Early civilizations have been wondering where everything came from and how everything began. For centuries, humanity has been wondering what our place in the vast world surrounding us is. Tied in with this was the fundamental question of how the world came into being. Early civilizations had a mythical view of the cosmos. For example, the Mesopotamian civilizations of Sumer and Babylon had a cosmology in which Earth was a disk surrounded by the underground waters of the Apsu and the underworld of the dead, with the heavens of the stars surrounding it all. Despite the highly sophisticated level of astronomy in the neo-Babylonian world and its inheritants, it is with the ancient Greeks that astronomy and cosmology entered the level of scientific inquiry. While many common Greeks still shared a mythical view of the world, impressive philosophical and scientific advances led to a world view based on mathematical and geometrical models of reality that remained virtually unchallenged until the 16th century. Building upon the observations carefully obtained and archived by the Babylonians, the Hellenistic Greeks were the first to use their geometric models to predict the observational reality, creating a quantitative as well as a qualitative model of the Universe. Eratosthenes measured the circumference of Earth, while Aristarchus measured size and distance of Earth and Moon. He even forwarded the suggestion that the Sun was at the center of our world, a view which only became the accepted view with Copernicus and Galilei in the 15th and 16th century.

It is with the Scientific Revolution of the 16th and 17th century that a truly scientific model of the Universe came into being. With Nicolai Copernicus in 1543 Earth finally lost its privileged central position in the cosmos. This prods us to refer to the Copernican principle when we wish to refer to the fact that we can not be at a special spatial or temporal position in space-time. Standing on the shoulders of Copernicus, Brahe, Kepler and Galilei, it was Isaac Newton who managed to frame the laws of gravity and mechanics. This formed the foundations of classical physics. His view of a static space-time and a gravitational force resulting from action at distance made it impossible to open the view on the dynamic cosmological world view which we presently hold. It was Einstein’s General Theory of Relativity that formed the final breakthrough towards turning cosmology into a scientific inquiry. His metric theory of gravity turned space-time into a dynamic medium in which gravity is a manifestation of the curvature of space-time. Soon it was realized that this implies that the Universe could not be static and instead should be expanding or contracting. Friedmann and Lemaître were the first who worked out the expanding solutions for a homogeneous and isotropic Universe. Their theoretical ideas were soon confirmed in the seminal discovery in 1929 by Edwin Hubble of the expanding system of galaxies around us. His “Hubble Law”, describing the fact that distant galaxies are receding with velocities proportional to their distance, is still the fundamental basis for present day cosmology.

It was Lemaître who realized the tremendous implications of this finding. At earlier times, the Universe would have been a lot smaller, a lot denser and much hotter than the present Universe. This gave rise to the Big Bang Theory, stating the fact that the Universe came into being at some finite point
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Figure 1.1 — The “Bullet Cluster”. It provides the best evidence to date for the existence of dark matter. It is also a nice example of hierarchical structure formation in action. Image Credit: X-ray: NASA/CXC/M.Markevitch et al. Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al. Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/D.Clowe et al.

in the past. We now know that the cosmos came into being 13.7 billion years ago in a seething sea of radiation and matter. In the meantime, this theory has gained widespread acceptance as our world view because of the mounting and impressive amount of observational evidence. It would explain the darkness of the night sky, the so-called Olber’s paradox and the abundance of the light chemical elements hydrogen and helium. Its validity got its final confirmation with the discovery of the cosmic microwave background (CMB) radiation in 1965 by Penzias and Wilson. By reading its signal to ever increasing sensitivity and accuracy, we have been able to read the value of the fundamental cosmological parameters to incredible precision, turning the CMB into the most important pillar of modern cosmology.

Even despite its tremendous successes, the standard Big Bang theory does involve several unsolved issues and coincidences. Some of the most precarious issues are that of the near flatness of the Universe and its almost perfect isotropy ($< 10^{-5}$), a fine tuning which is hard to understand within the context of standard Big Bang theory. Also, it does not explain the origin of structure in the Universe. All these issues may be solved simultaneously if the Universe underwent an inflationary exponential expansion phase. During this cosmic inflation, $10^{-34}$ seconds after the Big Bang, the Universe blew up by a factor of $10^{60}$.

While inflation has become an almost inescapable ingredient of our cosmological world view, we are still left with the puzzle of the identity of the energy content of the Universe. Only 4% of its energy content is in the form of known baryonic matter and radiation. Structure would never have been able to form if it had not been for the dominant gravitational influence of a mysterious dark matter component. Only sensitive to the force of gravity and perhaps to the weak nuclear force, its insensitivity to the electromagnetic force means it is invisible, hence its name of darkness. Fig. 1.1 shows the “Bullet Cluster”, an impressive collision of two cluster of galaxies. The presence of dark matter was detected indirectly by the gravitational lensing of background objects. Even though it may represent more than 85% of matter and its gravitational influence has been recognized over a range of scales, we have as yet been unable to pin down its identity. The presence of such a large amount of
dark matter forms one of the major challenges for present day cosmology.

Even more mysterious and intriguing is the presence of dark energy. Discovered only ten years ago, it has transformed our view of the Universe. Representing some 73% of the Universe’s energy, it is responsible for the accelerated expansion of the Universe and assures its flat geometry. Its identity is a total mystery for astronomers and physicists, though its overriding influence may provide the key towards unraveling the dichotomy between quantum theory and general relativity on the highest energy scales. While the most probable situation is that of the presence of a cosmological constant, i.e. a modifying curvature term, another reading is the possibility that it involves a strange dark energy medium. The pressure of this dark energy would be negative, translating into a repulsive gravitational impact. While we have recognized its dominant influence on the largest cosmological scales, it remains to be seen whether its impact can also be recognized in smaller structures.

One of the best studied and understood physical objects on extragalactic scales are the clusters of galaxies. They are the most massive and most recent fully developed objects in the Universe. One may therefore improve our understanding of dark matter and dark energy by studying their influence on the structure of galaxy clusters. It is this which we attempt to do within this thesis. One particular approach is to do this on the basis of a few model situations. We have chosen to investigate structure, and in particular clusters, formation in a variety of cosmological models. By comparing the outcome of the evolution of clusters in scenarios with different cosmological mass density and dark energy content, we look for significant differences between the equivalent clusters in each of these scenarios.

1.1 Friedmann-Robertson-Walker Universe

The general relativity theory of gravity is a metric theory. To describe the gravitational evolution of the Universe we therefore need to constrain the geometry of the Universe. It is within this context that the Cosmological Principle plays a fundamental role. Its statement that the Universe is homogeneous and isotropic is clearly valid on large scales of several hundreds of megaparsecs and beyond, homogeneity means that every region in space is the same and isotropy means that it looks the same in each direction. The direct implication is that the Universe can only have one of three geometries: negatively curved hyperbolic space, positively curved spherical space or flat space. The metric of these geometries translate into the Robertson-Walker (RW) space-time metric,

\[ ds^2 = c^2 dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \]

In this equation, \( r \) is the radial coordinate, \( t \) is cosmic time and \( \theta \) and \( \phi \) specify the angle towards an object. The curvature of space is specified in terms of a renormalized integer constant \( k \), which can attain three values: \(-1\) (hyperbolic), \(0\) (flat) and \(1\) (spherical). The expansion of the Universe is encapsulated in the cosmic scale factor \( a(t) \). By convention, at the present time \( t_0 \), \( a(t_0) = 1 \).

The Einstein field equations for a spacetime obeying the RW metric, assuming that the Universe can be described as an ideal fluid, leads to the Friedmann equations, which describes the expansion and evolution of the Universe:

\[ \frac{\dot{a}}{a} = -\frac{4\pi G}{3} \left( \frac{\rho + 3p}{c^2} \right) + \frac{\Lambda}{3} \]

and

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho}{3} - \frac{k c^2}{a^2} + \frac{\Lambda}{3} \]

where \( G \) is Newton’s gravitational constant, \( p \) is the pressure, \( \rho \) is the mass density and \( \Lambda \) is the vacuum energy or cosmological constant, which acts as an energy density \( \rho_\Lambda c^2 = c^4 \Lambda / 8\pi G \).

By extrapolating these equations backwards in time, it is possible to see that an expanding Universe should have had a beginning. Edwin Hubble confirmed the expansion of the Universe, when he discovered that galaxies recede from us with a velocity which increases with increasing distance,

\[ v = Hr; \quad H(t) = \frac{\dot{a}}{a} \]
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Figure 1.2 — An all-sky image of the Universe, when it had around three hundred thousands years (300,000 yrs), as measured by the WMAP. The shades of grey correspond to tiny temperature fluctuations. Courtesy NASA/WMAP Science Team.

$H(t)$ is known as the Hubble parameter, and it is the rate of expansion of the Universe. This universal relation is known as the Hubble’s law. The Hubble constant, $H_0$ is defined as the expansion rate at present time $t_0$, and is often expressed as $H_0 = 100 \, h \, \text{km s}^{-1} \, \text{Mpc}$, where $h$ is a dimensionless factor. Hubble’s discovery may be seen as the beginning of modern cosmology. To assess the fate of the Universe it is useful to express energy and matter densities in terms of the critical density $\rho_c$,

$$\rho_c = \frac{3H_0^2}{8\pi G},$$

the density at which the Universe would be flat, i.e. $k = 0$. A Universe with a density higher than $\rho_c$ will be spherically curved, i.e. it is spatially closed, while a density lower than $\rho_c$ would correspond to a hyperbolic geometry and a spatially open Universe. Depending on the precise value of the Hubble constant, currently estimated to be $H_0 = 71 \pm 1 \, \text{km/s/Mpc}$, the value of the critical density is $9.31 \times 10^{-27} \, \text{kg m}^{-3}$.

With the definition of the critical density, we can define other useful cosmological parameters:

$$\Omega_m = \frac{\rho_m}{\rho_c}, \quad \Omega_{\text{rad}} = \frac{\rho_{\text{rad}}}{\rho_c}, \quad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}, \quad \Omega_k = \frac{-kc^2}{a_0^2H_0^2},$$

(1.6)

to refer to the density of ordinary matter, relativistic matter (radiation), vacuum energy and curvature. Ignoring $\Omega_{\text{rad}}$, we have

$$\Omega_m + \Omega_k + \Omega_\Lambda = 1.$$  

(1.7)

Note that they concern at present time, with $\Omega_{m,0}$ the density parameter at the current epoch. The density parameter at other epochs will be denoted by $\Omega(z)$. Accordingly, the current density of the Universe may be expressed as

$$\rho_0 = 1.8789 \times 10^{-26} \Omega h^2 \, \text{kg m}^{-3}$$

(1.8)

$$= 2.7752 \times 10^{11} \Omega h^2 \, M_\odot \, \text{Mpc}^{-3}$$

(1.9)

Introducing the deceleration parameter

$$q = \frac{-\ddot{a}}{a^2}$$

(1.10)

In a Universe with matter and a cosmological constant, Eqn. 1.2 takes the form

$$q_0 = \frac{1}{2} \Omega_m - \Omega_\Lambda$$

(1.11)
where $q_0$ is evaluated at present time. The deceleration parameter describes the rate at which the expansion of the universe is slowing down. A matter dominated Universe ($\Omega_m = 1$) correspond to $q_0 = \frac{1}{2}$, while a negative value of $q_0$ corresponds to a universe in which the expansion is accelerating. In principle, it should be possible to determine the value of $q_0$ observationally. For example, for a population of uniformly luminous sources because of the luminosity distance dependence on $q_0$ (e.g., a set of identical supernovae within remote galaxies) the deceleration parameter tend to favor $q_0$ values of less than $\frac{1}{2}$. Since 1998 (Riess et al. 1998; Perlmutter et al. 1999), we know that $q_0$ is negative, therefore, we are living in an accelerating Universe.

Having presented all these definitions, it is possible to completely specify a FRW Universe by the cosmological parameters ($H_0$, $\Omega_0$, $\Omega_\Lambda$). Once these parameters are known, the evolution of a cosmological model is given by

$$H(a) = \frac{\dot{a}}{a} = H_0 E(a),$$

(1.12)

where $E$ is the normalized Hubble function defined as

$$E^2(a) = \Omega_0 a^{-3} + (1 - \Omega_0 - \Omega_\Lambda) a^{-2} + \Omega_\Lambda,$$

(1.13)

The hot Big Bang model is supported by a large amount of observational evidence. A few observations have become major pillars of the Big Bang model. One of them is that it explains Olber’s paradox. Only in a Universe with finite age and with finite velocity of light the sky at night is dark. Hubble’s law confirms the reality of an expanding Universe. The most important one, the Cosmic Microwave Background (CMB) radiation. After the first minutes of the Big Bang, the temperature of the Universe had cooled down to a few billion degrees. Photons were continually emitted and absorbed, giving the radiation a blackbody spectrum. As the Universe expanded, it cooled to a temperature at which photons could no longer be created or destroyed. When the temperature fell to some 3000 degrees, electrons and nuclei began to combine to form atoms, a process known as recombination. Almost coincidental is the resulting decoupling of radiation and matter, 380 000 years after the Big Bang. No longer scattered by freely floating electrons, photons assumed a long journey along the depths if a virtually transparent Universe. The photons make up the CMB that is observed today. Since then, gradual expansion of the Universe goes along with a proportional cooling down of the photon temperature, having reached a present day value of $T \approx 2.725^{\circ}K$. A map of the temperature distribution of the CMB radiation is seen in Fig. 1.2.

Current observations, in particular those of the microwave background temperature fluctuations by WMAP, indicate that our Universe is nearly, perhaps perfectly, flat. From light element abundances, in combination with primordial nucleosynthesis considerations, and from the WMAP determination of the second acoustic peak in the CMB power spectrum we have learned that normal baryonic matter can account for no more than 4.4% of the critical density. Numerous other observations indicate that there is a substantial amount of non-baryonic dark matter. This is already found on galactic scales from e.g. the rotation curves of disk galaxies. The dynamics of galaxies clusters indicate that the dark matter may account for $\sim$25-30% of the energy density of the Universe. The recent WMAP5 (Dunkley et al. 2008) results list a value of 23% hidden non-baryonic dark matter.

### 1.1.1 Effects of a cosmological constant

Dark energy, in the form of a cosmological constant, has several strong effects on the evolution of the Universe. Universes with a negative cosmological constant will recollapse due to the attractive gravity of matter. A positive cosmological constant will resist the attraction of matter due to its negative pressure. In most Universes, a positive cosmological constant will eventually dominate over the attraction of matter and will drive the Universe to expand exponentially. For a limited range of values, the cosmological constant will never dominate over the matter, and therefore the Universe will recollapse after some finite time. In the extreme case of a Universe with large cosmological constant, the Universe may not experience a Big Bang. These Universes collapse from an infinite size, they turn around and then expand to infinity again. They are called *bouncing Universes*. 
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Various pieces of evidence show that a Universe with a cosmological constant best fits observations. It was not until recently that observations started to confirm this issue. The most compelling pieces of evidence are:

- **Observations of Type Ia supernovae**: one of the most direct impacts of having a cosmological constant is its influence on the cosmic and dynamical timescales. In principle, given a set of objects with either a standard proper size or luminosity, one could determine the physical distance to the object. These particular objects are the supernovas type Ia. This type of supernovae exhibit a behavior that allows the absolute magnitude of the supernovae to be determined from the shape of their light curve and their time varying spectra. Once we know the absolute magnitude, it is possible to determine their actual distance from us. Riess et al. (1998) and Perlmutter et al. (1999) measured the light curves of distant type Ia supernovas and found direct evidence that the Universe is expanding.

- **Cosmic microwave background**: the discovery of temperature anisotropies in the cosmic microwave background by the COBE satellite started a new era in the determination of cosmological parameters.

  The cosmic microwave background (CMB) is made up of photons that are coming to us since they decoupled from matter. These photons are shifted to microwave wavelengths due to the expansion of the Universe. This is the oldest light in the Universe.

  The temperature fluctuations of the sky are decompose into spherical harmonics, yielding the angular spectrum of the CMB. The Wilkinson Microwave Anisotropy Probe (WMAP) has studied this fluctuations, investigating the physical processes that happened when the Universe was young. WMAP has matched the patterns of these fluctuations and matched them to the physics we know, providing convincing results on the contents of the Universe. It has determined the age of the Universe, the epochs of key transitions of the Universe, the geometry of the Universe and the value of the cosmological constant $\Omega_\Lambda$.

- **Integrated Sachs-Wolfe effect**: this is a property of the cosmic microwave background radiation. Photons of the CMB gain energy by falling into gravitational potential wells, and lose energy when they climb out again. In a Universe in which the full critical energy density comes
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from atoms and dark matter, the overall loss and gain of energy cancel out. However, in the presence of dark energy, the photons gain more energy as they fall into an overdense region and lose less energy as they come out due to the stretching of the potentials caused by the expansion of the Universe. This is the Integrated Sachs-Wolfe effect. Hotter CMB photons are due to overdense regions in the Universe, while cold CMB photons correspond to underdense regions. This behavior is seen in the CMB spectrum.

- Gravitational lensing: gravitational lensing is the process in which light from a very distant, bright source (e.g. a quasar) is “bent” around a massive object (such as a galaxy cluster) between the source and the observer. This process is one of the predictions of the general theory of relativity.

Figure 1.4 — A composite of images for the evidence of dark energy. Top left figure: Hubble diagram (distance modulus vs. redshift) from the SNIa observations. Top right image: a distant galaxy cluster. Bottom plot: the “angular spectrum” of the fluctuations in the WMAP full sky-map. These combined data puts constraints on the cosmological parameters, shown in the central figure. (WMAP spectrum: Courtesy NASA/WMAP Science Team.)
The volume of space back to a specified redshift is sensitively dependent on the cosmological constant $\Omega_\Lambda$. Therefore, counting the apparent density of observed objects provides a potential test for $\Omega_\Lambda$. Using the statistics of gravitational lensing of distant galaxies it is possible to infer the volume of space and, therefore, the value of the cosmological constant. An example of a gravitational lens can be seen in Fig. 1.3. The galaxy cluster Abell 2218 is so massive that gravity bends and focuses the light from galaxies that lie behind it. Images of background galaxies are observed, which are larger and brighter than normal, optical images. As a result, it is possible to observe a large amount of galaxies that otherwise would be extremely hard to observe. Abell 2218 has produces more than 120 images of galaxies that are members of a remote cluster.

- **Number counts of galaxy clusters**: they provide a direct probe of cosmology, complementary to supernova Ia and CMB measurements. Catalogs are built using clusters found by the Sunyaev-Zeldovich effect or with clusters in X-ray. The idea is to measure abundances of these objects as a function of redshift and compare this to theoretical predictions.

The mentioned observational evidence does not provide independent identification of the cosmological parameters. Each one of them gives pieces of evidence on the values of $\Omega_m$ and $\Omega_\Lambda$. Fig. 1.4 gives a more clear picture. SNIa gives information on the acceleration of the Universe via the deceleration parameter (see Eqn. 1.11), while the abundance of galaxy cluster tightly constrains the matter abundance $\Omega_m$. The CMB spectrum tell us that the Universe is flat. Combined, these data allows us to infer a “confidence interval” (image on the center of the figure) in the $\Omega_m - \Omega_\Lambda$ plane.

Using these combined data, WMAP5 inferred values of $\Omega_m \sim 0.258$, $\Omega_\Lambda \sim 0.742$ and $H_0 \sim 71.9 \text{ km/s/Mpc}$, resulting in an age of the Universe of 13.69 Gyrs. They are the main parameters defining the concordance model.

# 1.2 Growth of Structure

The observed Universe is far from being homogeneous: there is an enormous richness of structure ranging from dwarf galaxies to groups and clusters of galaxies.

## 1.2.1 Gravitational instability theory

The standard, and very important, ingredient of the theory of the evolution of the Universe is inflation (Guth 1981). Inflation basically states that shortly after the Big Bang, the Universe entered a phase of very rapid expansion.

In our current view of structure formation, we assume that the primordial Universe is not completely uniform. Instead, small density fluctuations were imprinted in the very early Universe. As the Universe evolved, the small quantum fluctuations that were present in the first instants of the Universe were blown up to cosmological scales. This implies that after this rapid expansion, matter in the Universe is inhomogeneously distributed, resulting in the plethora of structures we see in the Universe today (see Fig. 1.5). Inflation also predicts that the fluctuations have the character of a Gaussian random field.

For a description of the inhomogeneities in the primordial density field it is convenient to write

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \rho_b}{\rho_b}, \quad (1.14)$$

where $\rho_b$ is the background density. In comoving perturbation quantities, the continuity equation,
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Figure 1.5 — Gravitational collapse of primordial fluctuations. Tiny primordial fluctuations in the primordial density field (left frame) get amplified by gravity. They will end up forming the large scale structure seen in the right panel.

Euler equation and Poisson equation describing the evolution of a density perturbation are

\[ \frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot [(1 + \delta) \mathbf{v}] = 0, \]  
\[ \frac{\partial \mathbf{v}}{\partial t} + \frac{1}{a} (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{\dot{a}}{a} \mathbf{v} = -\frac{1}{\rho a} \nabla p - \frac{1}{a} \nabla \phi, \]  
\[ \nabla^2 \phi = 4\pi G \bar{\rho} a^2 \delta, \]  

in which \( \mathbf{v} \) is the peculiar velocity and \( \phi \) is the peculiar potential. The peculiar gravitational potential \( \phi \) is related to the peculiar gravitational field \( g \) by:

\[ g(x) = -\frac{\nabla \phi}{a} = G a \rho \int \frac{\delta(x') (x' - x)}{|x' - x|^3} \, dx'. \]  

1.2.2 The Power Spectrum

The initial density field is fully characterized by its power spectrum \( P(k) \), which specifies the amplitude of the fluctuations as a function of their spatial scale. In general, it is assumed that the power spectrum has a power-law behavior \( P(k) \propto k^n \), where the relative amplitude between scales is dictated by the index \( n \), i.e., \( n \) determines the balance between small and large scale power.

The initial perturbation spectrum is commonly assumed to be a power law,

\[ P(k) = k^n. \]  

There is a special case, where \( n = 1 \), in which Eqn. 1.19 has the property that the density contrast has the same amplitude on all scales when the perturbations enter the horizon. This special case is often referred to as the Harrison-Zeldovich (Zeldovich (1972), Harrison (1970)).

The primordial power spectrum is believed to change during the evolution of the early Universe until the end of the epoch of recombination by various processes, such as growth under self-gravitation, effects of pressure and dissipative processes. The overall effect can be encapsulated in the transfer
function, $T(k)$, which gives the ratio of the later-time amplitude of a mode to its initial value:

$$P(k,z) = P_0(k) T^2(k) \frac{D^2(z)}{D^2(z_0)}, \quad (1.20)$$

The evolution of linear perturbations back to the surface of last scattering obeys the simple growth laws given in Eqn. 1.20.

Calculations of the transfer function are a challenge, mainly because there is a mixture of matter and relativistic particles. For Cold Dark Matter spectrum, Bardeen et al. (1986) found the approximation

$$T(k) = \ln \left(1 + 2.34q\right) \left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4}, \quad (1.21)$$

with $q = kh/\Gamma$ Mpc, and $\Gamma$ is the shape parameter, $\Gamma = \Omega_m h$. Sugiyama (1995) obtained a more general form for the shape parameter, which is given by

$$\Gamma = \Omega_n h \exp \left[-\Omega_b \left(1 + \frac{\sqrt{2h}}{\Omega_m}\right)\right], \quad (1.22)$$

with $\Omega_b$ the baryon fraction density parameter.

To completely specify the power spectrum, we need to fix the overall amplitude. For $P(k)$ with a given shape, the amplitude is fixed if we know the value of $P(k)$ at any $k$. One prescription for normalizing a theoretical power spectrum involves the variance of the galaxy distribution when sampled with randomly placed spheres of radius $R$:

$$\sigma^2(R) = \frac{1}{2\pi} \int_0^\infty k^3 P(k) W^2(k,R) \frac{dk}{k}, \quad (1.23)$$

where $W(k,r)$ is the Fourier representation of a real space top-hat filter enclosing a mass $M$ at the mean density of the Universe, which is given by

$$W(kR) = 3 \left( \frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2} \right). \quad (1.24)$$

The value of $\sigma(R)$ derived from the distribution of normal galaxies is approximately unity in spheres of radius $R = 8h^{-1}$ Mpc, a quantity known as $\sigma_8$. An alternative is to calculate the present day abundance of rich clusters of galaxies: $\sigma_8 \Delta_{m}^{\alpha}$, with $\alpha \approx 0.5 - 0.6$. This quantity put constraints on both $\sigma_8$ and the matter density $\Omega_m$.

1.2.3 Linear Perturbation Theory

Assuming that the fluctuation field is small, one can linearize the equations of motion. One then obtains

$$\frac{\partial^2 \delta}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} = 4\pi G \rho \delta. \quad (1.25)$$

The general solution to this equation consists of two modes:

$$\delta(x,t) = A(x) D_1(t) + B(x) D_2(t), \quad (1.26)$$

where $D_1(t)$ and $D_2(t)$ are two time independent solutions. They are often called growing and decaying modes. Usually, analysis concentrates on the growing mode, evidently the decaying solution is damped. For a generic FRW Universe, the general expression for the a growing mode is given by

$$D_1(z) = \frac{5\Omega_{m0} H_0^2}{2} H(z) \int_{z}^\infty \frac{1 + z'}{H^3(z')} dz'. \quad (1.27)$$
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In a Universe with $\Omega_m = 1$, the growing mode is $D_1 \approx a \propto t^{2/3}$.

For matter dominated Universes, the growth of structure closely resembles that in a Universe with $\Omega_m = 1$. As it evolves and becomes increasingly empty, it enters a near free expanding phase. This happens around redshift

$$1 + z_{mf} = \frac{1}{\Omega_{m,0}} - 1.$$  \hspace{1cm} (1.28)

As a result, in matter dominated $\Omega_m < 1$ Universes at early times we see structure growing with a rate $D(a)$ proportional to $a(t)$, while it freezes out after $z_{mf}$.

In the case of of $\Lambda$ dominated Universes, structure formation comes to a halt when the Universe sets in its accelerated expansion at redshift $z_{Af}$.

$$1 + z_{Af} = \left( \frac{2\Omega_{\Lambda,0}}{\Omega_{m,0}} \right).$$  \hspace{1cm} (1.29)

In a Universe with $\Omega_m = 0.3$ and $\Omega_{\Lambda} = 0.7$, this corresponds to $z \sim 0.7$.
1.2.4 Nonlinear evolution

Once density fluctuations approach unity, linear theory is not longer valid. Since the full nonlinear solutions are in general too complex to solve analytically, one must rely on other alternatives, such as computer simulations.

Structure in the mildly nonlinear regime is marked by some important characteristics. Amongst these, the most important are:

- Hierarchical Clustering
- Anisotropic Collapse
- Cellular morphology

1.2.4.1 Hierarchical Clustering

In the cold dark matter scenario, for a power spectrum with $n(k) > 3$, small scale fluctuations collapse first to form bound objects before larger structures do, resulting in a gradual building up of successively larger structures by the clumping and merging of smaller structures. This process is called hierarchical structure formation. An example of this is seen in Figs. 1.6 and 1.7, where the evolution of the same cluster is seen in two different cosmologies, SCDM and ΛCDM.

A powerful description of hierarchical structure formation is the Press-Schechter theory (Press & Schechter 1974; Bond et al. 1991), which describes the sample average characteristics of an emerging population of nonlinear objects evolving from a linear density field of fluctuations in the primordial cosmos. For a Gaussian density field $\delta(M)$, with $\langle \delta \rangle = 0$ and $\langle \delta^2 \rangle = \sigma^2(M)$, one has that

$$p(\delta) = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left( -\frac{\delta^2 c}{2\sigma^2(M)} \right),$$

where $\delta_c$ is the density contrast associated with perturbations of mass $M$. Then, at any given time $t$, the fraction of an object of mass $M$ enclosed in a sphere of radius $R$ within which the mean overdensity exceeds $\delta_c$ is given by

$$f(\delta > \delta_c) = \frac{1}{2} \text{erfc}\left( \frac{\delta_c}{\sqrt{2\sigma(R)}} \right).$$

As $M \to 0$, $\sigma(R) \to \infty$ and thus $f \to 1/2$. The pure PS formula therefore predicts that only half of the Universe forms lumps of any mass. This was corrected by Bond et al. (1991) using the extended PS formalism or excursion set theory, showed that the factor of 2 can be justified with a sharp $k$-space filter. The PS formalist leads to the following comoving number density of halos of mass $M$ at time $Z$:

$$\frac{dn}{dM}(M,z) = \sqrt{\frac{2}{\pi}} \frac{\dot{\rho}}{M^2 \bar{\sigma}(M)} \left| \frac{d\ln \sigma(M)}{d\ln M} \right| \exp\left( -\frac{\delta_c^2(z)}{2\sigma^2(M)} \right),$$

where $\delta_c(z) = \delta_c/D(z)$ is the critical overdensity linearly extrapolated to the present time.

1.2.4.2 Anisotropic collapse

Another important characteristic of the nonlinear evolution is its anisotropic collapse: structures which at early stages are slightly non-spherical tend to become more and more anisotropic as time evolves. This characteristic was predicted by Zel’Dovich (1970), who explored the nonlinear regime by simply assuming that linear conditions remain valid in the early nonlinear regime. Icke (1973) investigated the evolution of homogeneous ellipsoidal configurations in an expanding FRW Universe and found that the predominant final morphologies are flatness and elongation.

We can easily observe this by considering the initial displacement of particles and assume that they continue to move in this initial direction. The physical position of a particle can be written as $\mathbf{r} = a(t)x$, where $a(t)$ is the scale factor.
1.2. GROWTH OF STRUCTURE

Figure 1.7 — Formation and evolution as a function of redshift of a single dark matter halo in a SCDM Universe. The hierarchical process is clearly visible: the halo gains mass by merging with smaller, surrounding objects.

where $\mathbf{x}$ is called the comoving position. Thus, for a given particle, the proper coordinate will be given by

$$x(t) = q + D(t)f(q).$$  \hfill (1.33)

This looks like Hubble expansion with some perturbation, which will become negligible as $t \to 0$. Therefore, the coordinates $q$ are equal to the usual comoving coordinates at $t = 0$, and $D(t)$ is the linear growth factor.

We concentrate here on the anisotropic collapse of a patch of matter. By applying a simple mass conservation of the form $\rho(x,t)d\mathbf{q} = \rho(q)d\mathbf{q} = \rho_0d\mathbf{q}$, we get

$$1 + \delta = \left| \frac{\partial x}{\partial q} \right|^{-1} = \frac{1}{(1 - D(t),\lambda_1)(1 - D(t),\lambda_2)(1 - D(t),\lambda_3)},$$  \hfill (1.34)

where $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the three eigenvalues of the deformation tensor $\partial f_i/\partial q_j$. The vertical bars denote the Jacobian determinant of the transformation between $\mathbf{x}$ and $\mathbf{q}$. Eqn. 1.34 describes the evolution of the density field in the Zeldovich approximation. It predicts the collapse of matter into planar sheets or pancakes. The subsequent collapse is determined by the second largest eigenvalue, which produces a filament. The final collapse along the axis defined by the third eigenvalue will result in the formation of galaxy clusters.
The Zeldovich approximation normally breaks down later than Eulerian linear theory, i.e., first order Lagrangian perturbation theory can give results comparable in accuracy to Eulerian theory with high order terms. It is therefore commonly used to set up quasi-linear initial conditions for $N$-body simulations. Also, it has been very successful in describing a variety of properties of the cosmic web.

### 1.2.4.3 Cellular Morphology

The third important characteristic of nonlinear regime is the appearance of a complex cellular geometry, consisting of a foam-like network of filamentary and wall-like structures surrounding extended empty regions, the Cosmic Web (Bond et al. 1996). Fig. 1.8 shows the galaxy distribution in the 2dF Galaxy Redshift Survey. The web-like structure is clearly visible. Bond et al. (1996) showed that the cosmic web is largely defined by the position and primordial tidal fields of rare events in the medium, with the strongest filaments between nearby clusters whose tidal tensors are nearly aligned.

The rare high peaks in the cosmic web corresponding to clusters play a fundamental role. They are the nodes that define the Cosmic Web.

### 1.3 Cluster of Galaxies

Galaxy clusters are the largest stable structures in the Universe. Typical properties of galaxy clusters include:

- They contain 50 to 1000 galaxies, hot gas and large amounts of dark matter.
- They have total masses of $\sim 10^{14} - 10^{15} h^{-1} M_\odot$.
- Their radius are in the order of $\sim 2-6 h^{-1} \text{Mpc}^2$

\[1 M_\odot = 1.989 \times 10^{30} \text{ kg}, \ \text{is the mass of the Sun}\]

\[1 \text{ Mpc} = 3.086 \times 10^{22} \text{ mts.}\]
1.3. CLUSTER OF GALAXIES

• Galaxy members have velocity dispersions in the order of \( \sim 800-1000 \) km/s.

Galaxy clusters have been key astrophysical objects in the development of our current understanding of the large scale Universe. It was in galaxy clusters that dark matter was first detected. Clusters are also very luminous X-ray sources, emitted by a tenuous extremely hot intracluster gas with a temperature of \( T \sim 10^7 - 10^8 \) K. The fact that they contain an atypical mixture of galaxies makes them into important probes of the study of galaxy evolution.

When observed visually, cluster of galaxies appear to be collections of galaxies held together by mutual gravitationally attraction (see Fig. 1.9). However, their velocities are too large for them to remain gravitationally bound by their mutual attraction. This implies that there must be an additional invisible mass component or an additional attractive force besides gravity. Most of the mass of galaxy clusters is in the form of hot gas, which emits in X-ray. In a typical cluster perhaps only \( \sim 5\% \) of the total mass in in the form of galaxies, \( \sim 10\% \) in the form of hot X-ray emitting gas and the rest is in the form of dark matter.

1.3.1 Spherical Collapse Model

The spherical collapse model (Gunn & Gott 1972) is a simple but very useful approximation to study the formation and evolution of structures. It describes the evolution of an isolated spherical overdense region in a homogeneous cosmological background of mean density \( \rho_b \). Although isolated spherical systems do not exist in reality, the spherical collapse model provides an excellent basis for understanding and interpreting considerably more complicated evolution of generic systems.

In this model, the isolated spherical region starts to expand at the same rate as the background. However, if its density is high enough, its rate of expansion will slow down sufficiently that it will eventually stop at some point, in which it reach a maximum radius, and then it will turn around, collapse and virialize. We can identify three stages of evolution in the spherical collapse model:

• **Turn around:** the galaxy cluster starts to slow down and eventually will decouple from the expansion of the Universe. It will reach a point in which it will stop expanding. In this point, its velocity is zero and has reached its maximum radius.

• **Collapse:** after reaching maximum expansion, the cluster starts to collapse and shrinks to a small size.

• **Virialization:** in reality, the cluster will not collapse to a point. Before that happens, the kinetic energy of the region is converted into random motions. The perturbation reaches a bound equilibrium state, virialization.

1.3.2 Cluster Catalogues

Galaxy clusters were first identified as overdensities in the spatial distribution of galaxies. This method has obvious systematic problems, in particular due to projection effects, and also because of the difficulties of identifying poor clusters.

The most extensive and widely used catalog of rich clusters was created by Abell (1958). Abell surveyed plates taken from the Palomar Observatory Sky Survey (POSS) by eye, identifying clusters such they contain at least 50 galaxies within a radius of \( \sim 3 \) Mpc (known as the Abell radius) and within two magnitudes of the third brightness member. Abell’s sample contained \( \sim 1700 \) objects which met his criteria. He also included an additional \( \sim 1000 \) clusters, but were not part of the statistical sample.

Later, Zwicky et al. (1968) created another catalog from POSS, the Catalog of Galaxies and Cluster of Galaxies (CGCG). His criteria was less strict than Abell’s. He drew isopleths at the level were the cluster density was twice that of the background density of galaxies. The number of cluster members was determined by counting all galaxies within the isopleth and within three magnitudes of the brightest member. His galaxy cluster had at least 50 members, making them rich galaxy cluster.
Clusters of galaxies contain substantially more mass in the form of hot gas, which is observable in X-ray band. Many catalogs of galaxy clusters are also selected through their X-ray emission. Catalogs are based on the ROSAT All Sky Survey (RASS). Amongst these, we can find the ROSAT Brightest Cluster Sample (BCS, Ebeling et al. (1998)), the Northern ROSAT All Sky Galaxy Cluster Survey (NORAS, Böhringer et al. (2000)) and the ROSAT-ESO Flux Limited X-ray catalog (REFLEX, Böhringer et al. (2001)), which contains clusters over a large part of the southern sky.

1.4 Galaxy Cluster as Cosmological Probes

Galaxy cluster have been (and are still) used to put constraints on cosmological parameters using a variety of methods:

- The mass function, the number density of objects of a given mass, provides constraints on the amplitude of the power spectrum at the cluster scale (e.g., Rosati et al. (2002)). Its evolution also provides constraints on the linear growth rate of density perturbations, which translates into dynamical constraints on the matter density parameter and dark energy density parameter.

- The power spectrum and the correlation function (clustering properties) of the large scale distribution of galaxy clusters provide direct information on the shape and amplitude of the underlying dark matter distribution. The evolution of these clustering properties is sensitive to the value of the density parameters through the linear growth rate of perturbations (e.g., Moscardini et al. (2001)).
1.5 Superclusters

We can extend further in the hierarchy of structures in the Universe and define superclusters of galaxies. As its name reveals, these structures consist of several galaxy clusters. Their sizes are in the order of $\sim 100$ Mpc. The supercluster formation is now at an early stage. They may be at the critical point of maximum expansion and starting to collapse under their own gravity into an increasingly dense superstructure. Due to their early formation stage, superclusters contain information on the large scale initial density field and their properties can be used as a cosmological probe to discriminate between different cosmological models.

Superclusters appear to surround large under-dense regions, called voids. The voids are of comparable sizes. Together, they create the cellular-like morphology of the Universe on large scales (see Fig. 1.8). Due to their recently formation, identifying superclusters is a very difficult task. Their overdensity is thought to be small in comparison to the overdensity of galaxies or cluster of galaxies. It is important to define a method that is best suited to identify superclusters. So far, superclusters have been defined as clusters of clusters, using catalogues of superclusters of galaxies. Recent redshift surveys, such as the 2dF Redshift Survey, have help to overcome the problem of identification, pinpointing potential superclusters.

One of the largest supercluster in our local Universe is the Shapley Supercluster, in the constellation of Centaurus. It consists of several hundred galaxies, and several of the clusters in Shapley are also strong sources of X-rays. Fig. 1.10 shows a radio map of galaxies in the core of Shapley.

- The mass-to-light ratio in the optical band can be used to estimate the matter density parameter once the mean luminosity density of the Universe is known. This is under the assumption that mass traces light with the same efficiency both inside and outside clusters.

- By measuring the baryon fraction in nearby clusters it is possible to constrain the matter density parameter, once the cosmic baryon density parameter is known. The baryon fraction of distant clusters provide a geometrical constraint on the dark energy content. Two assumptions are made: 1) clusters are fair containers of baryons and 2) the baryon fraction inside clusters does not evolve.
Chapter 1: Introduction

1.6 Outline of this thesis

The aim of the thesis is to achieve a more profound understanding of the role of the cosmological constant $\Lambda$ and dark matter in the formation and evolution of cluster of galaxies. To this end, we have performed a variety of $N$-body simulations. All simulations involve variants of the Cold Dark Matter scenario, embedded with a range of cosmological parameters.

In chapter 2 we extensively describe the simulations we use throughout this thesis. This simulations include open, flat and closed Universes, with or without a cosmological constant. An important property of these simulations is that they all start from primordial Gaussian conditions with the same Fourier phases. This allows us to follow the same structures in each of the simulations. We investigate the influence of the cosmological constant on global properties of dark matter halos, in particular the shape and evolution of the mass functions.

In chapter 3 we investigate the mass assembly and formation history of cluster halos in a range of cold dark matter cosmologies. We also investigate the virialization of these clusters halos. We look into the assembly history of identical clusters and assessed differences in its formation as a function of three different time scales: redshift, lookback time and cosmic time. By doing this, we expect to obtain lights on the influence of the cosmological constant in the formation of structures. We also compare the degree of virialization in the different simulated cosmologies.

Chapter 4 is devoted to the study of individual properties of galaxy cluster halos. These properties include the evolution of the angular momentum, morphology and density profile. This properties are studied as a function of the underlying cosmology.

In chapter 5 we explore the effects of dark matter and dark energy on the dynamical scaling properties of cluster halos. We investigate the Kormendy, Faber-Jackson and Fundamental Plane relation between the mass, radius and velocity dispersion of galaxy cluster halos. The validity and behavior of these relations in the different cosmological models should provide information on the general virial status of the cluster halo population.

In chapter 6 we probe the effects of future evolution in several properties of cluster sized halos. Towards the future, structures such as galaxy clusters will grow in complete isolation in physical coordinates (see Fig. 1.11). The effect of a nonzero cosmological constant drives the Universe towards unbounded exponential expansion, while a zero cosmological constant makes the expansion to decelerate. This expansion will have an effect on the internal evolution of galaxy cluster. In order to study...
1.6. OUTLINE OF THIS THESIS

Figure 1.12 — A simulated supercluster of galaxies defined as an overdensity of 2.36 times the critical density of the Universe at present time (left frame) and as an overdensity of 2 times the critical density in the far future (right frame), in a cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. At present time, the supercluster presents various substructures. In the far future, the supercluster has collapse, becoming a massive bound structure.

the influence of the cosmological constant, we extract information of the future gravitational growth of the large scale structure of the Universe and of physical quantities such as morphology, angular momentum, virialization and scaling relations. Global properties, such as the mass function and mass accretion history are also explored. These properties will tell us when and how clusters of galaxies reach dynamical equilibrium and, more importantly, they will allow us to determine the importance of the cosmological constant in the fate of the Universe.

Chapter 7 presents the study of the mass functions of gravitationally bound structures, in particular, the largest and most massive of these. The identification of the bound structures is done by using the criterion presented in Dünner et al. (2006). We compare the identification at present time and in the far future. We use this criterion as a physical definition for superclusters. Fig. 1.12 shows a massive bound structure (a supercluster) at present time (left frame) and in the far future (right frame). The supercluster has collapsed, forming the compact structure seen in the right frame of the figure. By investigating the mass functions, we can identify qualitatively and quantitatively the largest superclusters in our local Universe.