Strangeness in dense hadronic matter
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Chapter 4

Hyperon-Nucleon potentials

One of the main goals in nuclear physics is trying to understand the baryon-baryon interaction. The simplest baryons available are the nucleons the neutron and proton. So at first attempts were made to describe the NN interaction. This led to many phenomenological models for NN scattering.[52, 53, 54] Nowadays a lot more accurate data is available and these older models are no longer suitable for describing the data without refitting the parameters.

Later on in the 1970s and 1980s a number of so-called meson-exchange potentials were constructed. [55, 56, 57, 58, 59, 60, 61] Meson-exchange models are based on the assumption that the strong nuclear force between baryons is mediated by mesons. The differences in the models are first which mesons are used, secondly if two-meson exchange is taken into account and thirdly if the Pomeron is taken into account. Although they have been fitted to the newer data they still have only a $\chi^2/data \approx 2$. The refitting of the NSC78 model the NSC93 obtained a $\chi^2/data \approx 1.87$. When comparing this to [62] which has $\chi^2/data = 0.991$ which is about the best one can do, there seems room for improvement. One has to keep in mind that the Nijmegen partial wave analysis[62] uses 39 parameters, whereas the NSC93 has only 15 parameters. The large number of parameters for the PWA93 can be determined accurately because one has 1787(2514) pp(np) scattering datapoints. Which results in roughly 100 datapoints per parameter.

The next logical step in describing the baryon-baryon interaction, is describing the YN interaction. Contrary to the NN case one has only scarce data, with large errorbars. For YN channels one usually uses 35 selected datapoints. These low-energy data only provide s wave information. Additional there are some extra scattering data available which carry
almost no extra information. In a sense there is also "hidden" data, in that one has to avoid bound states which are not observed experimentally. Also hyperfragment data offer some extra insight in the YN interaction. To stress the difference in quality of data of the NN sector compared to the YN sector, total elastic cross sections are given. First in Fig. 4.1 one sees the elastic cross section for np and pp. In Fig. 4.2 the same for YN.

To construct a YN potential one cannot have too many free parameters 5 or 6 at most, if one wants to determine them reliably. Considerable theoretical input is therefore needed to construct a YN model. The strategy is then to start with a NN model and then apply $SU(3)$ flavor symmetry to this model in order to obtain a YN model. This means that such a NN model must already be consistent with $SU(3)$ symmetry. Not all models are suited for such a generalization, for instance [61] is not. For YN there are two groups with meson-exchange potentials the Jülich/Bonn potentials [69, 70] and the Nijmegen potentials.[56, 71, 72, 73] Apart from these meson-exchange models there is also the quark model [74, 75] from Tübingen and the hybrid-quark model [76, 77, 78, 79, 80, 81, 82] from Tokio.

### 4.1 Baryon-baryon potential

A baryon-baryon potential can both be described in momentum- and in configuration-space. The four-dimensional scattering equation is difficult to solve, so usually a reduc-
4.1. Baryon-baryon potential

A three-dimensional scattering equation is made to a three-dimensional scattering equation. In momentum space this integral equation is the so called Lippmann-Schwinger equation. The configuration-space equivalent differential equation is the Schrödinger equation. Both equations can either be relativistic or nonrelativistic. The difference being in the use of relativistic or nonrelativistic kinematics. The early potentials used the configuration-space potentials in a nonrelativistic Schrödinger equation. The potential must be invariant under rotations, reflections and time reversal. A general form [83] which satisfies these requirements is given by Eq. (4.1) in which the potential is written as a sum of 8 independent terms.

\[
V = V_C + V_\sigma \sigma_1 \cdot \sigma_2 + V_T [3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - (\sigma_1 \cdot \sigma_2)] + V_{SO} L \cdot S \\
+ V_{Q_2} \left[ (\sigma_1 \cdot L)(\sigma_2 \cdot L) + (\sigma_2 \cdot L)(\sigma_1 \cdot L) \right] + V_{AC} + V_{AS_1} \cdot \sigma_2 \frac{1}{2}(\sigma_1 - \sigma_2) \cdot L \\
+ \left[ (\sigma_1 \cdot \nabla) V_p (\sigma_2 \cdot \nabla) + (\sigma_2 \cdot \nabla) V_p (\sigma_1 \cdot \nabla) \right].
\] (4.1)

The first five terms are the well known central, spin-spin, tensor, spin-orbit and quadratic spin-orbit terms from nucleon-nucleon scattering. The last three terms are less well known and are usually neglected. The two antisymmetric terms \((V_{AC}, V_{AS_1})\) only contribute for \(L > 1\) partial waves. The \(V_i's\) are in configuration-space functions of \(p^2, L^2, r^2\). In practice one keeps only the \(r^2\) dependence and the \(p^2\) dependence is linear and is only kept in the central potential \(V_1\). In most potentials the \(V_i's\) are assumed to be the same for all partial waves and the differences between partial waves is accounted for by different expectation values of the operators \(P_i\). This means that the \(V_i's\) only depend on \(L^2, r^2\).
In contrast the Reid68 potential parameterizes every partial wave separately. Therefore
the $V_i$’s become also dependent on $S^2$ and $J^2$. Modern potentials of this form are NijmI,
NijmII and Reid93.

Since the 1960’s it is become practice to describe the baryon-baryon interaction as an
exchange of mesons. As a result potentials were written as a sum of one-boson-exchange
potentials. The OBE potentials are usually derived in momentum space. One can define
the following momentum vectors

\[ \mathbf{k} = \mathbf{p}_f - \mathbf{p}_i, \quad \mathbf{q} = \frac{1}{2}(\mathbf{p}_f + \mathbf{p}_i), \quad \mathbf{n} = \mathbf{q} \times \mathbf{k}, \]

where $\mathbf{p}_i$ and $\mathbf{p}_f$ are the initial and final momenta. The momentum-space equivalents of
the configuration-space operators are

\begin{align*}
    P_1 &= 1, & P_2 &= \sigma_1 \cdot \sigma_2, \\
    P_3 &= (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3}k^2(\sigma_1 \cdot \sigma_2), & P_4 &= \frac{i}{2}(\sigma_1 + \sigma_2) \cdot \mathbf{n}, \quad (4.2) \\
    P_5 &= (\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) & P_6 &= \frac{i}{2}(\sigma_1 - \sigma_2) \cdot \mathbf{n}.
\end{align*}

The $V_i$’s in momentum space are now functions of $\mathbf{k}$, $\mathbf{q}$, $\mathbf{n}$ and of the energy. The operators
$P_i$ in momentum space form a complete set, but the $Q_{12}$ operator in configuration space
is not the exact Fourier transform of $(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n})$. When one wants configuration and
momentum space versions of the potential to give the same phase-shift and bound states,
one has to redefine $P_5$ in momentum space as the inverse Fourier transform of $Q_{12}$. Also
the momentum space $V_i$’s should not depend on energy and the $\mathbf{q}$ dependence should be
of second order at most.

Potentials in momentum space are usually first regularized before they are Fourier transformed
to configuration space. This is done to remove the singularities at the origin. This can also be interpreted physically as an extended source. The regularisation is done
by introducing a form factor $F(k^2)$. An example of a Fourier transform which is often
encountered is

\[ \int \frac{d^3k}{(2\pi)^3} \frac{e^{ikr}}{k^2 + m^2} F(k^2) = \frac{m}{4\pi} (-m^2)^n \phi^n_0(r) \quad (4.3) \]
\[ = \frac{m}{4\pi} (-\nabla^2)^n \phi^n_0(r). \quad (4.4) \]

When no form factor is used, $F(k^2) = 1$ this leads to the familiar Yukawa function.

\[ \phi^n_0 = \frac{e^{-mr}}{m^r}, \quad (4.5) \]
4.2. The Nijmegen potentials

which has a singularity at the origin. Often multipole form factors are used,

\[ F(k^2) = \left( \frac{(\Lambda^2 - m^2)}{\Lambda^2 + k^2} \right)^n, \quad F(-m^2) = 1, \]  

(4.6)

where \( n=1,2,\ldots \) for monopole, dipole, ... form factors. This gives for monopole

\[ \phi^0_{C,r} = \frac{e^{-mr} - e^{-\Lambda r}}{mr}, \]

(4.7)

and for dipole

\[ \phi^0_{C,r} = \frac{e^{-mr} - e^{-\Lambda r} \left( 1 + \frac{\Lambda^2 - m^2}{2\Lambda^2} \right)}{mr}. \]

(4.8)

For the Nijmegen potentials Gaussian form factors are used with the exception of Reid93 where a dipole form factor is used,

\[ F(k^2) = e^{-k^2}/\Lambda^2, \]

(4.9)

which results in

\[ \phi^0_{C,r} = e^{m^2/\Lambda^2} \left[ e^{mr} \text{erfc} \left( \frac{m}{\Lambda} - \frac{\Lambda r}{2} \right) - e^{mr} \text{erfc} \left( \frac{m}{\Lambda} + \frac{\Lambda r}{2} \right) \right] /2mr \]

(4.10)

with \( \text{erfc}(x) \) is the complementary error function

\[ \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} dt e^{-t^2}. \]

(4.11)

4.2 The Nijmegen potentials

There are many different Nijmegen potentials. In Table 4.1 a overview of the most commonly used potentials is given. The \( \chi^2 \) per datapoint and if the potential is NN or YN is also noted. The first 6 models A-F are hard-core potentials which were essentially NN and YN potentials. Of these hard-core potentials model D and model F were the most successful in terms of describing the data.

The hard-core potentials were succeeded by a soft-core NN potential NSC78 and the extension to YN NSC89. After the partial wave analysis \([62]\) of experimental data was completed a updated soft-core NN potential was constructed NSC93 and it’s YN counterpart NSC97. In NSC97 some modifications were made compared to NSC89 to facilitate the extension to YY and to solve some deficiencies in the spin-spin interaction for the
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Figure 4.3: $1S_0$ and $3S_1$ phaseshift for different versions of NSC97 potential.

Λ − N channel. Of the NSC97 potential 6 different versions (a-f) which describe the scattering data YN data equally well were constructed. The potentials differ in their s-wave interaction. The optimal potential can be selected in a nuclear structure calculation. The optimal solution appears to be somewhere in between NSC97e and NSC97f. However for the p-shell hypernuclei NSC97a is favored. To show the variation possible in s-wave phaseshift, while retaining the same fit to the data Fig. 4.3 is given. The potentials I,II and Reid93 are special high-quality NN-potentials only which have no YN counterpart.

All previous models are based on One Boson Exchange (OBE). Starting with ESC02 not only OBE but also contributions from two-meson exchange and one-pair and two-pair diagrams are taken into account. The ESC02 is the NN-potential variant and ESC03 is the YN potential were for the first time a simultaneous fit of NN and YN is done.

4.3 $SU(3)$ structure

The standard theory of the strong interaction is of course QCD. Which has yielded many successes in the high energy perturbative regime. In QCD the fundamental constituents are quarks and gluons which have a quantum number color. The QCD Lagrangian has certain exact symmetries, it is relativistic invariant and also $SU_C(3)$ invariant. These
### Table 4.1: Nijmegen YN, NN scattering models.

<table>
<thead>
<tr>
<th>Model</th>
<th>NN $\chi^2$/data</th>
<th>Ref.</th>
<th>YN $\chi^2$/data</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>large</td>
<td>[55]</td>
<td>0.71</td>
<td>[55]</td>
</tr>
<tr>
<td>B</td>
<td>5.9</td>
<td>[84]</td>
<td>0.68</td>
<td>[84]</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>[85]</td>
<td>0.62</td>
<td>[85]</td>
</tr>
<tr>
<td>D</td>
<td>2.4</td>
<td>[56]</td>
<td>0.65</td>
<td>[86]</td>
</tr>
<tr>
<td>E</td>
<td>2.22</td>
<td>[87]</td>
<td>0.61</td>
<td>[87]</td>
</tr>
<tr>
<td>F</td>
<td>2.17</td>
<td>[71]</td>
<td>0.89</td>
<td>[71]</td>
</tr>
<tr>
<td>NSC(78,89)</td>
<td>2.09</td>
<td>[57]</td>
<td>0.58</td>
<td>[72]</td>
</tr>
<tr>
<td>NSC(93,97)</td>
<td>1.87</td>
<td>[88]</td>
<td>0.55</td>
<td>[73]</td>
</tr>
<tr>
<td>I</td>
<td>1.03</td>
<td>[88]</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>1.03</td>
<td>[88]</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Reid93</td>
<td>1.03</td>
<td>[88]</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ESC02</td>
<td>1.15</td>
<td>[89, 90]</td>
<td>1.35</td>
<td>[91]</td>
</tr>
<tr>
<td>ESC03</td>
<td>1.35</td>
<td>[91]</td>
<td>1.35</td>
<td>[91]</td>
</tr>
</tbody>
</table>

Table 4.1: Nijmegen YN, NN scattering models.
symmetries are not enough do describe the properties of hadrons. For this dynamical symmetries are needed which are only approximate. There are six flavors of quarks, so flavor symmetry is the first symmetry which comes to mind to use as a dynamical symmetry. For elastic low energy baryon-baryon scattering one is in the non-perturbative regime. For this one needs an effective field theory to describe the scattering processes. The effective degrees of freedom then are the light quarks (up, down and strange). The current masses of the light quarks are

\[ m_u = 1.5 - 4.0 \, \text{MeV}, \]
\[ m_d = 4 - 8 \, \text{MeV}, \]
\[ m_s = 80 - 130 \, \text{MeV}, \]  

(4.12)

where the renormalization scale \( \mu = 2 \, \text{GeV} \). For a renormalization scale of \( \mu = 1 \, \text{GeV} \) one has to multiply the masses by a factor 1.35. Taking \( \mu = \Lambda_\chi \approx 1 \, \text{GeV} \) one can easily see that the quark masses are substantially smaller than the mass scale. To first approximation one can take the light quarks as massless. For up and down quarks this symmetry is very well satisfied (0.4\% difference). Even the strange quark mass breaking is at a 14\% level. This chiral symmetry leads to an \( SU_R(3) \times SU_L(3) \) symmetry or \( SU_V(3) \times SU_A(3) \) symmetry. The \( SU_A(3) \) is spontaneously broken. This leads to a remaining \( SU_V(3) \) symmetry and 8 massless Goldstone bosons, and they have spin 0 and negative parity. The pseudoscalar mesons have the same exact quantum numbers as the Goldstone bosons. The pseudoscalar mesons have a relatively small mass compared to the other mesons. This can be attributed to the fact that the \( SU_A(3) \) is explicitly broken by the non-zero mass of the light quarks. The 'Goldstone bosons' acquire then a mass. Allowing the quark masses to be non-zero but keeping \( m_u = m_d = m_s \neq 0 \) the \( SU_V(3) \) is still obeyed. The strange quark mass is quite a bit larger than the up and down quark mass. This explicit breaking of \( SU_V(3) \) is the cause of the much larger kaon mass compared to the pion mass. The remaining \( SU(2) \) isospin symmetry is very well obeyed as can be seen from the almost equal proton and neutron mass and the small mass differences within the pion triplet. For low and intermediate energy processes the most relevant energy scale is \( \Lambda_\chi \). One has now two Langrangians the \( L_{\text{QCD}} \) with current quark fields and an effective Langrangian \( L_{\text{eff}} \) with other quark fields and non-perturbative contributions to the quark masses. These constituent quark masses contain important contributions from vacuum condensates [93]. Matching these two Langrarians ([94] provides a way to link the parameters from QCD to the parameters from the Effective Theory. .. The effective quark masses become

\[ m_u = m_d = 350 \, \text{MeV}, \]
\[ m_s = 500 \, \text{MeV}. \]  

(4.13)

In this picture is \( SU_f(3) \)-symmetry a natural symmetry. The gluons are flavour-blind.
4.3. SU(3) structure

4.3.1 Mesons

When one takes mesons as $q\bar{q}$ bound states of the three lightest quarks one has 9 possible combinations. In $SU_f(3)$ language this becomes an octet and a singlet

$$\{3\} \otimes \{\bar{3}\} = \{8\} \oplus \{1\}$$

There are various nonets with different quantum numbers one usually uses $J^{PC}$ to denote different nonets. For instance the nonet with the smallest mass is the pseudoscalar nonet $J^{PC} = 0^-$. The octet contains a isotriplet $I = 1$, two isodoublets $I = \frac{1}{2}$ and a isosinglet $I = 0$ which has the same quantum numbers as the $SU_f(3)$ singlet. Therefore one has a octet-singlet mixing. One usually takes as generic names; a for the the triplet, $K$ for the doublets and $f$ and $f'$ for the physical isoscalars. These physical isoscalars are thus mixtures of pure $SU(3)$ wavefunctions $\psi_8$ and $\psi_1$

$$f' = \psi_8 \cos \theta - \psi_1 \sin \theta,$$

$$f = \psi_8 \sin \theta + \psi_1 \cos \theta,$$

with $\theta$ the nonet mixing angle and

$$\psi_8 = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}),$$

$$\psi_1 = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}).$$

To decouple the $u\bar{u} + d\bar{d}$ and $s\bar{s}$ components one has to take the “ideal” mixing angle $\theta_i$, $\tan \theta_i = 1/\sqrt{2}$, which gives $\theta_i = 35.3^\circ$. For this ideal angle $f$ becomes purely $u\bar{u} + d\bar{d}$ and $f'$ becomes purely $s\bar{s}$. Alternatively starting with the masses one can calculate the mixing angle $\theta$. Using the linear Gellman-Okubo [95, 96] mass relation and the physical masses one obtains for the mixing angle

$$\tan^2 \theta = \frac{4m_K - m_a - 3m_f}{-4m_K + m_a + 3m_f}.$$  

4.3.2 Exotic mesons

Already in 1977 the possibility of a light exotic meson nonet consisting of four quarks was suggested by Jaffe [97]. Light in this case means a mass below 1GeV. In this scheme one couples two triplets of light quarks ($u$, $d$ and $s$) together. This leads to six symmetric states ($uu, dd, ss, ud+du, us+su, ds+sd$) and 3 antisymmetric states ($ud-du, us-su, ds-sd$). In $SU(3)$ dimensional representation

$$\{3\} \otimes \{3\} = \{6\} \oplus \{\bar{3}\}.$$
When combining $SU(3)$ with spin and color and requiring antisymmetry the lightest diquark resides then in the $\bar{3}$ representation in the spin singlet state. Coupling this $\bar{3}$ diquark with the 3 anti diquark gives again a so-called crypto-exotic meson nonet

$$\{3\} \otimes \{3\} = \{8\} \oplus \{1\}. \quad (4.20)$$

The $SU(3)$ symmetry is broken by the larger strange quark mass compared to the up and down quark mass. One has light states containing no strange (anti)quark, medium heavy states containing one strange (anti)quark and heavy states containing two strange (anti)quarks. Therefore the nonet is split in a light isosinglet $(u\bar{d} d\bar{d})$, two medium heavy isodoublets $(d\bar{s}u, u\bar{s}d, s\bar{d}u, s\bar{d}d, s\bar{u}u, s\bar{d}d)$ and a heavy isotriplet $(\bar{d}s u, \sqrt{2}s(u - d\bar{d}), u\bar{s}d, \sqrt{2}s(u - d\bar{d}))$ plus a heavy isosinglet $(\sqrt{2}(u\bar{u} + d\bar{d})\bar{s}s)$.

### 4.3.3 Pseudo scalar mesons

The pseudoscalar mesons with $J^{PC} = 0^{--}$ consist of $(\pi, \eta, \eta', K)$. First one identifies the triplet $a$ with $\pi$ and the two strange isodoublets $K$ with the $K$. The $f'$ becomes the $\eta$ and the $f$ becomes the $\eta'$. The mixing angle $\theta_{PV}$ is as mentioned earlier usually determined by the linear Gell-Mann-Okubo relation. Which gives a value of $\theta_{PV} = -24.6^\circ$. Alternatively $\theta_{PV}$ can be measured in $\gamma \gamma$ collisions. From the ratios of the partial widths one finds for

$$\frac{\Gamma(\eta' \to 2\gamma)}{\Gamma(\eta \to 2\gamma)}, \quad (4.21)$$

a mixing angle $\theta_{PV} = (-18 \pm 2)^\circ$ and for

$$\frac{\Gamma(\eta' \to 2\gamma)}{\Gamma(\pi^0 \to 2\gamma)}.$$ \quad (4.22)

a mixing angle $\theta_{PV} \approx -24^\circ$.

### 4.3.4 Vector mesons

The vector mesons are $J^{PC} = 1^{--}$ $(\rho, \phi, \omega, K^*)$. Here the triplet $a$ is identified with the $\rho(770)$ and the two strange isodoublets $K$ with the $K^*(892)$. The $f'$ is now associated with the $\phi(1020)$ and the $f$ with the $\omega(782)$. The vector mixing angle as determined from the linear Gell-Mann-Okubo mass relation $\theta_V = 36.0^\circ$ which is almost equal to the ideal mixing angle.
4.3.4 SU(3) structure

4.3.5 Scalar mesons

The scalar mesons $J^{PC} = 0^{++}$ are given by $(a_0(980), f_0(975), f_0(760), \kappa(880))$. The $f_0(975)$ meson will be denoted by $f_0$ and the $f_0(760)$ meson will be denoted by $\epsilon$. The scalar mesons can be seen as a conventional quark antiquark pairs, but also as two quark and two antiquark (crypto-exotic) states, or glueball states. In the conventional picture the scalar mesons are attributed to the $^3P_0$ $qq$ state. The triplet $a$ is then the $a_0(980)$ and the doublets $K$ are the $\kappa(880)$. The $f'$ is the $f_0(980)$ and $f$ is the $\epsilon(760)$. Again one has unitary singlet and octet states which mix with mixing angle $\theta_S$. For ideal mixing $\theta_S = 35.3^\circ$.

When the scalar mesons are crypto exotic mesons consisting of a diquark and anti diquark one identifies the light isosinglet with the $f_0(600)$. The heavy isodoublet is then identified with the $a_0$ and the isosinglet with the $f_0(980)$. Finally one can identify the two medium heavy isodoublets with the strange $\kappa(800)$ meson. Also here one has singlet octet mixing. But here the ideal mixing angle is $\theta_S = -54.8^\circ$. In general the scalar mesons could even be a mixing of $qq$ with $q^2\bar{q}^2$ states. When one assumes ideal mixing then one can say that for $\theta_S < 0$ the $q\bar{q}$ component dominates, and for $\theta_S > 0$ that the $q^2\bar{q}^2$ component dominates.

4.3.6 Pomeron, tensor mesons

In baryon-baryon scattering with $p_{lab} < 1$GeV one can describe the cross section very well with a potential model. For high energy $p_{lab} > 2$GeV the pomeron and tensor meson-exchange dominate the cross section as can be seen from Fig. 4.4. But even the region in between one sees already a significant contribution from the pomeron. So it seems important to include the pomeron in a meson-exchange potential model. The nature of the pomeron has not always been clear. Nowadays pomeron-exchange corresponds to a two-gluon (or multigluon) exchange in the QCD framework. Pomeron-quark coupling leads in the Low-Nussinov model to a repulsive Gaussian potential. Unitarity and exact $SU(3)$ symmetry give rise to a strong mixing between the isosinglet member of the tensor mesons and the “bare” pomeron. Breaking of this symmetry leads to the $f_2$ tensor meson and the physical pomeron.

4.3.7 Baryons

In QCD the baryon is a $SU_C(3)$ antisymmetric colorless singlet. Because baryons are three quark states $qqq$, the baryon is a fermion which has an antisymmetric total wave...
Figure 4.4: The total and elastic $pp$ cross section as function of $p_{lab}$. 
4.4 Coupling constants

The total baryon wave-function can thus be written as

$$\Psi_b = |qqq\rangle = |\text{color}\rangle_A \times |\text{space, spin, flavor}\rangle_S,$$  \hspace{1cm} (4.23)

where A and S denote antisymmetric and symmetric state of a quark antiquark pair. Again only using the lightest three quarks quarks (up, down, strange) and assuming $SU_f(3)$ symmetry leading to the following multiplets [98]

$$\{3\} \otimes \{3\} \otimes \{3\} = \{10\}_S \oplus \{8\}_M \oplus \{8\}_M \oplus \{1\}_A.$$  \hspace{1cm} (4.24)

Where the subscripts S,M and A denote symmetric, mixed symmetry and antisymmetric states. As with the mesons the $SU_f(3)$ singlet state $\Lambda_1 = uds$ can mix with a state in the octet $\Lambda_8$ providing they have the same spin and parity. The flavor singlet $\{1\}$ is in the ground state forbidden by Fermi-statistics. Therefore the lowest lying baryons are contained in an octet. These $J^P = \frac{1}{2}^+$ baryons are $(n, p, \Sigma^-, \Sigma^0, \Sigma^+, \Lambda, \Xi^-, \Xi^0)$. When $SU_f(3)$ is broken, but $SU(2)$ is still conserved this leads to a isospin triplet $\Sigma$ two isospin doublets $N$ and $\Xi$ and finally a isosinglet $\Lambda$.

4.4 Coupling constants

When one assumes $SU_f(3)$ symmetry one obtains a set of relations between the different coupling constants. When one takes Yukawa coupling of the mesons with the baryons one has a baryon current $J = \bar{B}B$ which is coupled to the meson field $\phi$ with a coupling constant $g$.

$$\mathcal{L}_{\text{int}} = gJ\phi$$ \hspace{1cm} (4.25)

When $\phi$ is a pseudo-scalar meson then the baryon current $J$ should be a pseudo-scalar, likewise when $\phi$ is 4-vector $J$ should be a 4-vector. The spin and space-time indices are ignored for the moment. For the interaction Lagrangian to be $SU(3)$ invariant it needs to be a unitary scalar. Now the following states are possible for the baryon-baryon current

$$\{8\} \otimes \{8\} = \{27\} \oplus \{10\} \oplus \{10^*\} \oplus \{8\}_A \oplus \{8\}_S \oplus \{1\}.$$  \hspace{1cm} (4.26)

The mesons are in $SU(3)$ octets or singlets, so the meson singlet has to couple to the singlet part of the baryon current or the meson octet has to couple to the octet part of the baryon current. The baryons can be combined in a traceless matrix $B$ which is invariant under $SU(3)$ transformations

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & \frac{2\Lambda}{\sqrt{6}} \end{pmatrix}.$$  \hspace{1cm} (4.27)
Likewise for the antibaryons

\[ \bar{B} = \begin{pmatrix} \Sigma^0 \sqrt{2} + \frac{\Lambda}{\sqrt{6}} & \Sigma^- - \Xi^- \\ \Sigma^- & \bar{p} \end{pmatrix} = \begin{pmatrix} \bar{\Sigma}_0 \sqrt{2} + \frac{\Lambda}{\sqrt{6}} & \Xi^0 \\ \Xi^0 & \bar{n} \end{pmatrix}. \] (4.28)

A meson nonet can be written as

\[ P = P_{\text{sin}} + P_{\text{oct}}, \] (4.29)

for the pseudoscalar mesons this becomes

\[ P_{\text{sin}} = \begin{pmatrix} \eta_1 \sqrt{3} & 0 & 0 \\ 0 & \pi_0 \sqrt{2} & \eta_8 \sqrt{6} \\ 0 & \eta_8 \sqrt{6} & K_0 \end{pmatrix}, \quad P_{\text{oct}} = \begin{pmatrix} \pi_0 \sqrt{2} + \eta_8 \sqrt{6} & \pi^- & K^+ \\ K^- & -\pi_0 \sqrt{2} + \eta_8 \sqrt{6} & K^0 \\ K^0 & -2\eta_8 \sqrt{6} & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}. \] (4.30)

In the singlet case a meson singlet is coupled to the singlet part of the baryonic current. The singlet part of the baryonic current is given by

\[ J^{(1)}_0 = \text{Tr}(\bar{B}B) = N^\dagger N + \Sigma^\dagger \cdot \Sigma + \Lambda^\dagger \Lambda + \Xi^\dagger \Xi, \] (4.31)

where

\[ N = \begin{pmatrix} p \\ n \end{pmatrix}, \quad \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma^0 \\ \Sigma^- \end{pmatrix}. \] (4.32)

Which gives a singlet coupling Lagrangian

\[ L^{\text{sin}}_{\text{oct}} = g^{\text{sin}}(N^\dagger N + \Sigma^\dagger \cdot \Sigma + \Lambda^\dagger \Lambda + \Xi^\dagger \Xi)\phi^{\text{sin}}. \] (4.33)

The unitary singlet mesons couples to all the baryons with the same \( g^{\text{sin}} \) coupling constant.

For octet coupling the meson octet is coupled to the octet part of the baryonic current. As seen before in Eq. (4.26) there a two different octets, the symmetric and the antisymmetric octet. The symmetrical baryonic current is

\[ J^{(8)}_S = \frac{1}{2}(\bar{B}B + B\bar{B}) - \frac{1}{3}I\text{Tr}(\bar{B}B), \] (4.34)
and the antisymmetrical baryon current is

\[ J_A^{(8)} = -\frac{1}{2}(B\bar{B} - B\bar{B}). \quad (4.35) \]

The octet coupling Lagrangian then becomes

\[ \mathcal{L}_{\text{oct}}^{\text{int}} = 2\sqrt{2} D \text{Tr}(J_S^{(8)} \phi^{\text{oct}}) + 2\sqrt{2} F \text{Tr}(J_A^{(8)} \phi^{\text{oct}}). \quad (4.36) \]

Defining the following \( SU(3) \) invariant couplings

\[ [\bar{B}BP]_F = \text{Tr}(\bar{B}P\bar{B}) - \text{Tr}(\bar{B}BP), \]
\[ [\bar{B}BP]_D = \text{Tr}(\bar{B}P\bar{B}) + \text{Tr}(\bar{B}BP) - \frac{2}{3} \text{Tr}(\bar{B}B)\text{Tr}(P), \]
\[ [\bar{B}BP]_S = \text{Tr}(\bar{B}B)\text{Tr}(P), \]

and

\[ D = g^{\text{oct}}(1 - \alpha), \quad F = g^{\text{oct}} \alpha, \quad \alpha = \frac{F}{F + D}. \quad (4.40) \]

With this the interaction Langrangian can be written as [98]

\[ \mathcal{L}_I = -g^{\text{oct}}\sqrt{2} \left\{ \alpha \left[ [\bar{B}BP]_F + (1 - \alpha) [\bar{B}BP]_D \right] - g^{\text{pv}} \sqrt{\frac{1}{3}} [\bar{B}BP]_S \right\}, \quad (4.41) \]

with \( \alpha \) the well known \( F/(F+D) \) ratio. Introducing the pseudoscalar isospin doublets

\[ K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad K_c = \begin{pmatrix} \bar{K}^0 \\ -K^- \end{pmatrix}, \quad (4.42) \]

and choosing the phases of the isovector pseudoscalar mesons fields such [98] that

\[ \Sigma \cdot \pi = \Sigma^+ \pi^- + \Sigma^0 \pi^0 + \Sigma^- \pi^+. \quad (4.43) \]

The coupling of the pseudoscalar mesons to the baryons can be described by two different Lorentz structures. The pseudoscalar(PS) coupling or the pseudovector(PV) coupling. Here the PV coupling is used where coupling constants are denoted by \( f \). When the Lagrangian is written out in all detail it is possible to derive relations between the various coupling constants

\[ f_{NN} \eta_1 = f_{\Lambda\Lambda} \eta_1 = f_{\Sigma\Sigma} \eta_1 = f_{\Xi\Xi} \eta_1 = f_{\text{pv}}^\text{sin}. \quad (4.44) \]
\[ f_{NN\pi} = f_{\text{octet}}, \quad f_{\Lambda N K} = -\frac{1}{\sqrt{3}} f(1 + 2\alpha), \quad f_{NN\eta} = \frac{1}{\sqrt{3}} f(4\alpha - 1), \]
\[ f_{\Sigma\Sigma\pi} = 2f\alpha, \quad f_{\Xi\Lambda K} = \frac{1}{\sqrt{3}} f(4\alpha - 1), \quad f_{\Lambda\Lambda\eta} = -\frac{2}{\sqrt{3}} f(1 - \alpha), \]
\[ f_{\Lambda\Sigma\pi} = -\frac{2}{\sqrt{3}} f(1 - \alpha), \quad f_{\Sigma N K} = f(1 - 2\alpha), \quad f_{\Sigma\Sigma\eta} = \frac{2}{\sqrt{3}} f(1 - \alpha), \]
\[ f_{\Xi\Xi\pi} = -f(1 - 2\alpha), \quad f_{\Xi\Sigma K} = -f, \quad f_{\Xi\Xi\eta} = -\frac{1}{\sqrt{3}} f(1 + 2\alpha). \]

The charged-pion mass is used to make the pseudovector coupling constants \( f \) dimensionless. For the pseudoscalar mesons there are 3 coupling constants \( f_{\pi\pi}, f_{\text{octet}} = f_{NN\pi}, \alpha \). Where \( f_{NN\pi} \) is very well known from the pion nucleon interaction. \( \alpha \) can be calculated using the Cabibo theory of weak interaction and the Goldberger-Treiman relation. For the vector mesons things become a bit more complicated, because there are two different coupling constants. First there is the electric coupling with coupling constants \( g_V(e) \) and \( \alpha_{V}^{(c)} \). There is also the magnetic coupling with coupling constants \( g_V(m) \) and \( \alpha_{V}^{(m)} \). Generally one assumes that \( \alpha_{V}^{(c)} = 1 \) which results in a universal coupling. The \( I = 1 \) meson \( \rho \) couples then universally to the isospin current, likewise the \( I = 0 \) meson \( \phi_0 \) couples universally to the hypercharge current.

### 4.5 Nijmegen Soft-Core potential

The remaining part of this chapter will primarily be devoted to the SC soft-core potentials.

#### 4.5.1 One Boson Exchange

The Nijmegen soft-core model is derived from Regge-pole theory. The exchange of the lowest-lying trajectories in the complex-J plane reduces for low energy to the exchange of pseudoscalar, vector and scalar mesons. In addition to this One Boson Exchange (OBE) one has contributions from the \( J = 0 \) parts of the Pomeron and the tensor-mesons. In the Nijmegen soft-core models the following complete nonets are exchanged,

\[ J^{PC} = 0^{-+}: \pi; \eta; \eta'; K; \]
\[ J^{PC} = 0^{++}: \alpha_0; f_0; K^*_0, \]
\[ J^{PC} = 1^{--}: \rho; \omega; \phi; K^*, \]
\[ J^{PC} = 2^{++}: \alpha_2; P \oplus f_2; f'_2; K^*_2. \]

The non-strange members of the nonets with hypercharge \( Y = 0 \) come in isospin triplets \( I = 1 \) and singlets \( I = 0 \). The \( \pi, \rho, \alpha_0 \) are isospin triplets and the \( \eta, \eta'; \omega, \phi; \epsilon, f_0 \) are
isospin singlets. The strange members of the nonets come in two isospin doublets \( Y = \pm 1 \) and \( I = \frac{1}{2} \). The two isospin doublets are \((K^*, K^0)\) and \((\bar{K}^0, K^-)\), where for the pseudoscalar nonet \( K = K(495) \), for the vector nonet \( K = K^*(892) \) and for the scalar nonet \( K = K^*(880) \).

For the NN-channels one has hypercharge \( Y = 0 \) and two isospin channels \((I = 0, I = 1)\). So the following bosons from each nonet are exchanged.

1. for \( I = 1 \) one meson like the \( \pi, \rho, a_0 \),
2. for \( I = 0 \) two mesons like the \( \eta, \eta', \omega, \phi, \epsilon, f_0 \).

For the description of the YN-channels one also needs to consider the exchange of two isospin doublets \((K^*, K^0), (K^0, K^-)\) hypercharge \( Y = \pm 1 \) and \( I = \frac{1}{2} \) strange mesons like the \( K(495), K^*(892), K^*(880) \). As mentioned earlier the \( I = 0 \) mesons are mixed, due to breaking of \( SU(3) \) symmetry. Therefore a mixing angle \( \theta \) is introduced for every nonet.

\[ \text{Figure 4.5: Two one boson exchange diagrams.} \]

\[ Y = 0 \quad Y \neq 0 \]

\[ \text{direct} \quad \text{exchange} \]

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4.6 Symmetry-breaking

Already at the quark level it is known that $SU(3)$ is badly broken, caused by differences in quark masses. The strange quark is much heavier than the up and down quark. But there are different ways in which $SU(3)$ symmetry is broken.

First the breaking caused by the medium strong interaction which conserves $SU(2)$ symmetry. Which results in different masses for particles with different hypercharge and total isospin.

The $SU(2)$ symmetry is broken by the electromagnetic interaction which splits the isospin multiplets. Also the mass differences between members of the same isospin multiplet break the $SU(2)$ isospin symmetry.

There is another interesting source of isospin breaking. The physical $\Lambda_{\text{phys}}$ is not a pure $I = 0$ state, because the electromagnetic interaction mixes a small part of $\Sigma^0$ in. This results in

$$\Lambda_{\text{phys}} = \cos(\theta)\Lambda + \sin(\theta)\Sigma^0 \quad \Sigma^0_{\text{phys}} = -\sin(\theta)\Lambda + \cos(\theta)\Sigma^0.$$  \hspace{1cm} (4.47)

The $\pi^0$ which does not couple to the bare $\Lambda$, couples to the physical $\Lambda_{\text{phys}}$, because it couples to the $\Sigma^0$. The coupling constants of the $\pi^0$ to the neutron and proton have opposite sign, resulting in a isospin symmetry breaking. The result is a isospin breaking OPE potential in the $\Lambda N$ channel.

4.6.1 $SU(3)$ breaking in Nijmegen Soft-core model

The $SU(3)$ symmetry in the model is broken by solving the multichannel Schrödinger equation on a physical particle basis and including the Coulomb interaction. The nuclear potential is calculated on an isospin basis so one includes only the medium strong $SU(3)$ breaking.

4.7 Nijmegen Soft-core Models in detail

As mentioned earlier all Nijmegen soft-core models have the same general features, but differ in details. In general three meson nonets are exchanged. For every nonet one
4.7. Nijmegen Soft-core Models in detail

has a singlet and a octect coupling constant. Plus a mixing angle and a $\alpha = \frac{F}{F+D}$ ratio gives 4 parameters per nonet. For the vector nonet one also has to distinguish between the electric and magnetic coupling which adds to 7 parameters. For the scalar mesons the mixing angle is not known. For $NN$ one has then no constraints on the coupling constants and therefore one fits the three physical coupling constants. For $YN$ there is $\theta_S$ as additional parameter which has to be fitted. When one also includes the Pomeron $P$ and the $J = 0$ tensor contributions one obtains in the simplest case a single mass parameter for $P; f_2, f'_2, K^*$. Plus two coupling constants for the $I = 0$ and $I = 1$ contributions.

Gaussian form-factors are used at the baryon-baryon-meson vertices, to parameterize the short range interaction. These form-factors are parameterized by so-called cutoff masses, which leads to at least one other parameter. This gives a total of 19 parameters some of which are fixed by theoretical values.

4.7.1 NSC78

As stated earlier the NSC78 is the first soft-core NN potential model of the Nijmegen group. It is calculated in configuration space and all momentum dependence in the invariant potential forms is neglected except for the nonlocal terms in the central potentials. This allows for high computer speed and accuracy and easy inclusion of the Coulomb effects.

For the pseudoscalar mesons the the value $\alpha_P = 0.361$ is taken. For the mixing of the singlet and octet the value of the linear Gell-Mann-Okubo mass formula $\theta_P = -23^\circ$ is used. This leaves the singlet coupling constant $f_\eta$ and the octet coupling constant $f_\pi$ as parameters.

For the vector mesons $SU(3)$ relations for the electric and magnetic coupling are assumed. Furthermore universal coupling of the $\rho$ to the isospin current is assumed, leading to $\alpha^\rho_V = 1$. Again for the singlet-octect mixing angle the linear Gell-Mann-Okubo mass formula is used. This gives for the mixing angle $\theta_V = 37.5^\circ$. The singlet $g_\rho$ and the octet $g_\pi$ coupling constants are parameters. For the magnetic coupling constants no sound theoretical value for $\alpha^\rho_M$ is known. The magnetic coupling constants are given by $g^m = g + f$, but $f_\phi$ cannot be determined, because the fit of the parameters is not sensitive to variations in $f_\phi$. Therefore $f_\rho \equiv 0$. The coupling constants $g_\rho^m$ and $g_\pi^m$ become parameters and together with $g_\phi^m = g_\phi$ they determine $\alpha^\phi_M = 0.449$.

The nature of the scalar mesons is still controversial. The singlet-octet mixing angle is also still an unsolved problem. A value for $\alpha_S$ does not constrain the three coupling constants $g_\omega, g_\eta, g_\phi$ and so they become parameters.
Table 4.2: Meson-nucleon coupling constants.

<table>
<thead>
<tr>
<th>Meson</th>
<th>Mass(MeV)</th>
<th>$g^2/4\pi$</th>
<th>$f^2/4\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>138.041</td>
<td>13.676</td>
<td>$7.566 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>548.8</td>
<td>3.433</td>
<td>$1.899 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\eta'$</td>
<td>957.5</td>
<td>3.759</td>
<td>$2.080 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>770, $\Gamma = 146$</td>
<td>0.795</td>
<td>14.157</td>
</tr>
<tr>
<td>$\omega$</td>
<td>783.9</td>
<td>8.683</td>
<td>0.960</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1019.5</td>
<td>0.099</td>
<td>0</td>
</tr>
<tr>
<td>$a_0$</td>
<td>962</td>
<td>1.632</td>
<td></td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>760, $\Gamma = 640$</td>
<td></td>
<td>22.371</td>
</tr>
<tr>
<td>$f_0$</td>
<td>993</td>
<td>0.704</td>
<td></td>
</tr>
<tr>
<td>$P, f_2, f_2'$</td>
<td>307.81</td>
<td>8.778</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>307.81</td>
<td>0.197</td>
<td></td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>964.52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the Pomeron and the tensor mesons a single mass parameter $m_P$ is used. Two coupling constants for the $I = 0$ and the $I = 1$ contributions are parameters.

One Gaussian form-factor with one cutoff mass is used for all baryon-baryon-meson vertices.

Summarizing there are 11 coupling constants, one Pomeron mass and one universal cutoff parameter. So in total there are 13 parameters. The values of the physical coupling constants are given in Table 4.2

### 4.7.2 NSC89

The NSC89 model is the YN extension of the NSC78 model. It uses 8(6) additional parameters which are fitted to 35 YN data.

The changes compared to the NSC78 model are given below. For the pseudoscalar mesons in stead of fixing $a_{PV}$ it becomes a parameter which is fixed by the YN-fit. The magnetic
4.7. Nijmegen Soft-core Models in detail

<table>
<thead>
<tr>
<th>Mesons</th>
<th>Coupling constants</th>
<th>$\frac{F}{F+D}$-ratios mixing angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudoscalar</td>
<td>$f_n = 0.18455$</td>
<td>$f_\pi = 0.27204$</td>
</tr>
<tr>
<td></td>
<td>$f_\pi = 0.27204$</td>
<td>$\alpha_{PV} = 0.355$</td>
</tr>
<tr>
<td></td>
<td>$\theta_P = -23.00^\circ$</td>
<td></td>
</tr>
<tr>
<td>Vector</td>
<td>$g_{a_1} = 2.52934$</td>
<td>$g_\rho = 0.89147$</td>
</tr>
<tr>
<td></td>
<td>$g_\rho = 0.89147$</td>
<td>$\alpha_V = 1.0$</td>
</tr>
<tr>
<td></td>
<td>$f_{a_1} = 0.97982$</td>
<td>$f_\rho = 3.76255$</td>
</tr>
<tr>
<td></td>
<td>$f_\rho = 3.76255$</td>
<td>$\alpha_V^0 = 0.275$</td>
</tr>
<tr>
<td></td>
<td>$\theta_V = 37.50^\circ$</td>
<td></td>
</tr>
<tr>
<td>Scalar</td>
<td>$g_{a_1} = 3.75548$</td>
<td>$g_{a_0} = 1.27734$</td>
</tr>
<tr>
<td></td>
<td>$g_{a_0} = 1.27734$</td>
<td>$\alpha_S = 1.28555$</td>
</tr>
<tr>
<td></td>
<td>$\theta_S = 40.895^\circ$</td>
<td></td>
</tr>
<tr>
<td>Diffractive</td>
<td>$g_1 = 2.85507$</td>
<td>$g_{a_2} = 0.44372$</td>
</tr>
<tr>
<td></td>
<td>$g_{a_2} = 0.44372$</td>
<td>$\alpha_D = 1.02267$</td>
</tr>
<tr>
<td></td>
<td>$\Psi_D = 15.50^\circ$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Coupling constants, $\frac{F}{F+D}$-ratios mixing angles and cutoff masses, bold entries are from YN-fit, other entries are theoretical input or determined by NN-fit.

The four extra parameters come from the fact that as in NSC78 one general form factor for baryon-baryon-meson vertices is no longer sufficient. The form factors in the NSC89 model are assigned per BB $SU(3)$ irrep. For NN one has then two $SU(3)$ irreps, the 27 for the $J = $ even and the $10^*$ for the $J=odd$ partial waves. These can be fixed in the NN-fit. For YN one has 3 additional irreps the $8_s,8_a$ and the 10. Because of various $SU(3)$ breaking mechanisms the nuclear potentials have to be calculated on a $SU(2)$ isospin basis. States can now become combinations of different irreps. When this was the case a separate form factor was used. Using strict $SU(3)$ symmetry for form factor assignment lead to friction.
between the $\Sigma^+ - p$ and the $\Sigma^- - p$ channel. Fitting the YN-data with one $\Lambda$ for the 10 produces an unobserved bound state in the $^3S_1 \Sigma^+ p$ state. So an $SU(2)$ breaking was introduced for the 10. Also a different value was used for the $\Lambda_{27}$ for YN compared to NN introducing a breaking of $SU(3)$ in the 27 channel. Summarizing four form factors were used: for $\Sigma^+ p$ a $^1S_0$ and a $^3S_1$-form factor, for $\Lambda N = \Sigma N$ a $^1S_0$- and a $^3S_1$-form factor. Because the low energy YN-data is mainly total cross section data, the $S$-waves are the dominant waves for fitting.

### 4.7.3 NSC93

The NSC93 model has fourteen parameters. For the pseudoscalar mesons the four parameters are reduced to one the singlet coupling constant $f_{\eta 1}$. In contrast to NSC78 where the octet coupling constant was fitted, here the octet coupling constant is taken from the partial wave analysis PWA93 as $f_{\pi}^2 = 0.075$. For the other the parameters the same values as in NSC78 are taken. The $F/(F + D)$ ratio is taken as $\alpha_{PV} = 0.355$. For the singlet-octet mixing angle the value $\theta_{PV} = -23^\circ$ is taken.

For the vector mesons nothing is changed compared to NSC78. There are not only electric but also magnetic couplings. This leads to 7 parameters of which 3 are fixed. The 7 parameters are the electric singlet and octet coupling constants $g_{V1}, g_{V8}$, the electric $F/(F + D)$ ratio $\alpha_{V}$, the magnetic coupling constants $f_\rho, f_\phi, f_\omega$ and the singlet-octet mixing angle $\theta_{V}$. It is assumed that the $\rho$ meson is universally coupled to the isospin current ($a_{eV} = 1$, this and $g_{V}$ determines $g_{V8}$. Taking the singlet-octet mixing angle $\theta_{V} = 37.5^\circ$ fixes the physical coupling constants $g_\omega, g_\rho$ in terms of $g_{V1}$ and $g_{V8}$. The magnetic coupling of the $\Phi$ meson $f_\Phi \equiv 0$ Also for the scalar mesons nothing is changed for the fitted coupling constants. The singlet-octet mixing angle is not known so there are no constraints on the coupling constants. This leads to three coupling constants $(a_0, f_0, \epsilon)$ which have to be fitted. Likewise for the Pomeron and tensor mesons nothing is changed. So one fits the Pomeron mass $m_p$ and two coupling constants the $I = 0$ $g_\rho$ and the $I = 1$ $g_{a2}$ coupling constant. The big difference with NSC78 is that in NSC93 three cutoff masses are used. For every meson species a cutoff mass, so one has $\Lambda_P, \Lambda_V, \Lambda_S$ as cutoff parameters. For $NN$ this extra freedom is not needed but for the extension to $YN$ and $YY$ one now needs no extra cutoff parameters as one had in the case of NSC78/89 where cutoff mass were attributed to $SU_F(3)$ irreps.

### 4.7.4 NSC97

The parameters in the pseudoscalar meson sector are completely determined by the $NN$ fit. For the vector mesons 6 different values are taken for $\alpha_{V}^{m}$ and the $NN$ fit determines
4.7. Nijmegen Soft-core Models in detail

Mesons coupling constants

<table>
<thead>
<tr>
<th>Mesons</th>
<th>physical</th>
<th>singlet</th>
<th>octet</th>
<th>$\alpha$</th>
<th>mixing angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudovector(Pseudoscalar)</td>
<td>$f_\pi(g_\pi)$</td>
<td>$f_{\eta_1}(g_{\eta_1})$</td>
<td>$f_{\eta_8}(g_{\eta_8})$</td>
<td>$\alpha_P$</td>
<td>$\theta_P$</td>
</tr>
<tr>
<td>Vector</td>
<td>$g_\rho$</td>
<td>$g_{\omega_1}$</td>
<td>$g_{\omega_8}$</td>
<td>$\alpha_V^e$</td>
<td>$\theta_V$</td>
</tr>
<tr>
<td>Scalar</td>
<td>$f_\rho$</td>
<td>$f_{\omega_1}$</td>
<td>$f_{\omega_8}$</td>
<td>$\alpha_V^m$</td>
<td></td>
</tr>
<tr>
<td>Diffractive</td>
<td>$f_{\delta}$</td>
<td>$f_\epsilon$</td>
<td>$f_{\delta^*}$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Pomeron mass cutoff parameter

$m_P$ | $\Lambda$

the rest of the parameters for the vector mesons. In the scalar meson sector already three physical coupling constants where determined, for $YN$ one has one remaining fit parameter the mixing angle $\theta_S$. The Pomeron and the tensor mesons parameters are completely described by the $NN$ fit. In contrast to NSC93 not one cutoff parameter was taken per meson nonet, but three cutoff parameters per meson nonet. One for the isovector part $\Lambda_8$, one for the isoscalar $\Lambda_1$ and one for the two strange isodoublets $\Lambda_K$. The first two can be determined by the $NN$ fit but the last has to be determined by the $YN$ fit. This leads to three cutoff parameters for the $YN$ fit. To allow for flavour symmetry breaking for the coupling constants a parameter per meson nonet is introduced leading to three extra parameters $\Lambda_P^{fsb}, \Lambda_V^{fsb}, \Lambda_S^{fsb}$ which have to be determined in the $YN$ fit.