manifestation of the Virial Theorem. As such, it is expected to hold for any virialized clump of matter, be it a galaxy or a cluster of galaxies.

In order to appreciate this, we first look at the expected relationship between the mass $M$ of a virialized halo, its size $R$ and its velocity dispersion $\sigma_v = \langle v^2 \rangle^{1/2}$. The kinetic energy $K$ and the gravitational potential energy $U$ of this halo are given by

$$K = \frac{M \langle v^2 \rangle}{2}, \quad U = -\frac{GM^2}{R}. \quad (5.5)$$

The Virial Theorem establishes the following strict relationship between $U$ and $K$,

$$2K + U = 0, \quad (5.6)$$

which implies that

$$\langle v^2 \rangle = \frac{GM}{R}. \quad (5.7)$$

This would imply the following scaling relation between $M$, $R$ and $\sigma_v$,

$$\log M = 2 \log \sigma_v + \log R + \gamma_M, \quad (5.8)$$

in which $\gamma_M$ is a constant.

In hierarchical scenarios of structure formation halos build up by subsequent merging of smaller halos into larger and larger halos. Some of these mergers involves sizeable clumps, most of it consists of a more quiescent accretion of matter and small clumps from the surroundings. This process is certainly leaving its mark on the phase-space structure of the halos. Indeed, these dark halo streams are a major source of attention in present day studies of the formation of our Galaxy (Helmi & White 1999; Helmi 2000). It remains an interesting question whether we can also find the influence of these merging events on the Fundamental Plane. For example, González-García & van Albada (2003) did look into the effects of major mergers on the Fundamental Plane. They did find that the Fundamental Plane does remain largely intact in the case of two merging ellipticals. However, what the effects will be of an incessant bombardment of a halo by material in its surroundings has not been studied in much detail. Given that this is a sensitive function of the cosmological scenario, we will study the influence on FP parameters and width in more detail.

5.2.2 Fundamental Plane Evolution: Mass-to-Light ratio and Galaxy Homology

In case the object of mass $M$ has a luminosity $L$, the observed Fundamental Plane not only provides information on the dynamical state of the object but also on the evolution of its stellar content. Encapsulating the relation between $M$ and $L$ in terms of the mass-to-light ratio,

$$M = \left(\frac{M}{L}\right) L, \quad (5.9)$$

we find that the radius $R$, $\sigma$ and the mean surface brightness $I = L/4\pi R^2$ are related according to

$$R \propto \langle v^2 \rangle I^{-1} \left(\frac{M}{L}\right)^{-1}, \quad (5.10)$$

which yields the following theoretical Fundamental Plane relation

$$\log R = 2 \log \sigma_v - \log I - \log (M/L) + C_s, \quad (5.11)$$
5.2. SCALING RELATIONS

in which $C_\gamma$ is a constant dependent on the structure of the object. Although the measured Fundamental Plane of elliptical galaxies is rather thin, it does have an intrinsic scatter. The latter is not completely explained and may be a manifestation of the formation process.

When looking at the observed parameter values for elliptical galaxies (see 5.4), we do find a difference with the “predicted” Fundamental Plane: this it not only due to a (constant) mass-to-light ratio $M/L$, but also involves an angle between the two planes known as the tilt of the Fundamental Plane (González-García 2003; Robertson et al. 2006).

Several explanations have been given to account for this tilt. One is homology of the galaxies, i.e. that all galaxies are structurally equivalent, in combination with a mass-dependent $M/L$ ratio. It would imply a formation process involving a tight fine tuning of $M/L$. It might also be that there is no such systematic variation of $M/L$, implying that elliptical are not homologous systems so that the tilt would be due to variations in the structure parameters of the galaxy. Recent semi-analytical modelling of galaxy formation do suggest a more complex relation between the mass-to-light ratio and luminosity, involving a minimum $M/L$ for galaxies with $M \approx 10^{11} - 10^{12} h^{-1} M_\odot$.

Following the first view, the parameters inferred by Jørgensen et al. (1996) (Eqn. 5.4) would imply a mass-to-light ratio

$$ (M/L) \propto M^{0.25}, \quad (5.12) $$

using $M \propto \sigma^2 R_e$ and $L \propto l e^2$ (see e.g. Faber (1987)). By tracing the possibly systematic evolution of the galaxy FP, e.g. amongst a population of rich clusters spanning a range of redshifts (van Dokkum & Franx 1996), one might be able to infer the evolution of the mean $M/L$ ratio. A range of physical processes may underlie such systematic shifts. For example, Jørgensen et al. (1995) suggested a relation with the metallicity of the galaxies.

5.2.3 Cluster Fundamental Plane

Instead of assessing the FP of galaxies in clusters, one might also investigate the question in how far clusters themselves do follow a Fundamental Plane relation. After all, they are fully virialized objects.

The first to address this question were Schaeffer et al. (1993). They inferred an FP relation from a sample of 29 Abell clusters, $L \propto R_e^{0.89} \sigma_e^{1.28}$, which would translate into an equivalent relation

$$ \log R_e = 1.15 \log \sigma_e - 0.90 \log l_e, \quad (5.13) $$

in which $l_e$ is the hypothetical mean surface brightness of the cluster. The corresponding FJ relation is $L \propto R_e^{1.87}$ and the Kormendy relation is $L \propto R_e^{1.34}$. Similar numbers were inferred by Lanzoni et al. (2004), $L \propto R_e^{0.90} \sigma_e^{1.31}$. Adami et al. (1998) also found a FP relation for a sample of ENACS Clusters, although their inferred parameters do show some marked differences, $L \propto R_e^{0.87} \sigma_e^{0.70}$.

In studies of simulated galaxy clusters, which are known to be dark matter dominated, we may wonder whether scaling relations similar to those inferred from observable quantities do also exist for the dark matter host. To infer these relations we base ourselves on the mass $M$ of the object. If the selected objects have the same average density, we would expect an equivalent Kormendy relation given by

$$ M \propto R_e^3. \quad (5.14) $$

Any difference in slope should be ascribed to a dependence of mean density $\langle \rho(R_e) \rangle$ on the size $R_e$ of the object. The equivalent Fundamental Plane relation will be that of Eqn. 5.8, while the Faber-Jackson relation would then be

$$ M \propto \sigma_e^3, \quad (5.15) $$

Note that this is based on the assumption of constant mean density $\rho$ of the selected objects, in line with HOP overdensity criterion.

In line with the above, Lanzoni et al. (2004) analyzed the scaling relations on the basis of a sample of 13 massive dark matter halos identified in a high resolution ΛCDM N-body simulations. They did
Table 5.1 — Cosmological parameters for the runs. The columns give the identification of the runs, the present matter density parameter, the density parameter associated with the cosmological constant, the age of the Universe in Gyr since the Big Bang, the mass per particle in units of $10^{10}h^{-1}M_\odot$, the mass cut of the groups given by HOP in units of $10^{10}h^{-1}M_\odot$, the value of the density needed to have virialized objects with respect to the background density, and the same as before, but now with respect to the critical density.

confirmed the existence of FP relations for the dark matter clusters but also found that these have a significant different slope. On the basis of this, they suggest a mass dependent cluster $M/L$ ratio

$$(M/L) \propto M^{0.8}. $$

Interestingly, this is markedly different from that inferred for early types galaxies.

In this study, we systematically investigate this issue within the context of a range of different cosmologies. These concern both different values for the mass density $\Omega_m$, for dark energy $\Omega_\Lambda$ and for the implied power spectrum of density perturbations and the related merging and accretion history of the clusters.

### 5.3 The Simulations

The simulations and the method to identify halos are extensively described in chapter 2. Here, we summarize this description.

We perform thirteen N-body simulations that follows the dynamics of $N = 256^3$ particles in a periodic box of size $L = 200h^{-1}$Mpc. The initial conditions are generated with identical phases for Fourier components of the Gaussian random field. In this way each cosmological model contains the same morphological structures. For all models we chose the same Hubble parameter, $h = 0.7$, and the same normalization of the power spectrum, $\sigma_8 = 0.8$. The principal differences between the simulations are the values of the matter density and vacuum energy density parameters, $\Omega_m$ and $\Omega_\Lambda$. By combining these parameters, we get models describing the three possible geometries of the Universe: open, flat and closed. The effect of having the same Hubble parameter and different cosmological constants translates into having different cosmic times.

The initial conditions are evolved until the present time ($z = 0$) using the massive parallel tree N-body code GADGET2 (Springel 2005). The Plummer-equivalent softening was set at $\epsilon_{pl} = 15h^{-1}$kpc in physical units from $z = 2$ to $z = 0$, while it was taken to be fixed in comoving units at higher redshifts. For each cosmological model we wrote the output of 100 snapshots, from $a = 0.2$ ($z = 4$) to the present time, $a = 1$ ($z = 0$), equally spaced in $\log(a)$.
5.3. THE SIMULATIONS

Figure 5.1 — Comparison between the mean harmonic radius and the half-mass radius of the halos in ΛCDM scenario.

5.3.1 Halo identification

We use the HOP algorithm (Eisenstein & Hut 1998) to extract the groups present in the simulations. HOP associates a density to every particle. In a first step, a group is defined as a collection of particles linked to a local density maximum. To make a distinction between a high density region and its surroundings, HOP uses a regrouping procedure. This procedure identifies a group as an individual object on the basis of a specific density value. Important for our study is the fact that for this critical value we chose the virial density value $\Delta_c$ following from the spherical collapse model. In order to have the proper $\Delta_c$ we numerically compute its value for each of the cosmologies (see appendix 2.A). Table 5.1 lists the values of the cosmological parameters and the values of the virial density for each cosmology at $z = 0$. For the latter we list two values: the virial overdensity $\Delta_{\text{vir},b}$ with respect to the background density $\rho_b$ of the corresponding cosmology, and the related virial overdensity $\Delta_{\text{vir},c}$ with respect to the critical density.

Note that we only consider groups containing more than 100 particles. Because the particle mass depends on the cosmological scenario, this implies a different mass cut for the halos in each of our simulations. As a result, SCDM does not have groups with masses lower than $10^{13}h^{-1}M_\odot$. We have to keep in mind this artificial constraint when considering collapse and virialization in hierarchical scenarios at high redshifts. When structure growth is still continuing vigorously at the current epoch, the collapsed halos at high redshifts will have been small. Our simulations would not be able to resolve this.

5.3.2 Halo properties

In our study, we limit ourselves to cluster-like halos. A galaxy cluster is defined as a dark matter halo with a mass $M > 10^{14}h^{-1}M_\odot$. Of these clusters, we measure three quantities and test their scaling relation.

Scaling relations of collapsed and virialized objects relate two or three fundamental characteristics of those objects. The first involves a quantity measuring the amount of mass, often in terms of the amount of light emitted by the object. The second quantity involves the size of the object, while the third one quantifies its dynamical state.

- **Mass**: defined as the number of particles multiplied by the mass per particle present in each
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Table 5.2 — Scaling relations parameters: inferred parameters for the Kormendy relation, the Faber-Jackson relation and Fundamental Plane for the galaxy clusters in each of the simulated cosmological models.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Omega_m$</th>
<th>$\Omega_\Lambda$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$\sqrt{\chi^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>1</td>
<td>0</td>
<td>2.27</td>
<td>0.37</td>
<td>2.85</td>
<td>0.27</td>
<td>1.05</td>
</tr>
<tr>
<td>OCDMO1</td>
<td>0.1</td>
<td>0</td>
<td>2.01</td>
<td>0.57</td>
<td>2.36</td>
<td>0.34</td>
<td>0.94</td>
</tr>
<tr>
<td>OCDMO3</td>
<td>0.3</td>
<td>0</td>
<td>2.33</td>
<td>0.41</td>
<td>2.73</td>
<td>0.29</td>
<td>1.09</td>
</tr>
<tr>
<td>OCDMO5</td>
<td>0.5</td>
<td>0</td>
<td>2.27</td>
<td>0.40</td>
<td>2.81</td>
<td>0.29</td>
<td>1.03</td>
</tr>
<tr>
<td>$\Lambda$CDMO1</td>
<td>0.1</td>
<td>0.5</td>
<td>1.87</td>
<td>0.58</td>
<td>2.28</td>
<td>0.38</td>
<td>0.96</td>
</tr>
<tr>
<td>$\Lambda$CDMO2</td>
<td>0.1</td>
<td>0.7</td>
<td>1.91</td>
<td>0.52</td>
<td>2.47</td>
<td>0.38</td>
<td>1.00</td>
</tr>
<tr>
<td>$\Lambda$CDMF1</td>
<td>0.1</td>
<td>0.9</td>
<td>1.97</td>
<td>0.57</td>
<td>2.39</td>
<td>0.36</td>
<td>0.98</td>
</tr>
<tr>
<td>$\Lambda$CDMO3</td>
<td>0.3</td>
<td>0.5</td>
<td>2.35</td>
<td>0.41</td>
<td>2.72</td>
<td>0.30</td>
<td>1.12</td>
</tr>
<tr>
<td>$\Lambda$CDMF2</td>
<td>0.3</td>
<td>0.7</td>
<td>2.40</td>
<td>0.39</td>
<td>2.76</td>
<td>0.29</td>
<td>1.14</td>
</tr>
<tr>
<td>$\Lambda$CDMC1</td>
<td>0.3</td>
<td>0.9</td>
<td>2.50</td>
<td>0.39</td>
<td>2.72</td>
<td>0.28</td>
<td>1.16</td>
</tr>
<tr>
<td>$\Lambda$CDMF3</td>
<td>0.5</td>
<td>0.5</td>
<td>2.38</td>
<td>0.40</td>
<td>2.75</td>
<td>0.27</td>
<td>1.06</td>
</tr>
<tr>
<td>$\Lambda$CDMC2</td>
<td>0.5</td>
<td>0.7</td>
<td>2.42</td>
<td>0.40</td>
<td>2.76</td>
<td>0.26</td>
<td>1.06</td>
</tr>
<tr>
<td>$\Lambda$CDMC3</td>
<td>0.5</td>
<td>0.9</td>
<td>2.43</td>
<td>0.41</td>
<td>2.74</td>
<td>0.27</td>
<td>1.08</td>
</tr>
</tbody>
</table>

As a measure for the size of the halos, we have explored two options: the virial radius and the mean harmonic radius.

- **Virial radius**: $r_{vir}$ is the size of the entire virialized clump identified by HOP. It does suffer from several artifacts, such as occasional unbound particles that happen to fly by or bridges between groups. Also, it suffers from the precise location of the outermost particles.

- **Half-mass radius**: $r_{half}$ is the radius that encloses half of the mass of the clump. This radius is closest in definition to the half-light radius used in observational studies.

- **Mean harmonic radius**: $r_h$ is defined as the inverse of the mean distance between all pairs of particles in the halo:

\[ \frac{1}{r_h} = \frac{1}{N} \sum_{i<j} \frac{1}{|r_{ij}|}, \quad N = \frac{n_{part}(n_{part} - 1)}{2}, \]

where $r_{ij}$ is the separation vector between the $i$th and the $j$th particle. The great virtue of this radius is that it is a good measure of the effective radius of the gravitational potential of the clump, certainly important when assessing the virial status of the clump. Also, it has the practical advantage of being independent of the definition of the cluster center. To some extent, it is also
5.3. THE SIMULATIONS

Figure 5.2 — Kormendy Relation. Each panel plots the relation between mass $M$ and mean harmonic radius $r_h$ of the cluster-sized dark halos in the simulations corresponding to one particular cosmology. Going from top left to bottom right these are: SCDM, $\Lambda$CDM02, $\Lambda$CDMF2 and $\Lambda$CDMC2. In each of the panels we have superimposed the fitted Kormendy relation, the style of each fit is given in the top left panel.

an indicator of the internal structure of the halo because it put extra weight to close pairs of particles.

Most of the results presented in this chapter refer to the mean harmonic radius of the halos. We have also compared the results which follow for the half-mass radii of the halos, and in a few cases we have also looked at the virial radius. We did find that indeed the mean harmonic radius is physically a better defined and therefore preferred radius. In Fig. 5.1 we plot the mean harmonic radius vs. the half-mass radius of the halos in the $\Lambda$CDMF2 model. In particular, for small size halos there can be a considerable difference between the two radii. This reflects strongly in the different Kormendy relations, but turned out to be less bearing on the inferred Fundamental Plane parameters. Also we found that the mean harmonic radius relations are more solid, beset by less scatter and noise.

5.3.3 Determination of Scaling Relations

Given the inferred mass $M$ (Eqn. 5.17), velocity dispersion $\sigma$ (Eqn. 5.18), from here onward we write $\sigma = \sigma_v$ when writing the velocity dispersion) and the mean harmonic radius $r_h$ (Eqn. 5.19) of the cluster halos, we find the scaling relation parameters by linear fitting of the relations. In order, these
are the Kormendy relation
\[ \log M = a \log r + C_a. \] (5.20)
For the Faber-Jackson relation,
\[ \log M = b \log \sigma + C_b. \] (5.21)
And finally, for the Fundamental Plane,
\[ \log M = c \log r + d \log \sigma + C_{fp}. \] (5.22)
The perpendicular distance of the cluster halos to the fitted plane we take as a measure for its width \( w_{fp} \):
\[ w_{fp} = \sqrt{\frac{\sum D_\perp^2}{N}}, \] (5.23)
where \( N \) is the number of cluster halos in the sample and \( D_\perp \) is the perpendicular distance of a point to a plane
\[ D_\perp = \frac{c \log r_h + d \log \sigma + C_{fp} - \log M}{(c^2 + d^2 + 1)^{1/2}}. \] (5.24)

## 5.4 Scaling Relations in Different Cosmologies: \( z=0 \)

We first investigate our cosmological models at the current epoch, \( z=0 \), for the viability of the scaling relations of the cluster dark matter halos and for possible systematic differences between the parameter values and FP width as a function of the cosmology.

The parameters of the resulting linear fits, to be discussed in the ensuing subsections, are listed in Table 5.2.

### 5.4.1 Kormendy Relation

Fig. 5.2 shows the relation between the mass \( M \) of each cluster halo and their mean harmonic radius \( r_h \). Each of the four panels depicts the relation for the halos in one particular simulated cosmology. The top left panel concerns the SCDM cosmology, the top right one, the \( \Lambda \)CDMO2, the bottom left one, the \( \Lambda \)CDMF2 and the bottom right one, the \( \Lambda \)CDMC2.

A visual comparison between the SCDM (top left panel) and the \( \Lambda \)CDMF2 (bottom left panel) also shows that clusters of comparable mass have a larger size in the low \( \Omega_m \) cosmology than in the ones with a higher density value. In other words, clusters are more compact in the SCDM cosmology. Perhaps not unexpected, in higher \( \Omega_m \) models we find objects of a higher density.

Fig. 5.2 clearly shows that in each cosmology there is a strong and systematic almost linear relation between \( \log M \) and \( \log r_h \): the Kormendy relation appears to be a good description for all situations. There is a difference in the width of this relation: the spread in the SCDM cosmology is far more substantial than that in the case of the other cosmologies, with the possible exception of the closed \( \Lambda \)CDMC2 cosmology. This hints at a systematic trend as a function of the cosmological density parameter \( \Omega_m \). Along with this, we discern a widening of the relation as we go to smaller masses.

When fitting the plotted point distributions, we infer the parameter values listed in Table 5.2. In each of the panels in Fig. 5.2 we plotted the linear fits for all of the four depicted cosmologies. We can immediately see systematic differences. While the high density cosmologies – SCDM, \( \Lambda \)CDMF2 and \( \Lambda \)CDMC2 – have slopes in the order of 2.4, the low density \( \Lambda \)CDMO2 has a considerable flatter Kormendy relation with \( a \) in the order \( a \approx 1.9 \). This we find to be the case for all \( \Omega_m = 0.1 \) cosmologies. This seems to imply that the mean density \( \langle \rho(r_h) \rangle \propto r_h^{-0.6} \) in the higher density Universes: larger halos have a proportionally lower average density. The contrast is much stronger in the open model, where \( \langle \rho(r_h) \rangle \propto r_h^{-1.1} \). This indicates a strong dependence of halo concentration as a function of its mass (also see chapter 3). If this is indeed so, it is a strong evidence of weak, instead of a perfect, homology
amongst the halos in all cosmologies. This might be in line with some of the findings of Lanzoni et al. (2004).

We investigated the dependence of the Kormendy parameter $a$ on the cosmology. In Fig. 5.3 we have plotted $a$ as a function of the average mass density parameter $\Omega_m$ (top left panel), as a function of the cosmological constant $\Omega_\Lambda$ (top right panel) and as a function of the cosmic curvature, in terms of $\Omega_{total} = \Omega_m + \Omega_\Lambda$.

The one outstanding influence is the dependence of $a$ on $\Omega_m$. We find the marked difference in $a$ for all low density Universes. There seems to be a significant change going from $\Omega_m = 0.1$ to $\Omega_m = 0.3$. It is likely that this is a consequence of the fact that nearly all existent structures and objects in these Universes did form at very high redshift. As a result, nearly all objects have experienced a very quiescent life since these early epochs. In an ongoing study we are investigating this in detail.

There is no evidence for any systematic trends of the Kormendy parameter as a function of the value of the cosmological constant. This is a good argument for the absence of any influence of $\Omega_\Lambda$ on the internal structure of the halos. The picture is somewhat confusing for the impact of cosmic curvature and we do not feel enabled to draw any firm conclusions with respect to its influence.

### 5.4.2 Faber-Jackson Relation

Fig. 5.4 shows the Faber-Jackson relation, the relation between the mass $M$ and the velocity dispersion $\sigma$ of the cluster halos. Like in Fig. 5.2, each of the four panels corresponds to one particular simulated cosmology. And as in Fig. 5.2, these are the SCDM cosmology (top left panel), the $\Lambda$CDMO2 (top right panel), the $\Lambda$CDMF2 (bottom left panel) and the $\Lambda$CDMC2 (bottom right panel).

For comparison, in each of the panels we show the four lines corresponding to the linear fits of this relation in each of the depicted four cosmologies. The $M - \sigma$ relation is clearly well fitted by the Faber-Jackson like relation. It is considerably tighter than the equivalent Kormendy relation. Also the spread around the relation seems to be less dependent on the mass of the halo.

![Figure 5.3](image-url) — Inferred parameters for the Kormendy relation in the cosmological models as a function of $\Omega_m$ (top left panel), $\Omega_\Lambda$ (top right panel) and $\Omega_m + \Omega_\Lambda$ (bottom panel).
Also interesting is to note that the inferred FJ relation does not vary significantly as a function of the underlying cosmology: the slope $b$ in all cases is in the order of $\sim 2.75$ (see Table 5.2). As may be inferred from Fig. 5.5, the only possible deviant cases are the low $\Omega_m$ Universes. However, even this is hardly significant while any dependence on $\Omega_\Lambda$ or $\Omega_{\text{total}}$ can be immediately discarded.

A likely explanation for the fact that $b \sim 2.75$ in most situations, instead of the expected value of $b = 3$ for virialized perfectly homologous systems (see Eqn. 5.15), is that halos do form a weakly homologous population along the lines described in, e.g., Bertin et al. (2002).

### 5.4.3 Fundamental Plane

The Kormendy relation and the Faber-Jackson relation are two dimensional projections of an intrinsically three dimensional relation between mass $M$, size $r_h$ and velocity dispersion $\sigma$ of the halos. By implication, the spread of the Fundamental Plane relation should be less than that of each of the two previous relations.

We have illustrated the Fundamental Plane relation in Fig. 5.6, for the same cosmologies as in Fig. 5.2 and 5.4 (SCDM, $\Lambda$CDMO2, $\Lambda$CDMF2 and $\Lambda$CDMC2 going from top left to bottom right panel). In each of the frames we have plotted the mass $M$ of the halos against the quantity $r_h^2 \sigma^d$. The

![Figure 5.4 — Faber-Jackson relation. Each panel plots the relation between mass $M$ and the velocity dispersion $\sigma$ of the cluster-sized dark halos in the simulations corresponding to one particular cosmology. Going from top left to bottom right these are: SCDM, $\Lambda$CDMO2, $\Lambda$CDMF2 and $\Lambda$CDMC2. In each of the panels we have superimposed the fitted Faber-Jackson relation, the style of each fit is given in the top left panel.](image-url)
parameters $c$ and $d$ in the latter quantity, combining velocity dispersion $\sigma$ and mean harmonic radius $r_h$ of each halo, are the best fit FP parameters for the corresponding cosmology.

The galaxy clusters in each cosmology do indeed seem to populate a tightly defined plane. The point clouds in each of the frames confirm our expectation that they do have a much lower scatter around the plane than in the case of the Kormendy and Faber-Jackson relation.

From Table 5.2 we find a surprising level of consistency between the Fundamental Planes that we find in each of the cosmologies. We find that the inferred parameters are close to the one theoretically expected for perfectly homologous virialized clusters halos, $M \propto r_h \sigma^2$. In particular, the scaling parameter $c$ for the radius $r_h$ is very close to unity. The difference is somewhat larger for the parameter $d$, showing that the velocity dispersion scaling has a difference of $\sim 0.15 - 0.25$ from the theoretical value of 2.

As can be seen in both Table 5.2 and Fig. 5.6, there is hardly any variation between the FP relations in each of the cosmologies: they almost all coincide. This is indeed true when it concerns the FP parameters $c$ and $d$. The two top panels of Fig. 5.7 do confirm the impression that there is no systematic difference as a function of $\Omega_m$ and/or $\Omega_\Lambda$. This in itself is a strong argument against differences in the scaling relations parameters being due to a partial or incomplete level of virialization, as was claimed by Adami et al. (1998).

One outstanding difference in Fundamental Plane between the different cosmologies concerns its width. Inspection of Fig. 5.6 does suggest a somewhat larger width of the FP for high density Universes. This turns out to be a very systematic effect: the lower left hand panel of Fig. 5.7 shows a nearly linear increase of the FP width with $\Omega_m$, in combination with a near independence of cosmological constant $\Omega_\Lambda$. We may try to understand this difference in terms of the ongoing evolution of the cluster population. In low $\Omega_m$ Universes all clusters formed at high redshift and have since had ample time to reach full virialization. In high $\Omega_m$ Universes, clusters still undergo a substantial levels of merging and accretion, both of which may affect the virial state of the cluster. We investigate this question in more detail in section 5.6.

Figure 5.5 — Inferred parameters for the FJ relation according to three different parameters: $\Omega_m$ (top-left panel), $\Omega_\Lambda$ (top-right panel) and $\Omega_m + \Omega_\Lambda$ (bottom panel).
Chapter 5: Cluster Halo Scaling Relations

Figure 5.6 — Fundamental Plane. Each panel plots the relation between mass $M$ and the quantity $r_h^c \sigma^d$, the combination of mean harmonic radius $r_h$ and velocity dispersion $\sigma$, with $r_h$ to the power $c$ of the FP linear fit and $\sigma$ to the power $d$ of the FP linear fit. The dots in the top left panel represent the cluster halos in the SCDM simulation, in the top right panel the ones in the $\Lambda$CDMO2 simulation, in the bottom left panel the ones in the $\Lambda$CDMF2 simulation while the bottom right panel contains the ones in the $\Lambda$CDMC2. The superimposed lines in each panel represent the fitted Fundamental Plane for the four different cosmologies (see insert top left panel). Note that by definition each of these lines has slope unity and only differs in amplitude.

Finally, if we relate the Fundamental Plane $M - r_h - \sigma$ of our simulated cluster samples to the observationally measure one $L - R_e - \sigma$, e.g. $L \propto R_e^{0.90} \sigma^{1.31}$ found by Lanzoni et al. (2004), we may try to see whether the difference can be solely ascribed to a mass dependent mass-to-light ratio $M/L$. We do find a significant dependence in that more bright clusters would have a larger $M/L$. However, in each of our cosmologies it is not possible to get an agreement only on the basis of a mass dependent $M/L$. Although our results suggest that $M/L \sim L^\alpha$ with $\alpha \sim 0.2 - 0.4$, other factors of structural or dynamical origin do seem to be of importance.

5.4.4 Halo size: radius definition and scaling relation

Apart from the mean harmonic radius that we have used as a measure of halo size in the previous sections, we have also assessed the viability of the scaling relations in case of alternative size definitions. In Table 5.3 we list the resulting parameters for the Kormendy relation and the Fundamental Plane in the case of using 1) half-mass radius $r_{half}$ and 2) virial radius $r_{vir}$.

The parameters for the Kormendy relation and the Fundamental Plane are significantly different from the ones inferred on the basis of the mean harmonic radius. This is true for both half-mass
radius and virial radius. The difference is most stark for the low \( \Omega_m \) ACDMO2 cosmology. Note the substantial increase in the width of the Fundamental Plane for the SCDM cosmology. The other cosmologies do not seem to yield substantially wider FP relationships. The Kormendy relation also does not appear to widen.

Interestingly, the differences nearly all concern the scaling parameter for the size while the scaling for the velocity dispersion \( \sigma \) in the Fundamental Plane remains well behaved, and seems to yield a value \( d \) closer to the ideal virial value \( d = 2 \).

The change in scaling parameter values may be ascribed to the use of quantities that probe different aspect of the structure and dynamics of the halos. In an extreme situation, this might have disrupted the scaling relations. However, our finding shows that scaling relations do still hold but in a slightly different disguise. It may be an indication for our contention that halos do not form a perfectly homologous population. Size measures sensitive to different aspects of the halos' internal mass distribution may then result in somewhat different scaling properties. In this respect, we agree with the conclusions of Adami et al. (1998) and Lanzoni et al. (2004).

### 5.5 Evolution of Scaling Relations

In the previous sections we have extensively studied the scaling relations at the current cosmic epoch \( z = 0 \). We have also noted that there are differences between the scaling relation parameters that we find in our simulations and those for perfect virialized and homologous systems. This makes it interesting to trace the evolution of the different scaling relations.

In this section we have investigated the evolution of the scaling relations as a function of redshift and as a function of cosmic time. Cosmic time is the time that has passed since the Big Bang. While observers usually think in terms of redshift, it is good to realize that a given redshift corresponds to an entirely different dynamical epoch in different cosmologies. Given the same Hubble parameter, the

![Figure 5.7](image_url)

Figure 5.7 — Top panels: inferred parameters \( c \) and \( d \) of the Fundamental Plane relation as a function of \( \Omega_m \) (top left) and as \( \Omega_\Lambda \) (top right). Bottom panels: rms scatter of the FP relation as a function of \( \Omega_m \) (bottom left) and as \( \Omega_\Lambda \) (bottom right).
Figure 5.8 — Top: Kormendy relation of dark halos in a SCDM (left) and ΛCDMO2 (right) cosmology, using the virial radius $r_{\text{vir}}$ (upper row) and the half-mass radius $r_{\text{half}}$ (lower row). Each of the panels plots mass versus radius of the halos. The lines represent the best fit relations: SCDM, solid line, and ΛCDMO2, dashed line. Bottom: Fundamental Plane relation of dark halos in the SCDM (left) and ΛCDMO2 (right) cosmology, using the virial radius $r_{\text{vir}}$ (upper row) and the half-mass radius $r_{\text{half}}$ (lower row). Plotted are mass $M$ versus the FP quantity $r_c \sigma_d$. The lines represent the best fit FP relations.
age of the Universe is a sensitive function of the cosmic density parameter $\Omega_m$ and even more so of the cosmological constant. As for the latter, we have to realize that the change in cosmic time as a function of the cosmological constant is the most important influence of $\Lambda$. To give an impression of the differences in cosmic time for a given redshift between the different cosmologies, we refer to Table 5.4.

We have probed the scaling relations over a range of redshifts from $z = 4$ to $z = 0$ and over a range of cosmic time going from $1 - 10$ Gyr. The evolution of the obtained scaling parameters as a function of redshift is shown in the left column of Fig. 5.10. The corresponding evolution as a function of cosmic time can be found in the right hand column. The Kormendy parameter $a$ is shown in the top panels, the Faber-Jackson parameter $b$ in the center panels and the FP parameters $c$ and $d$ in the bottom panels. Each different cosmology is represented by a different linestyle, listed in the insert of the top left hand frame.

The Kormendy relation clearly evolves as a function of redshift, at least for the three high density cosmologies. Only in the case of the low $\Omega_m$ $\Lambda$CDMO2 we can not discern any significant increase of $a$, partially due to the large uncertainties in the calculated parameter as a result of the low number of halos in this simulation. The evolution of the Kormendy parameter in the other cosmologies is considerably more interesting. Up to a redshift of $z \sim 1$, we find a continuous increase of $a$ towards a value of $a \sim 2.5$. At later times the impression is one of a mild decline in the value of $a$ in the case of SCDM. In the $\Lambda$CDMF2 and $\Lambda$CDMC2 the Kormendy parameter seems to remain constant. This is particularly clear when assessing the evolution in terms of the cosmic time, as can be seen in the top right panel.

Evolutionary trends for the Faber-Jackson relation are comparable to that seen in the Kormendy relation. No discernible trends are found in the open cosmology, while all of the other high density Universes do show a uniform increase over the full redshift range 4 to 0. At the most recent redshifts $b$ reaches a value in the range $b > 2.5$, from a value in the order of $b \sim 2$ at higher redshifts. There is an indication that the increase of $b$ is slowing down at more recent epochs. This is the impression obtained when looking at the evolution of $b$ as a function of cosmic time (see center right panel).

Interestingly, there is absolutely no trend whatsoever in the values of the Fundamental Plane parameters. When it comes to the evolution of the Fundamental Plane, it mainly or even exclusively concerns the width of the fitted plane. In Fig. 5.9 we show the development of the FP width as a function of cosmic expansion factor $a(t) = 1/(1+z)$. We see a systematic increase of FP width over the whole cosmic evolution in the case of the high $\Omega_m$ SCDM cosmology. While we do see a rise of the FP width before $a < 0.5$ in the $\Lambda$CDMF2 and $\Lambda$CDMC2 cosmologies, after that time this increase

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Omega_m$</th>
<th>$\Omega_\Lambda$</th>
<th>Radius</th>
<th>$M \propto r^a$</th>
<th>$M \propto r^c \sigma^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Half-mass</td>
<td>$a$</td>
<td>$c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Virial</td>
<td></td>
<td>$d$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{\chi^2/(n-1)}$</td>
<td>$\sqrt{\chi^2/(n-2)}$</td>
</tr>
<tr>
<td>SCDM</td>
<td>1</td>
<td>0</td>
<td>Half-mass</td>
<td>1.88 0.52</td>
<td>0.72 2.13 0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Virial</td>
<td>2.09 0.40</td>
<td>0.80 1.89 0.25</td>
</tr>
<tr>
<td>ACDMO2</td>
<td>0.1</td>
<td>0.7</td>
<td>Half-mass</td>
<td>1.49 0.69</td>
<td>0.69 2.15 0.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Virial</td>
<td>1.56 0.62</td>
<td>0.62 1.72 0.37</td>
</tr>
<tr>
<td>ACDMF2</td>
<td>0.3</td>
<td>0.7</td>
<td>Half-mass</td>
<td>2.04 0.53</td>
<td>0.82 2.06 0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Virial</td>
<td>2.21 0.60</td>
<td>0.76 2.01 0.27</td>
</tr>
<tr>
<td>ACDMF2</td>
<td>0.5</td>
<td>0.9</td>
<td>Half-mass</td>
<td>2.00 0.54</td>
<td>0.75 2.13 0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Virial</td>
<td>2.23 0.46</td>
<td>0.71 2.01 0.27</td>
</tr>
</tbody>
</table>

Table 5.3 — Scaling relations parameters: inferred parameters for the Kormendy relation and the Fundamental Plane relation for the galaxy clusters when considering half-mass radius and virial radius.
levels off and may even flatten completely. It may be of relevance that these simulations do not attain sufficient halo mass resolution at higher redshifts: in these cosmologies halos still are low mass objects at these epochs. The one outstanding cosmology is that of the low $\Omega_m$ Universe $\Lambda$CDM2. Except for a rather abrupt and sudden jump in FP width at $a \sim 0.3$, there is no noticeable change at later epochs. By $a = 0.3$ nearly all its clusters are in place and define a Fundamental Plane that does not undergo any further evolution.

On the basis of their study of galaxy merging, Nipoti et al. (2003) argued that the origin of the Fundamental Plane is more due to a structural non-homology than a dynamical non-homology. Our finding that the development of the Kormendy and FJ relations somehow appear to compensate each other in reproducing the same Fundamental Plane at every redshift, but with a stronger and more systematic evolution of the Kormendy relation, seems to point at the same conclusion. The systematic evolution of the Kormendy and the Faber-Jackson relation that we find, in combination with the unchanging character of the Fundamental Plane certainly hints at the role of the structural and dynamical properties of halos in defining the Fundamental Plane.

The latter clearly indicates that although the halo population remains solidly within the Fundamental Plane, its location within the plane clearly shifts as time proceeds. Apparently the evolution of halos consists of a gradual shift along an almost universal Fundamental Plane. This is clearly illustrated in

Table 5.4 — Cosmic times in Gyr and its corresponding redshift for each cosmological model.

<table>
<thead>
<tr>
<th>Cosmic Time</th>
<th>SCDM</th>
<th>$\Lambda$CDM2</th>
<th>$\Lambda$CDMF2</th>
<th>$\Lambda$CDMC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.36</td>
<td>1.49</td>
<td>4</td>
<td>2.71</td>
<td>2.21</td>
</tr>
<tr>
<td>3.26</td>
<td>1.01</td>
<td>2.92</td>
<td>1.98</td>
<td>1.60</td>
</tr>
<tr>
<td>4.06</td>
<td>0.74</td>
<td>2.35</td>
<td>1.56</td>
<td>1.24</td>
</tr>
<tr>
<td>5.07</td>
<td>0.50</td>
<td>1.83</td>
<td>1.19</td>
<td>0.93</td>
</tr>
<tr>
<td>9.31</td>
<td>0</td>
<td>0.71</td>
<td>0.38</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 5.9 — Evolution of the width of the Fundamental Plane for four different cosmologies. Note the almost consistently tighter FP for the low $\Omega_m$ Universe and the modest decrease of FP width in the other cosmologies.
Figure 5.10 — Evolution of the obtained scaling relations parameters as a function of redshift (left column) and as a function of cosmic time (right column). Top: Kormendy parameter $a$. Center: Faber-Jackson parameter $b$. Bottom: FP parameters $c$ and $d$. 
fig. 5.11. It shows that the halo population seems to evolve from a rather scattered and loose one into a tightly elongated point cloud at later epochs, providing interesting clues towards understanding the cluster virialization process.

### 5.6 Merging and accretion dependence

In the above, we have studied the scaling relations as a manifestation of the virial state of the cluster halos in each of the cosmologies. In the real world of hierarchical structure formation scenarios the formation and evolution of halos is hardly a quiescent and steadily progressing affair. Instead, halos grow in mass by steady accretion of matter from its surrounding as well through the merging with massive peers. Even the accretion is not a continuous and spherically symmetric process: most matter flows in in a strongly anisotropic fashion through filamentary extensions into the neighboring large scale matter distribution. As a result, we do expect that numerous halos will not have settled in a

![Figure 5.11 — Shifting location of the cluster halo population within the Fundamental Plane. The depicted halo sample is the one in ΛCDMf2 cosmology, and is shown at four different redshifts: $z = 2.61$ (top left panel), $z = 1.61$ (top right panel), $z = 0.89$ (bottom left panel) and $z = 0$ (bottom right panel). The abscissa and ordinate axis are arbitrarily chosen axes within the FP plane $\log M - \log r_h - \log \sigma$.](image-url)
perfect virial state. This will certainly be the case when it recently suffered a major merger with one or more neighboring clumps.

The detailed accretion and merging history is a function of the underlying cosmology. Low density cosmologies or cosmologies with a high cosmological constant will have frozen their structure formation at early epochs. The halos that had formed by the time of that transition will have had ample time to settle into a perfect virialized object. Also, there is a dependence on the power spectrum of the corresponding structure formation scenario. Power spectra with a slope $n < -1.5$ (at cluster scales) will imply a more homologous collapse of the cluster sized clumps, less marked by an incessant bombardment by smaller clumps. It may be clear that a more violent life history of a halo will usually be reflected in a substantial deviation from a perfect virial state.

In order to investigate the implications of a difference in accretion or merging history of halos, we have split the samples of cluster halos in each of our cosmologies into a merging sample and an accretion sample. The mergers were those halos which suffered a merger with another halo that contained at least 30% of its mass. Possible differences in virial state should be reflected in the quality of the scaling relations, in particular that of the width of the Fundamental Plane.

In Fig. 5.12 we show the evolution of the width of the Fundamental Plane for each of the two samples in the four indicated cosmologies. Note that our simulations do not have sufficient resolution for reconstructing the precise merging or accretion history before $a = 0.3 - 0.4$, so that we may not draw conclusions on the rise of the FP width up to that epoch.
In the more recent history we do find some significant differences between merging and accretion-only halos in the different cosmologies. In the case of the $\Lambda$CDMO2 scenario we do not have enough cluster halos to detect any systematics differences between the merging and accreting halos. There does not seem to be a systematic difference between these groups in the $\Lambda$CDMF2 cosmology. The story is quite different for the $\Lambda$CDMC2 and SCDM cosmology. While the cluster halos that undergo a major merger do reveal a constantly growing FP width, their accretion-only clusters do not display such a systematic increase. Instead, their FP width remains lower and levels off. In other words, accretion halos (dotted lines) do on average display a tighter FP relation. This is particularly true at the current epoch. Apparently, the absence of violent mass gain in the case of accretion halos implies them to have more time to relax and virialize.

5.7 Conclusions

We have studied three structural scaling relations of galaxy clusters in thirteen cosmological models. These relations are the Kormendy relation, the Faber-Jackson relation and the Fundamental Plane. Their validity and behavior in the different cosmological models should provide information on the general virial status of the cluster halo population. The cosmological models that we studied involved a set of open, flat and closed Universes with a range of matter density parameter $\Omega_m$ and cosmological constant $\Omega_{\Lambda}$.

The cluster samples are obtained from a set of corresponding $N$-body simulations in each of the cosmologies. These simulations concerns a box of $200h^{-1}$Mpc with $256^3$ dark matter particles. The initial conditions were set up such that the phases of the Fourier components of the primordial density field are the same for all simulations. In this way, we have simulations of a comparable morphological character: the same objects can be recognized in each of the different simulations (be it at a different stage of development).

After running the simulations from $z = 4$ to the current epoch, with the help of the GADGET2 code, we used HOP to identify the cluster halos. Of each halo population we investigated whether it obeyed a mass-radius relation akin to the Kormendy relation, a mass-velocity dispersion relation similar to the Faber-Jackson relation and a two parameter family between mass, radius and velocity dispersion that resembles a Fundamental Plane relation. We studied the dependence of the obtained scaling parameters as a function of the underlying cosmology and investigated their evolution in time.

Our results can be summarized as follows:

- In each cosmological model we do recover Kormendy, Faber-Jackson and Fundamental Plane relations for the population of cluster halos. This is a strong indications for the virialized state of the halos, as we do expect in hierarchical clustering scenarios.

- There are significant differences between the measured parameters of the various scaling relations and those seen in the observational reality. Given the different behavior of the Kormendy relation, Faber-Jackson relation and Fundamental Plane relation, we tend to agree with claims that these differences are not only due to simple difference in mass-to-light ratio $M/L$ but that we are dealing with a weakly homologous population of objects. This maybe more due to a structural non-homology to a dynamical non-homology.

- We find that the parameters of the Kormendy and Faber-Jackson relations are mildly sensitive to the value of $\Omega_m$: $a$ and $b$ are somewhat larger in high density Universes. By far, the largest impact is that of $\Omega_m$ on the width of the Fundamental Plane. The width is almost directly proportional to the value of $\Omega_m$. There is no indication for any influence of $\Omega_{\Lambda}$ on the scaling relations.

- The Fundamental Plane parameters do not show any sign of evolution: the scaling parameters $c$ and $d$ remain constant over the investigated redshift range. However, the width of the Fundamental Plane does evolve significantly. In all high density Universes we see a uniform but mild
5.7. CONCLUSIONS

decrease of width as time proceeds. This reflects the gradually virializing tendency of the cluster population.

- With the exception of low $\Omega_m$ Universes, we do find a systematic increase of the Kormendy and Faber-Jackson parameters $a$ and $b$ from $z = 4$ up to $z = 1$. From $z = 1$ to the present epoch, the Faber-Jackson parameter $b$ continues to grow, while the situation is less clear for $a$. In a SCDM cosmology $a$ shows a slight decrease, while in the other models seems to remain constant.

- The combination of evolution of the Kormendy and Faber-Jackson with the apparent universal nature of the Fundamental Plane reveals that the evolution of the cluster population is constrained to paths within the FP.

- Given our expectation that there is a difference in virial state between quiescently accreting clusters and those experiencing massive mergers, we have investigated the evolution of the Fundamental Plane width for samples of merging clusters and samples of accreting clusters. We find that accreting clusters at recent epochs do appear to be better virialized than the merging population in that the FP width is smaller in the former.