We investigate the formation history and virialization of cluster sized dark matter halos in a range of cosmologies. With the help of a set of thirteen $N$-body cosmological simulations, we seek to identify the influence of the cosmological density parameter $\Omega_m$ and the cosmological constant $\Omega_\Lambda$ on the formation of clusters. We find that the accretion and merging history of clusters is sensitive to the cosmological density parameter $\Omega_m$. The only identifiable influence of a cosmological constant is that via the corresponding dynamical timescales. The formation redshift of clusters is substantially higher in low $\Omega_m$ Universes. We compare our simulations to a few analytical prescriptions of the halo formation history and find that they manage to reasonably reproduce the complete mass assembly history of clusters. We also address the virialization of clusters in the various cosmologies. In none of the cosmologies we find perfectly virialized clusters. Low mass halos display a large range of virial states, ranging from highly virialized to hardly bound. High mass cluster halos have a virial state $|U| \sim 1.60 – 1.65 K$ which turns out to be the same for all simulated cosmologies. The one difference concerns the spread around this relation: it is larger in high $\Omega_m$ Universes.
Chapter 3: Galaxy Cluster Evolution: Mass Growth and Virialization

3.1 Introduction

Clusters of galaxies are the most massive and most recently collapsed and virialized objects in our Universe. As such, they represent ideal probes for understanding the complex formation history of objects in the Universe. In the currently favored structure formation scenarios, mostly involving a dominant cold dark matter component, structure arose through the gravitational growth of tiny primordial Gaussian density and velocity perturbations.

This process involves a complex hierarchical evolution in which small scale density fluctuations are the first to collapse and form objects. As larger and larger fluctuations start to mature, the small clumps merge with each other into ever larger objects. Along with the more pronounced mergers, the process also involves a gradual accretion of more modest mass concentrations. By means of the gradual accretion and merging, massive structures finally emerge out of the almost pristine and featureless primordial Universe.

Turning to individual objects like clusters, we may wonder to what extent their internal structure and dynamics is affected by the global cosmological background. By identifying possible influences – of which those of the cosmic density $\Omega_{\text{m}}$, the cosmological constant $\Omega_{\Lambda}$ and the amplitude of mass fluctuations quantified by $\sigma_8$ are the most straightforward ones – potentially we would be able to infer the cosmology from observations of clusters.

In this chapter we are particularly interested in three closely related aspects of cluster evolution. These are the mass assembly history of clusters, distinguished in terms of the specific merging and gradual accretion mode of mass growth, and the final dynamical state of the emerged cluster halos. The latter is usually quantified in terms of the virialization of the cluster.

Merging and accretion, together with virialization, are tightly related to the formation time. It is important to note that there is no proper definition of formation time. This makes it difficult to compare studies between authors. A few definitions of formation time are based on idealized analytical prescriptions of the mass accretion history of halos. None of these definitions are compelling and rather arbitrary. As we will show in this study, they not only fail to describe the mass accretion history over the complete formation time of cosmic objects but also appear to conflict with each other (see also, Lacey & Cole 1993; Wechsler et al. 2002; van den Bosch 2002; Tasitsiomi et al. 2004; Cohn & White 2005).

Virialization is the ultimate dynamical state of any collapsing structure turning into an individual and recognizable cosmic object. During the virialization process, the matter contained in the object exchanges energy to the extent of reaching an equilibrium state. Virialization would offer the possibility to infer the mass of the object on the basis of the thermal velocity of its matter content. Nevertheless, the presence of substructure tells us that this assumption is usually incorrect and not warranted. X-ray observations of galaxy clusters show a large variety of substructure and morphology (e.g. Mohr et al. 1995). There is a variety of studies addressing the virialization of galaxy clusters, usually involving simulated clusters (e.g., Knebe & Müller 1999; Macciò et al. 2003; Shaw et al. 2006). Knebe & Müller (1999) studied virialization of their simulated cluster halos and found that the virial theorem does not hold in any of their simulated cosmologies, arguing that this is due to the influence of an outer pressure of radially infalling particles into the halos. Macciò et al. (2003) also applied the virial theorem, but they added a pressure term to take into account the external material. Shaw et al. (2006) also work with the virial theorem plus a surface pressure term. They showed that by adding this term, the virial theorem holds for their clusters.

In this chapter we investigate the formation history and virialization of galaxy cluster halos and its dependence on the background cosmology. This formation history is tightly related to the internal kinetic and potential energy they acquire during their evolution. The structure of the chapter is as follows. In section 3.2 we briefly describe the simulations we use. Section 3.3 presents the study of merger and accretion of cluster halos. Section 3.4 presents the formation time and their dependence on the background cosmology. In section 3.5 we study the virial theorem of galaxy clusters. Conclusions are presented in section 3.6.
3.2. THE SIMULATIONS

The simulations and the method to identify halos are extensively described in chapter 2. Here, we summarize this description.

We perform thirteen N-body simulations that follows the dynamics of \( N = 256^3 \) particles in a periodic box of size \( L = 200h^{-1}\text{Mpc} \). The initial conditions are generated with identical phases for Fourier components of the Gaussian random field. In this way each cosmological model contains the same morphological structures. For all models we chose the same Hubble parameter, \( h = 0.7 \), and the same normalization of the power spectrum, \( \sigma_8 = 0.8 \). The principal differences between the simulations are the values of the matter density and vacuum energy density parameters, \( \Omega_m \) and \( \Omega_\Lambda \). By combining these parameters, we get models describing the three possible geometries of the Universe: open, flat and closed. The effect of having the same Hubble parameter and different cosmological constants translates into having different cosmic times.

The initial conditions are evolved until the present time (\( z = 0 \)) using the massive parallel tree N-body code GADGET2 (Springel 2005). The Plummer-equivalent softening was set at \( \epsilon_{pl} = 15h^{-1}\text{kpc} \) in physical units from \( z = 2 \) to \( z = 0 \), while it was taken to be fixed in comoving units at higher redshifts. For each cosmological model we wrote the output of 100 snapshots, from \( a = 0.2 \) (\( z = 4 \)) to the present time, \( a = 1 \) (\( z = 0 \)), equally spaced in \( \log(a) \).

### 3.2.1 Halo identification

We use the HOP algorithm (Eisenstein & Hut 1998) to extract the groups present in the simulations. HOP associates a density to every particle. In a first step, a group is defined as a collection of particles linked to a local density maximum. To make a distinction between a high density region and its surroundings, HOP uses a regrouping procedure. This procedure identifies a group as an individual object on the basis of a specific density value. Important for our study is the fact that for this critical value we chose the virial density value \( \Delta \), following from the spherical collapse model. In order to have the proper \( \Delta \), we numerically compute its value for each of the cosmologies (see appendix 2.A).

Table 3.1 lists the values of the cosmological parameters and the values of the virial density for each cosmological model.
cosmology at $z = 0$. For the latter we list two values: the virial overdensity $\Delta_{\text{vir}, b}$ with respect to the background density $\rho_b$ of the corresponding cosmology, and the related virial overdensity $\Delta_{\text{vir}, c}$ with respect to the critical density.

Note that we only consider groups containing more than 100 particles. Because the particle mass depends on the cosmological scenario, this implies a different mass cut for the halos in each of our simulations. As a result, SCDM does not have groups with masses lower than $10^{13}h^{-1}M_\odot$. We have to keep in mind this artificial constraint when considering collapse and virialization in hierarchical scenarios at high redshifts. When structure growth is still continuing vigorously at the current epoch, the collapsed halos at high redshifts will have been small. Our simulations would not be able to resolve this.

### 3.2.2 Halo properties

In this chapter we analyze the complete sample of halos, although in some situations we limit ourselves to those dark matter halos that would correspond to rich galaxy clusters. A galaxy cluster is defined to have a dark matter mass $M$ of $M > 10^{14}M_\odot$.

The particle mass is a function of the cosmology. With the $N$ particles contained within a simulation volume $V = L^3$, the mass $m_{\text{part}}$ of a particle in a cosmology with density parameter $\Omega_m$ is simply

$$m_{\text{part}} = \Omega_m \rho_c \left( \frac{V}{N} \right) = \Omega_m \left( \frac{3H^2}{8\pi G} \right) \left( \frac{V}{N} \right) = \Omega_m h^2 \left( \frac{V}{N} \right) 2.7755 \times 10^{11} M_\odot$$

(3.1)

To investigate the virial properties of the halos we determine of each halo the mass $M$, kinetic energy $K$ and gravitational potential energy $U$. Given a halo of $N$ particles we compute these quantities as follows:

- **Mass**: the number of halo particles multiplied by the mass per particle present in each halo:
  $$M = N_{\text{halo}} m_{\text{part}}$$
  (3.2)
  where $N_{\text{halo}}$ is the number of particles in the halo.

- **Kinetic Energy**: the total sum of the particle kinetic energies (with respect to the center of the halo):
  $$K = \frac{1}{2} \sum_{i=1}^{N_{\text{halo}}} m_i (v_i - v_{\text{center}})^2$$
  (3.3)
  where $v_i$ is the physical velocity of particle $i$ and $v_{\text{center}}$ the physical velocity of the halos’ center of mass.

- **Potential Energy**: defined as
  $$U = - \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{G m_i m_j}{|r_i - r_j|}$$
  (3.4)
  with $r_i$ and $r_j$ the locations of particles $i$ and $j$.

- **Virial ratio**: defined as the ratio of the kinetic energy and the potential energy,
  $$\mathcal{V} \equiv \frac{2K}{U}$$
  (3.5)
3.2.3 Halo merger trees

The hierarchical scenario of structure formation states that small clumps form first, and gradually merge and accrete into larger structures.

Under this process, the formation history of a halo is a complex story of the formation and evolution of many individual clumps that ultimately end up in a given halo at some cosmic epoch. To this, we have to add all the field material that meanwhile accreted quiescently on to these proto-halos. One practical approach to deal with this is to trace back in time all the particles that have ended up in the halo. By identifying the halos to which they previously belonged we may infer the merging and accretion history of the halo.

By continuing the proto-halo identification process until the epoch at which all contributing particles are field particles that do not belong to any proto-halo, we have produced what is commonly known as a merging tree (Kauffmann & White 1993; Lacey & Cole 1993). In Fig. 3.1 we depict the first stage of this process: a halo and the four proto-halos that finally merged into this halo.

Folklore calls the most recent halo the child halo. Of course, this poses the question which of the four proto-halos - or even all them - should be regarded as the real progenitor of the present halo. We assign, arguably a somewhat artificial choice, this quality to the most massive progenitor of a given dark matter halo. In Fig. 3.1 the progenitor of the present halo is proto-halo C.

![Diagram of halo merger tree](image)

Figure 3.1 — Simple description of the merging tree of a dark matter halo. In this case, a given halo has four progenitors at a previous redshift, but only one, C, is the most massive one.

Given the groups found by HOP, it is straightforward to construct the merging tree of every single dark matter halo in each cosmological model.

3.2.4 Timing the simulations

When evaluating the growth of cluster halos, we do so in terms of three time related quantities.

The first is the redshift \( z \), the prime observational time related cosmological quantity. It directly relates to what an observational cosmologist would infer from his/her observations. Cosmic redshift \( z \) is directly related to the cosmic expansion factor,

\[
    z = \frac{1}{a} - 1. \tag{3.6}
\]

The time \( t(z) \) is related to redshift, but in a way which depends on the cosmology. The same redshift \( z \) corresponds to different times for different cosmologies. Here we distinguish two different
time measures. The first, lookback time, relates directly to the redshift $z$: it is the time interval between the present epoch and the time of “emission” by the observed object at redshift $z$. For a FRW Universe, with a Hubble constant $H_0$, with a matter density parameter $\Omega_{m,0}$, a cosmological constant $\Omega_{\Lambda,0}$, the lookback time $t_l(z)$ is given by

$$ t_l(z) = \frac{1}{H_0} \int_{1/(1+z)}^1 \frac{x \, dx}{\sqrt{\Omega_{m,0} x + \Omega_{\Lambda,0} x^4 + (1 - \Omega_0) x^2}}, $$

with $\Omega_0 = \Omega_{m,0} + \Omega_{\Lambda,0}$.

An equally interesting quantity for dynamically evolving systems is what we call cosmic time, $t_c(z)$. It is nothing else than the age of the Universe at that redshift, i.e. the time that passed since the Big Bang. It is a measure for the available dynamical time of the system. For a FRW Universe, with the same parameters as specified above, it is simply given by

$$ t_c(z) = \frac{1}{H_0} \int_0^{1/(1+z)} \frac{x \, dx}{\sqrt{\Omega_{m,0} x + \Omega_{\Lambda,0} x^4 + (1 - \Omega_0) x^2}}. $$

To appreciate the relation between redshift, cosmic time and lookback time, we have plotted redshift as a function of these time definitions for four different cosmologies in Fig. 3.2. Objects at a particular redshift $z$ in the SCDM Universe are considerably younger than those in the Universes with a cosmological constant. Also, we see that Universes with a lower $\Omega_m$ are older than the ones with a higher $\Omega_m$ (left hand frame). Conversely, the lookback time to an object with redshift $z$ in a SCDM cosmology is substantially smaller than that in the Universes dominated by a positive $\Omega_{\Lambda}$. When comparing the $\Lambda$CDM cosmologies we see that the one with the higher $\Omega_m$ has a shorter lookback time for a given redshift.

Figure 3.2 — Redshift as a function of cosmic time (in units of Gyr, left panel) and lookback time (in units of the Hubble time, right panel for the four cosmological models described in the text.

### 3.3 Cosmological Cluster Formation

The formation of structure in the Universe is the result of the gravitational growth of primordial tiny density and velocity perturbations. Perhaps the most important aspect of the formation process is its hierarchical character. The first objects to form are small, these small clumps subsequently merge into
ever larger objects. Hence, clusters of galaxies emerged as the result of some massive mergers with peers and a more continuous process of gradual accretion of mass from their surroundings.

The history of this process is highly dependent on the cosmology. The first factor of importance is the amount of dark matter in the Universe. The structure formation process is driven by the growth of fluctuations in the density of dark matter. In low \( \Omega_m \) Universes, the rate of growth is therefore slower than that in high \( \Omega_m \) Universes. In addition, the cosmic expansion history bears strongly on the formation process. Negatively curved, matter dominated Universes tend to evolve towards a free cosmic expansion, while the Universe would even assume an exponential De Sitter expansion in case a cosmological constant assumes dominance over the cosmic dynamics. In both cases, the fast expansion rate of the Universe stifles the growth of matter perturbations so that the structure formation process comes to a halt. As a result, low \( \Omega_m \) and/or high \( \Omega_{\Lambda} \) Universes will stop forming clusters at some specific epoch.

The presence of a cosmological constant may also leave its imprint in a few alternative ways. One effect is that of the lengthening of the age of the Universe. Given sufficient amounts of dark matter this would lead to a more substantial level of structure formation at higher redshifts. On the other hand, it might have a negative influence because of the repulsive nature of the cosmological constant. However, this effect does seem to be of minor influence: e.g. Lahav et al. (1991) found it would have no more than \( \sim 1/70 \) of the influence of matter perturbations.

In this section we follow the evolution of one particular cluster in a range of cosmologies in order to appreciate the evolutionary status of that cluster at similar redshifts, lookback time and cosmic time.

### 3.3.1 Cluster formation: the role of \( \Omega_m \)

To get a visual appreciation of the cosmological influences on the hierarchical buildup of a cluster sized halo, we follow the evolution of one particular cluster in a range of simulated cosmologies. We are able to do so as we can identify the same cluster halo in all our simulations as we crafted our initial conditions such that they would contain the same morphological make-up, by taking the same Fourier phases for the initial Gaussian density fields.

In the following three figures, we follow the evolution of this dark matter halo, with mass \( M > 10^{14} \, h^{-1} M_{\odot} \), in four different cosmologies. These are the ACDMO2, ACDMF2, ACDMC2 and SCDM. In each of these cosmologies we show the evolution of the halo at six different time steps. We show the mass distribution in and around the cluster, and its progenitors, in a box of comoving size \( 5h^{-1}\text{Mpc} \). Circles enclose halos identified by HOP, with the circle radius proportional to virial radius of the group (i.e. the distance from the center of mass to the outermost particle of the group). Sometimes we see circles within circles, this is just a projection effect. In Fig. 3.3 we show the state of the cluster at the same \( z \) in all four cosmologies. The equivalent Fig. 3.4 does the same thing, but then at the same lookback time \( t_l(z) \). Finally, in Fig. 3.5 we try to do the same in terms of cosmic time by depicting the cluster at a more or less comparable cosmic time.

Perhaps most illustrative for the evolutionary trends in the different cosmologies is the redshift evolution in Fig. 3.3. The sequence ACDMO2, ACDMF2, ACDMC2 and SCDM clearly corresponds to a sequence in which the formation of the halo shifts to later and later epochs. Taking into account that all models were normalized to the present epoch, so that it comes as no surprise that the cluster halo at \( z = 0 \) looks similar in all four cosmologies, we find that at all depicted redshifts the cluster in the ACDMO2 is the most pronounced and evolved mass concentration.

In all four cosmologies, we clearly see that the buildup of the halo involves the merging of several smaller mass clumps, some of which are identified as genuine proto-halos by means of circles. Certainly at the first two to three time steps, we see that particles and protohalos fall in into the cluster via the filamentary structure running from the top to the bottom of the box. There is a substantial difference between the coherence and prominence of the filament in the ACDMO2 cosmology and the equivalent one in the higher \( \Omega_m \) Universes of the ACDMF2, ACDMC2 and SCDM. Over the whole depicted redshift range in the ACDMO2 cosmology we see a strong massive filament connecting the most massive clumps in the environment of our (proto) cluster. By contrast, the matter distribution
Figure 3.3 — Cluster evolution: $\Omega_m$ influence. Evolution as a function of redshift of a single dark matter halo in different cosmological models: ACDM02, ACDMF2, ACDMC2 and SCDM.
3.3. COSMOLOGICAL CLUSTER FORMATION

Figure 3.4 — Cluster evolution: $\Omega_m$ influence. Evolution as a function of lookback time of a single dark matter halo in different cosmological models: $\Lambda$CDMO2, $\Lambda$CDMf2, $\Lambda$CDMC2, and SCDM.
Figure 3.5 — Cluster evolution: $\Omega_\Lambda$ influence. Evolution as a function of cosmic time of a single dark matter halo in different cosmological models: ACDMO2, ACDMF2, ACDMC2 and SCDM.
around the cluster in the SCDM cosmology seems to be much more clumpy and less concentrated in the filament. Particularly at $z \sim 1.5$ and $z = \sim 1$ the clump distribution around the central cluster is quite isotropic, and we still see the imprint of this distribution at the more recent redshifts ($z \sim 0.5$ and $z = 0$). The prominence of the filamentary mass distribution is clearly related to the underlying cosmology, and will depend on the slope of the power spectrum at cluster scales.

<table>
<thead>
<tr>
<th>Model</th>
<th>%</th>
<th>Model</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>80</td>
<td>ΛCDMO1</td>
<td>86</td>
</tr>
<tr>
<td>OCDM01</td>
<td>84</td>
<td>ΛCDMC1</td>
<td>82</td>
</tr>
<tr>
<td>OCDM03</td>
<td>84</td>
<td>ΛCDMF3</td>
<td>82</td>
</tr>
<tr>
<td>OCDM05</td>
<td>84</td>
<td>ΛCDMC2</td>
<td>82</td>
</tr>
<tr>
<td>ACDM01</td>
<td>84</td>
<td>ΛCDMC3</td>
<td>81</td>
</tr>
<tr>
<td>ACDM02</td>
<td>82</td>
<td>ΛCDMF1</td>
<td>82</td>
</tr>
</tbody>
</table>

Table 3.2 — Percentage of halos formed via accretion in every cosmological model.

At later times we see that most matter in the surroundings has accreted onto the massive central cluster. Interesting in this respect is the question how much of the surrounding material has accreted quiescently and how much came in with merging subclumps. We distinguish merging and accretion on the basis of the relative amount of mass gain by an absorbing cluster halo. A merger is one where the mass of the protocluster grows by more than 30%. From Table 3.2 we learn that most of its mass has accreted gradually onto the forming cluster. In all cosmologies this concerns at least 80% of the cluster mass.

It is also clear that the protohalo in the flat cosmology and the SCDM cosmology at the first redshift bin $z \sim 3$ is a rather underdeveloped mass clump. In these cosmologies the growth proceeds almost linearly in redshift, in each frame we notice a strong development of the cluster with respect to the previous frame. By contrast, the lower density ΛCDMO2 cosmology, and to some extent also ΛCDMF2 after $z \sim 0.5$, testify of a significant slow down in cluster growth. The cluster in ΛCDMO2 at $z \sim 0.5$, and even that at $z \sim 1$, looks pretty much like the cluster nowadays.

There is a clear trend of a more advanced cluster state as we go from the left hand column (ΛCDMO2) to the right hand column (SCDM). Evolution in the SCDM cosmology is considerably stronger in the last few Gyrs than in the lower density ΛCDM models. Comparison between the four columns reveals the dominant influence of the matter density in the growth of structure: the larger $\Omega_m$, the stronger the evolution has been in the same time interval.

Fig. 3.4 shows the same comparison, but as a function of the lookback time. It provides the same impression, but at a larger contrast. Because cosmic time is stretched in the more open cosmologies a time span in the ΛCDMO2 cosmology corresponds to a shorter redshift interval than that in the SCDM (see Fig. 3.2). In the last 0.6 Gyr the cluster in ΛCDMO2 hardly underwent any significant development. In the same time span, we see a substantial amount of evolution in the ΛCDMC2 cosmology, an evolution which is mimicked to a weaker extent in the ΛCDMF2 cosmology. The situation is radically different for the SCDM clustering: 0.6 Gyr ago there were hardly any massive clusters and the ones of today were still minute clumps of matter embedded within a faint filamentary environment.

The differences between the cluster evolution as a function of redshift or lookback time to a substantial extent may be ascribed to the differences in the evolutionary state of the clusters at any one of these particular epochs. Taking the reverse view and starting at the same cosmic time, we follow the clusters at comparable dynamical stages of evolution and during the same time span in each of the cosmologies. Fig. 3.5 nicely confirms our expectations: the evolution in the higher density SCDM cosmology is more rapid than in the other cosmologies. In fact, we see a nice sequence of more prominent evolution as we go from the right hand column (SCDM) to the left hand column (ACDM02). It is like a mirror for our earlier assessment in terms of lookback time. The reason remains the same: driven by
Chapter 3: Galaxy Cluster Evolution: Mass Growth and Virialization

the higher mass density the structure evolution in high density Universes proceeds more rapidly in the same time interval.

While these three sets of images do illustrate the decisive role of $\Omega_m$ in the buildup of clusters, the role of $\Omega_\Lambda$ remains more subtle.

3.3.2 Cluster formation: the role of $\Omega_\Lambda$

Figs. 3.6 and 3.7 show the evolution of the same cluster as in the previous subsection, but then for a set of four cosmologies with the same $\Omega_m = 0.3$, yet systematically different $\Omega_\Lambda$.

Clearly, the differences between these four cosmologies are not very large. The one noticeable trend is that of the more rapid evolution in the higher $\Lambda$ cosmologies than in one with a lower $\Lambda$. At $z \sim 3$, the cluster in the OCDM03 model is clearly more substantial than the one in the ACDMC1 model. This remains true at nearly all redshift steps, except of course at the current epoch.

We also note that the surrounding mass distribution in the different cosmologies is comparable, be it at different redshifts. For example, the configuration at $z \sim 2.3$ in OCDM03 is almost the same as the one at $z \sim 1.5$ in the ACDMC1 cosmology. There does not seem to be a difference in local large scale geometry between these models. Earlier, we had seen that there was a substantial difference in filamentary character of the large scale structure between cosmologies with different $\Omega_m$. Apparently, $\Omega_\Lambda$ does not bear strongly on the coherence of the cosmic web.

The same impression is obtained from Fig. 3.7, where we compare the same cosmologies over a similar cosmic time. Also here we see a more rapid evolution of the ACDMC1 cluster in comparison to the OCDM03 cluster. The only conclusion we may draw here is that of $\Omega_\Lambda$ having some impact through its influence on the timescales in the Universe. We can not infer any significant dynamical influence.

The factor that we need to assess in somewhat more detail is the history of the accretion of mass towards the buildup of the cluster. The merging trees of the cluster halos may contain some more information on the underlying cosmology. We investigate this in the next section.

3.4 Formation time and mass accretion history

In hierarchical structure formation scenarios the buildup of a galaxy or cluster halo is a complex process of accretion and merging. It brings up the question what exactly the nature is of a protocluster. Any cluster combines the matter content of many previous mass clumps. Which of these or how many of these should be considered progenitors. And at which state should we consider such a halo as a matured object. It is clear that in such a situation the concept of progenitor or formation time may not be clearly and unequivocally defined. In other words, such a definition is imperfect and in many cases arbitrary.

To some extent, formation time involves a measure of arbitrariness. There are a few possible definitions of formation time in use, certainly not always in agreement with each other. The most idealized one is the one based on the spherical collapse model. Given a particular density threshold it would immediately provide a theoretical expression for collapse or virialization time (Gunn & Gott 1972; Lahav et al. 1991). However, spherical systems do not exist in reality. We know that the primordial density peaks are anisotropic, close to triaxial (Bardeen et al. 1986). Better approximations for collapse time(s) for such objects are provided by the ellipsoidal model (Icke 1973; Eisenstein & Loeb 1995; Bond & Myers 1996; Sheth et al. 2001). Even though it leads to considerable improvement of, e.g. the mass spectrum of halos, it does not fully take into account the complex, non local hierarchical buildup of halos (with the exception of the Peak Patch description of Bond & Myers (1996)).

To take into account the more realistic circumstances of the hierarchical clustering evolution, we will mostly follow Lacey & Cole (1993) in adopting the general definition of formation time being the time (redshift) $z_f$ at which the parent protocluster contains half (or more) of its current mass. An alternative definition of formation time is the time at which the potential well of the halo becomes deep
Figure 3.6 — Cluster evolution: $\Omega_\Lambda$ influence. Evolution as a function of redshift of a single cluster sized dark matter halo in different cosmological models: OCDM03, $\Lambda$CDMO3, $\Lambda$CDMF2 and $\Lambda$CDMC1. Note that each of these cosmologies have the same $\Omega_m = 0.3$. 
Figure 3.7 — Cluster evolution: $\Omega_\Lambda$ influence. Evolution as a function of cosmic time of a single cluster sized dark matter halo in different cosmological models: OCDM03, $\Lambda$CDM03, $\Lambda$CDM02 and $\Lambda$CDMC1. Note that each of these cosmologies have the same $\Omega_m = 0.3$. 
enough to be considered a cluster, for which e.g. one could assume an X-ray emission criterion when looking at rich clusters. Probably the most extensive discussion of the various cluster formation time definitions is the one by Cohn & White (2005).

### 3.4.1 Formation time and mass accretion history

In the context of hierarchical scenarios, possibly the most objective path towards defining the formation time of a particular object is to assess its complete history of merging and accretion. Pursuing the path of the most massive progenitor of a halo yields the so called mass accretion history (MAH). Different studies have found that MAHs may affect the final properties of halos, something which is to expect given the hierarchical scenario of structure formation (e.g., Wechsler et al. 2002; van den Bosch 2002; Zhao et al. 2003; Tasitsiomi et al. 2004).

On the basis of the MAH, through modelling by an idealized analytical expression, a few different definitions of formation time were forwarded. We have assess three of these.

Wechsler et al. (2002) (hereafter W02) proposed a fit to the MAHs by an exponential function:

$$M(a) = M_0 e^{-\alpha \left( \frac{1}{a} - 1 \right)},$$

(3.9)

where $a$ is the expansion factor and $M_0$ is the final mass. The single parameter $\alpha$ in Eqn. 3.9 is related to a characteristic formation epoch $a_f$. It is defined as the expansion scale factor $a$ at which the logarithmic slope of the accretion rate, $d\log M / d\log a$, falls below some specified value $S$ (Wechsler et al. 2002). The value of $S$ is arbitrary, we follow W02 and take $S = 2$. The resulting expression for the formation redshift $z_f$ is given by

$$z_f = \frac{2}{\alpha} - 1.$$  

(3.10)

In our fitting procedure we choose to keep the $M_0$ fixed, and not use it as an extra free parameter as in e.g. Allgood et al. (2006); Aragón-Calvo (2007).

van den Bosch (2002) (hereafter VDB02) defined the formation redshift $z_f$ from a theoretical fit to the mass accretion history inferred from the extended Press-Schechter formalism (Bond et al. 1991). This fitting formula is given by

$$\log (\Psi(M_0, z)) = -0.301 \left( \frac{\log(1+z)}{\log(1+z_f)} \right)^\nu,$$

(3.11)

where $z_f$ and $\nu$ are parameters that follow from fitting the expression to the mass history of a halo. The parameter $z_f$ would then correspond to the formation redshift, the epoch at which the protohalo contains half or more of the mass of the current cluster.

Arguing that the MAH description by W02 (see Eqn. 3.9) provides a poor fit in a variety of situations, Tasitsiomi et al. (2004) (hereafter T04) proposed a more general fitting equation,

$$M(a) = M_0 a^p e^{-\alpha (1/a - 1)},$$

(3.12)

defining $p = 0$.

### 3.4.2 Global formation epoch

The first step of our procedure is the construction of the merging tree of each halo. We do this by tracking backwards in time every progenitor of the present day halo (see section 3.2.3).

Once we obtain the merger tree of a given halo, we track down its most massive progenitor. In a first approximation of its formation time, we determined the redshift at which its most massive progenitor contains half or more of its present mass. By averaging the formation times of all cluster halos we obtain an *average* formation redshift and formation time for any particular cosmology. These are listed in Table 3.3.
From Table 3.3 that the formation redshift in low matter Universes \((\Omega_m = 0.1)\) is high compared to models with high density values. This is in line with our expectations based on structure formation in low density Universes. We also notice an effect of the cosmological constant. For cosmologies with the same \(\Omega_m\), the formation redshift is higher for models with decreasing \(\Omega_\Lambda\). This conclusion is slightly modified when we assess formation epoch in terms of formation time. Illustrative is the fact that the formation time in the SCDM model is significantly shorter than in the ΛCDMF1 model. This is in line with what we discussed in section 3.3, where we noticed the same trend of a more rapid evolution driven by a higher matter density.

While these estimates of average formation epoch do provide some impression of general trends, they do not take into account important factors like the accretion and merging history of the halos and their mass. These will bear strongly on the spread around the mean formation epoch. In order to investigate this issue, we turn towards fitting the full MAH on the basis of the model equations discussed in the previous section 3.4.1.

### 3.4.3 Single halo MAH

The mass accretion histories of four individual halos in a set of four cosmologies are shown in Fig. 3.8. In all situations we see a steady increase of the mass as a function of expansion factor \(a(t)\). It is clear that all four halos formed first in the open cosmology. Later, after \(a \sim 0.5\), it is these halos that hardly grow in mass anymore.

There are some interesting differences between these halos. The halos in the lefthand panels evolved by steady accretion. The ones in the righthand panels do suffer major mass jumps as a result of a massive merger. In the case of the latter, the merger happened recently at \(a \sim 0.9\) for the one in the upper panel, at least for the high density cosmologies. The cluster in the low density Universe experienced a similar merger at a considerably earlier time. The merger in the other halo (bottom panel) happened at different times for each of the cosmologies: the earliest in the ΛCDMO2, last in the SCDM cosmology.

The validity of the approximation by the W02, VDB02 and T04 expression for the theoretical mass accretion history may be appreciated from Fig. 3.9. As long as the halo formed via gentle accretion (lefthand frame), these fits seem to be quite reasonable. The VBD02 formula produces the best results of all three. The difference with reality is substantially larger for the halo that underwent a massive merger.
merger. The W02 expression fails over nearly the entire formation history. Although VDB02 and T04 represent better fits, they do not manage to accurately reproduce the entire formation history of this individual halo.

### 3.4.4 General MAH

We determine the general mass accretion history by averaging all individual mass accretion history of the halos in each cosmology sample. The average MAH of these galaxy clusters is shown in Fig. 3.10.

Careful inspection of the inferred MAHs reveals a few facts. Not surprisingly, galaxy clusters tend to form earlier as $\Omega_m$ is lower. This is particularly clear when comparing the sequence $\Lambda$CDMO2, $\Lambda$CDMF2, $\Lambda$CDMC2. Also we find that the scatter in mass accretion histories is more substantial in the higher $\Omega_m$ $\Lambda$CDMC2, and even more so in the SCDM cosmology. This may be tied in with the observation by Wechsler et al. (2002) that the MAH scatter is tightly correlated with the concentration parameter of the halos. In SCDM the clusters are much less concentrated than the halos in the low $\Omega_m$ Universes. This we also found in chapter 4.

In each cosmology we also found that the general MAH is shifted to earlier times for samples of lower mass halos. It reflects the general tendency of low mass halo to form earlier.

<table>
<thead>
<tr>
<th>Model</th>
<th>Average $z_f$</th>
<th>W02 $z_f$</th>
<th>T04 $z_f$</th>
<th>VDB02 $z_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>0.61 ± 0.34</td>
<td>0.86 ± 0.08</td>
<td>1.00 ± 0.46</td>
<td>0.62 ± 0.04</td>
</tr>
<tr>
<td>OCDM01</td>
<td>1.08 ± 0.54</td>
<td>2.20 ± 0.12</td>
<td>2.92 ± 0.92</td>
<td>1.00 ± 0.07</td>
</tr>
<tr>
<td>OCDM03</td>
<td>0.77 ± 0.40</td>
<td>1.37 ± 0.09</td>
<td>2.21 ± 0.73</td>
<td>0.73 ± 0.05</td>
</tr>
<tr>
<td>OCDM05</td>
<td>0.69 ± 0.37</td>
<td>1.14 ± 0.09</td>
<td>1.82 ± 0.70</td>
<td>0.67 ± 0.05</td>
</tr>
<tr>
<td>ACDMO1</td>
<td>1.03 ± 0.42</td>
<td>1.70 ± 0.10</td>
<td>1.06 ± 0.29</td>
<td>0.99 ± 0.06</td>
</tr>
<tr>
<td>ACDMO2</td>
<td>0.98 ± 0.38</td>
<td>1.41 ± 0.08</td>
<td>0.65 ± 0.20</td>
<td>0.95 ± 0.05</td>
</tr>
<tr>
<td>ACDMF1</td>
<td>0.86 ± 0.35</td>
<td>0.93 ± 0.06</td>
<td>0.21 ± 0.12</td>
<td>0.88 ± 0.05</td>
</tr>
<tr>
<td>ACDMO3</td>
<td>0.69 ± 0.33</td>
<td>0.95 ± 0.07</td>
<td>0.86 ± 0.30</td>
<td>0.68 ± 0.04</td>
</tr>
<tr>
<td>ACDMF2</td>
<td>0.66 ± 0.30</td>
<td>0.75 ± 0.06</td>
<td>0.37 ± 0.19</td>
<td>0.67 ± 0.04</td>
</tr>
<tr>
<td>ACDMC1</td>
<td>0.61 ± 0.25</td>
<td>0.44 ± 0.04</td>
<td>-0.07 ± 0.10</td>
<td>0.67 ± 0.04</td>
</tr>
<tr>
<td>ACDMF3</td>
<td>0.62 ± 0.31</td>
<td>0.78 ± 0.07</td>
<td>0.63 ± 0.29</td>
<td>0.64 ± 0.04</td>
</tr>
<tr>
<td>ACDMC2</td>
<td>0.60 ± 0.29</td>
<td>0.60 ± 0.06</td>
<td>0.24 ± 0.19</td>
<td>0.63 ± 0.04</td>
</tr>
<tr>
<td>ACDMC3</td>
<td>0.56 ± 0.26</td>
<td>0.37 ± 0.05</td>
<td>-0.09 ± 0.12</td>
<td>0.62 ± 0.04</td>
</tr>
</tbody>
</table>

Table 3.3 — Formation redshifts for cluster halos in all cosmological simulations. The second column indicates the average formation redshift and the third column indicates the cosmic time of this average formation redshift. The last three columns indicates the formation redshift inferred from the W02, T04 and VDB02 models.
While we do find these general trends as a function of underlying cosmology, perhaps as important is the observation that the width of the MAHs is so large that there is a large overlap between the MAHs of different cosmologies. The impression is that it is possible to discriminate between cosmologies of different $\Omega_m$, but not between comparable cosmologies with different $\Omega_\Lambda$. This makes attempts to infer cosmological parameters like $\Omega_\Lambda$ on the basis of the growth history of cluster sized objects, in other words the mass spectrum as a function of redshift, a nontrivial affair.

Figure 3.10 — Average mass accretion histories for galaxy clusters in four cosmological models. The shaded area denotes the standard deviations.

Figure 3.11 — Average mass accretion histories for galaxy clusters in four cosmological models. The fitting functions of W02, T04 and VDB02 are also shown.
3.4.5 General MAH: formation times

In Fig. 3.11 we compare the three model fits – W02: Eqn. 3.9, VBD02: Eqn. 3.11 and T04: Eqn. 3.12 – to the average MAH in each cosmology shown in Fig. 3.10. In general, we find that the models fits manage to reproduce the mass accretion history over a substantial timespan of our simulations. The only significant deficiencies occur at the earliest simulated epochs in the higher \( \Omega_m \) Universes. This may be ascribed for a substantial extent to the poor low mass resolution of these simulations.

However, when we infer the formation redshifts \( z_f \) implied by each of these model fits we find strong inconsistencies (see table 3.3, three last columns). Comparing each of the inferred model formation redshifts to the directly inferred one (section 3.4.2 and second column of table 3.3) there is some similarity with VDB02. This may be related to the fact that the latter also concerns the epoch at which the halos contain half of their final mass, i.e. according to Press-Schechter theory.

3.5 Virialization

The end stage of any evolving halo is its final and complete collapse, followed by virialization. During virialization the internal energy of the object is distributed such that it attains a perfect equilibrium configuration.

Virialization is a complex dynamical process. The simple and idealized spherical model leads to a definitive prediction of virial state and virial time. However, in reality the process will be less straightforward. Factors such as the nonspherical shape of the halo as well as the substantial level of substructure expected in hierarchical models will modify the virialization time and the virialization process itself. This will be exacerbated by the fact that the halo will not be an isolated island in the Universe but an organic part of the Cosmic Web, responsible for a constant influx of matter and matter clumps from the immediate surroundings to the halo.

In this section, we investigate the cluster virialization process in our set of cosmological simulations. This should provide information on the global cosmological as well as environmental influences on the virialization.

3.5.1 Virial Theorem

The exact expression of the virial equation for a self gravitating system, not necessarily isolated, is given by:

\[
\frac{1}{2} \frac{d^2 I}{dt^2} = 2K + U - E_p, \quad (3.13)
\]

where \( K \) is the kinetic energy, defined as

\[
K = \frac{1}{2} \sum_{i=1}^{N} \rho_i (v_i - v_{center})^2, \quad (3.14)
\]

where the sum is over all particles velocities within any given region. The potential energy \( U \) of the system is given by

\[
U = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{Gm_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|}, \quad (3.15)
\]

where the sum of the distances is over all particle pairs. The environmental influences are incorporated via a surface pressure term \( E_p \) (see e.g. Chandrasekhar 1961)

\[
E_p = \int P_s(r) \mathbf{r} \cdot d\mathbf{S} \quad (3.16)
\]
Limiting ourselves to an isolated object, i.e. $E_p = 0$, a system is virialized if it fulfills the condition $I \to 0$

$$2K + U = 0.$$  \hspace{1cm} (3.17)

This is known as the Virial Theorem for a perfect isolated self gravitating system.

To get a good measure for the virial state of an object may therefore obtained from the virial ratio

$$\mathcal{V} = \frac{2K}{|U|}.$$  \hspace{1cm} (3.18)

This expression should approach unity for virialized systems, the virial ratio $1 < \mathcal{V} < 2$ for a bound system, while $\mathcal{V} > 2$ implies the system is unbound.

Figure 3.12 — Evolution of the kinetic energy (left hand panel) and potential energy (right hand panel) as a function of expansion factor for four different halos in four cosmological models. Units are arbitrary.
3.5.2 Case studies: single halos

A first impression of the virialization process should concern that of a set of individual halos in each of the the cosmological simulations. On the basis of the SCDM simulation we identified four halos and we extracted them, and their equivalents, from each of the simulations. We select cluster sized halos with masses larger than \(6 \times 10^{14} h^{-1} M_\odot\).

Following our definition that a merger is a halo that at least at one particular epoch absorbed an infalling clump of at least 30% of its mass, we selected two merging halos and two accreting halos. One of these halos is a halo that formed by truly quiescent accretion (D), the other accreting halo did suffer some major impact in one or more of the cosmologies (A).

Fig. 3.12 shows the evolution of the kinetic energy (left column) and the potential energy (right column) of the four halos in four of the simulated cosmologies (SCDM, \(\Lambda\)CDMO2, \(\Lambda\)CDMF2 and \(\Lambda\)CDMC2). The first impression is that of the kinetic energy and potential energy evolution following each other quiet well.

It is interesting to see the influence of major mass infall on both the kinetic energy and potential energy of a halo. Major mergers are always reflected in sudden jumps in kinetic and potential energy (see halos B and C, and A in SCDM). In cases of merger or violent mass gain, the potential drops substantially while accretion manifests itself by a gradual decrease of the potential. Also note that the \(\Lambda\)CDMO2 halos stop evolving their potential and kinetic energy at an early epoch, and in the case of halo C even starts to lose energy after \(a \sim 0.6\), following a substantial merger. It is a clear reflection of the global cosmological influence on the development of individual clusters. In that spirit we do not find any trace of a role of the cosmological constant.

![Figure 3.13](image)

**Figure 3.13** — Evolution of the virial ratio as a function of expansion factor for four different halos in four cosmological models. Units are arbitrary.

To evaluate the virial state, and its evolution, of each halo we turn to Fig. 3.13, which depicts the virial ratio \(V\) for each of the analyzed halos. Most halos have a virial ratio consistently near unity but never really converging towards unity. Instead, they seem to linger erratically around values \(V \sim 1.1 - 1.4\). Halo A did seem to have reached virial equilibrium at a very early stage, but subsequently it started to drift away from \(V \sim 1\). If we wish to relate the behavior of \(V\) to the nature of the infall of matter it is perhaps that the fluctuations in \(V\) seem to be somewhat milder for the accreting halo D.

The fact that unity is never reached might be an indication for environmental influences. One factor that will play a role is that of the gradual infall of matter from the surroundings, which would account for the pressure term \(E_p\). On the other hand, we should not exclude the influence of numerical artifacts.
Chapter 3: Galaxy Cluster Evolution: Mass Growth and Virialization

such as that of force softening in the $N$-body simulations. The fact that we also do not observe a convergence for the case of the $\Lambda$CDM02 clusters might be indicative of such a factor. We are in the process of evaluating this systematically.

### 3.5.3 General view: virialization at $z = 0$

Turning from the virial state of individual halos we wish to see what the state of affairs is for the entire sample of halos and investigate whether we can detect any systematic trends with cosmology.

As a first step we investigate in how far the whole sample fits to the relation between potential and kinetic energy predicted by the virial theorem. In order to do so, we followed a procedure that remains close to the full form of the virial theorem for a self gravitating system:

$$\frac{1}{2} \frac{d^2 I}{dt^2} = 2K + U - E_p. \quad (3.19)$$

This we achieved by modelling this virial relation in the form of the linear "virial line" $|U| - K$

$$|U| = \mu K + \lambda. \quad (3.20)$$

For a perfect, isolated virialized object, $\mu = 2$ and the pressure term $\lambda = 0$. Also note that for a marginally bound object, $\mu = 1$ and $\lambda = 0$. If a halo would not be bound, i.e., $K > |U|$ we would have $\mu < 1$.

Each cosmological simulation provides us with a set of cluster halos and their kinetic energy $K$ and potential energy $U$ (Eqns. 3.14 and 3.15). On the basis of the resulting $|U| - K$ plot (Fig. 3.14) we seek to infer the corresponding virial line, i.e. its parameter $\mu$ and $\lambda$. In our analysis we assume $\lambda = 0$, as we found that variation of this parameter did not lead to results that would imply significantly different physical interpretations.

In order to prevent unphysical fits and sensitivity to halos that were evidently not bound or virialized, we proceeded as follows. We choose to discard the 10% halos that would fall below our infer virial line: they would be considered unbound. According to this choice, we simply determining the value $\mu$ such that it is the value for which 10% of the sample points fall below the line $|U| = \mu K$. One can think of this virial line as a lower virial relation (see also Knebe & Müller 1999).

We applied the described procedure to the complete set of dark matter halos. This also includes halos with a mass less than $M < 10^{14}h^{-1}M_\odot$, halos we do not necessarily brand as clusters. In Fig. 3.14

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mu$</th>
<th>Unvirialized Halos</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCDM</td>
<td>1.54</td>
<td>19%</td>
</tr>
<tr>
<td>OCDM01</td>
<td>1.63</td>
<td>10%</td>
</tr>
<tr>
<td>OCDM03</td>
<td>1.56</td>
<td>16%</td>
</tr>
<tr>
<td>OCDM05</td>
<td>1.54</td>
<td>18%</td>
</tr>
<tr>
<td>ACDM01</td>
<td>1.55</td>
<td>17%</td>
</tr>
<tr>
<td>ACDM02</td>
<td>1.51</td>
<td>22%</td>
</tr>
<tr>
<td>ACDMF1</td>
<td>1.46</td>
<td>29%</td>
</tr>
<tr>
<td>ACDM03</td>
<td>1.52</td>
<td>22%</td>
</tr>
<tr>
<td>ACDMF2</td>
<td>1.49</td>
<td>26%</td>
</tr>
<tr>
<td>ACDMC1</td>
<td>1.46</td>
<td>32%</td>
</tr>
<tr>
<td>ACDMF3</td>
<td>1.50</td>
<td>24%</td>
</tr>
<tr>
<td>ACDMC2</td>
<td>1.48</td>
<td>28%</td>
</tr>
<tr>
<td>ACDMC3</td>
<td>1.46</td>
<td>32%</td>
</tr>
</tbody>
</table>

Table 3.4 — Virial ratio $\mu$ of the halo sample in each cosmology, (see Eqn. 3.20) and percentage of unvirialized halos with respect to the OCDM01 virial ratio $\mu = 1.63$. 
Figure 3.14 — The virial relation for simulated halo samples in four cosmologies. Plotted is the potential energy $|U|$ vs. the kinetic energy $K$ of each halo in SCDM (top left frame), $\Lambda$CDMO2 (top right frame), $\Lambda$CDMF2 (bottom left frame) and $\Lambda$CDMC2 (bottom right frame). The solid line is the fitted relation $|U| = \mu K$ (see text), the dashed lines are the relations $|U| = 2K$, the perfect virial relation, and $|U| = K$, the criterion for a gravitationally bound configuration.

we have plotted the potential energy $|U|$ vs. the kinetic energy $K$ for all halos in four different cosmologies: SCDM, $\Lambda$CDMO2, $\Lambda$CDMF2 and $\Lambda$CDMC2, all at the present epoch. The solid line in each frame is the fitted line $|U| = \mu K$, the dashed lines delineate the region between perfect virialization (upper line) and mere boundness (lower line). We see that in all four cases the halo sample is quite close to the ideal virial state. The exception are the low mass halos: we find many that lie so far away from the virial line that obviously they are not even bound. Given that this even occurs in the case of a few moderately massive halos means that this can not be ascribed only to resolution effects.

Fig. 3.15 is a complementary image of the virial state of our halo sample. It plots the virial ratio $|U|/K$ as function of the mass of the halos. The plots are scatter plots, with each point representing a halo in the simulation. The density of the points in the scatter plot can be inferred from the superimposed density grey scale plot. By means of the horizontal bars the perfect virial state, $|U|/K = 2$, and the criterion for a gravitationally bound configuration, $|U|/K = 1$. In each of the four cosmologies we see that over the whole mass range the majority of halos linger around a similar virial ratio of $|U|/K \sim 1.5 - 1.6$.

Interesting to see is the wide spread in the case of the low mass halos: some of these are highly virialized ($|U|/K \sim 2$), while there is a group of low mass halos that may not even be considered as gravitationally bound. In how far this maybe ascribed to an artifact of the HOP halo identification procedure or to some real intrinsic physical effect cannot be judged within the context of this work. Also interesting would be the question in how far it could be related to the formation history and time of this halos, with the strongly bound ones presumably corresponding to early formed halos. Also e.g.
Figure 3.15 — The virial ratio $|U|/K$ as function of the mass of the halos in our simulated halo samples. The four represented cosmologies are: SCDM (top left frame), $\Lambda$CDM02 (top right frame), $\Lambda$CDMF2 (bottom left frame) and $\Lambda$CDMC2 (bottom right frame). The plots are scatter plots, with each point representing a halo in the simulation. The density of the points in the scatter plot can be inferred from the superimposed density grey scale plot. In each frame we indicate by means of the horizontal bars the perfect virial state, $|U|/K = 2$, and the criterion for a gravitationally bound configuration, $|U|/K = 1$.

Knebe & Müller (1999) found similar trends, leading them to suggest that the loosely bound halos are the result of recent soft mergers into halos that as yet are more anisotropic than the major share of halos. They would virialize at a later time. Also, it remains a particularly interesting speculation whether there are environmental factors at play: isolated halos in low density areas may evolve into substantially stronger bound objects. These aspects are the subject of a presently ongoing study.

3.5.4 Virialization of Galaxy Clusters

Limiting ourselves to the subset of halos that may be branded as genuine cluster sized halos with a mass larger than $M > 10^{14}M_\odot$, we find an interesting contrast with respect to the virial state of the complete sample of halos: all cluster halos center around the virial line $|U| = \mu K$, with $\mu \sim 1.60 - 1.65$ (Fig. 3.16). There is hardly any noticeable difference between the different cosmologies. The one important exception is the spread around the virial line: the high $\Omega_\text{m}$ cosmologies, SCDM and $\Lambda$CDMC2 have a larger width. Later on we will see that this is reflected in the related width of the cluster Fundamental Plane (see chapter 5).
3.6 Conclusions

In this chapter we have studied the mass assembly and formation history of cluster halos in a range of CDM dominated cosmologies. These cosmologies all concern hierarchical formation scenarios. The simulations all start from primordial Gaussian conditions with the same Fourier phases. This allows us to follow the same structures, and therefore clusters, in each of the simulations.

We first looked into the assembly history of a few identical clusters and assessed differences in its formation as a function of redshift, lookback time and also cosmic time, i.e. the time since the Big Bang. We found that nearly all the differences have to be ascribed to the difference in density of the cosmological background, cq. $\Omega_m$. The formation redshift of clusters is substantially higher in low $\Omega_m$ Universes. The only noticeable influence of $\Omega_\Lambda$ on the evolution of the clusters is its impact on the cosmic time corresponding to a particular cosmology. It either stretches or compresses the available dynamical timescales for cluster evolution.

We proceeded with a study of the mass accretion history (MAH) of the halos in our sample. We evaluated the MAH of a few individual cluster halos, and that of the average MAH in each of the cosmologies. As for the individual halos, we did find that the accretion and merging history of halos is the one dominant noticeable influence. This appears to be regulated to some extent by the background
cosmology: in low $\Omega_m$ Universes most evolution takes place at early times, often accompanied by massive mergers at high redshifts. In high $\Omega_m$ Universes such mergers seem to be more frequent at recent epochs.

When comparing the average mass accretion history with the predictions of analytic model descriptions such as Wechsler et al. (2002); van den Bosch (2002); Tasitsiomi et al. (2004), we find that they manage to reproduce the MAH over a large range of the cosmic expansion history. The only major differences occur at the earliest epochs, presumably the product of the poor mass resolution of the simulations. The spread of the MAHs around the average appears to be significant. It is somewhat larger in high $\Omega_m$ Universes than in low $\Omega_m$ ones, and most of them do overlap with the MAH in other cosmologies. It will render any conclusions on cosmological parameters like $\Omega_\Lambda$ on the basis of the cluster population rather cumbersome.

Finally, we studied the virialization of the emerging halos. In general, the halo population appears to be close to a virial state. However, almost independent of cosmology the halos attain a relation $|U| = \mu K$ with $\mu \sim 1.5 - 1.6$ instead of the perfect virial ratio $\mu = 2$. Low mass halos display a large spread, with some halos being highly virialized, while others can hardly be characterized as a singly gravitationally bound objects.

Cluster halos, i.e. halos with $M > 10^{14} h^{-1} M_\odot$, are perhaps the most well behaved halos. Nearly all of them obey the same virial relation, independent of cosmology. The only noticeable influence of cosmological background is through the spread in the virial relation. In high $\Omega_m$ Universes it is somewhat larger than in low $\Omega_m$ ones.

Neither in the mass accretion history of in the virial state of cluster halos have we been able to find any influence of a cosmological constant.