Chapter 2

Model calculations

In the first section of this chapter the BUU model [Cas90] will be described. In this framework also the different types of dissipation mechanisms will be explained. In the last two sections the abrasion-ablation code ABRABLA [Gai91] and the statistical-decay code GEMINI [Cha88] will be described briefly. The latter two codes were used to perform acceptance corrections to the data while the results of the BUU code helped to interpret the experimental results.

2.1 BUU

It is the aim of this section to outline the BUU transport equation and in particular focus on those aspects of the BUU-code that are important for the interpretation of the experimental data as will be shown later in this work. The BUU equation is based on the Boltzmann equation, a classical kinetic equation for dilute gases first formulated by Boltzmann in 1872. It describes the distribution function of a dilute gas under the assumption that the particles move under the influence of an external force, except when they collide with each other in which case it is also assumed that only two particles take part in the collision. In 1933 the Boltzmann equation was modified by Uehling and Uhlenbeck to include Pauli blocking. The resulting equation therefore goes by the name BUU. For a more detailed description of the BUU equation and code see e.g. [Ber88, Cas90].

The BUU transport equation is given by

\[
\left[ \frac{\partial}{\partial t} + \mathbf{\vec{v}} \cdot \nabla_r - \nabla_r \mathbf{U} \cdot \nabla_p \right] f_i = I_{col}
\]

(2.1)
\[ I_{\text{col}} = \frac{1}{(2\pi)^3} \int d^3 \vec{p}_2 d^3 \vec{p}_4 d^\Omega \frac{d\sigma}{d\Omega} v_{12} \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \]
\[ \times \left[ f_3 f_4 (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_3)(1 - f_4) \right] \] (2.2)

The left part of the BUU equation 2.1 describes the evolution of the phase-space density distribution, \( f(\vec{r}, \vec{p}, t) \). If there are no collisions between particles, the propagation in time of a given particle, defined as \( f_1 = f(\vec{r}_1, \vec{p}_1, t) \), is described according to Hamilton's equations:

\[ \dot{\vec{p}}_i = -\vec{\nabla} U(\rho(\vec{r}_i)) \] (2.3)
\[ \dot{\vec{r}}_i = -\vec{v}_i \] (2.4)

where \( U \) is an external potential.

In the collision term 2.2 individual nucleon-nucleon collisions are treated. This term contains the difference in scattering rate into and out of a phase-space element \( (\vec{r}, \vec{p}) \), taking Pauli blocking into account via the factors \( (1 - f_i) \). The \( \delta \)-term in the equation accounts for momentum conservation.

### 2.1.1 BUU code

The BUU transport equation cannot be solved analytically. Therefore, the solution is simulated using the test-particle method, where each nucleon is represented by \( N \) test particles, such that there are \( N \) parallel ensembles of \( N_p \) and \( N_t \) test particles (\( N_p \) and \( N_t \) are the number of nucleons of projectile and target). The density distribution is then calculated after each time step \( \delta t \) as an average over all the test particles.

\[ f(\vec{r}, \vec{p}, t) = \frac{1}{N} \sum_{i=1}^{N(N_p+N_t)} \delta(\vec{r} - \vec{r}_i(t))\delta(\vec{p} - \vec{p}(t)) \] (2.5)

The test particles can collide with a cross section \( \sigma_{NN}/N \). For computational reasons the collisions have been constrained to test particles in the same ensemble and for the reaction cross section \( \sigma_{NN} \) is used. If the test particles do not collide they propagate according to the Hamilton equations given above.

For the mean field a Skyrme potential is taken which has the form

\[ U(\rho) = A(\rho/\rho_0) + B(\rho/\rho_0)^\sigma \] (2.6)
where $\sigma > 1$, $A$ is attractive and $B$ is repulsive.

We have run the BUU code for the $^{36}\text{Ar} + ^{159}\text{Tb}$ case at 44 MeV/nucleon for different impact parameters with 1000 test particles per nucleon. For an impact parameter of 8 fm the typical time behaviour is shown in figure 2.1.

![Figure 2.1: Time evolution in coordinate space of a collision of $^{36}\text{Ar} + ^{159}\text{Tb}$ at 44 MeV/nucleon at an impact parameter of 8 fm.](image)

At a time of 100 fm/c after starting the code we will assume that projectile- and target-like fragments have been formed and we will assign the test particles via a simple geometrical cut either to the projectile- or to the target-like fragment. Furthermore, the assumption is made that the interaction between projectile and target stops when the projectile has passed the target which is after 75 fm/c. The collisions taking place after this time will be ignored. (If one would allow the calculation to go on, all test particles will eventually collide and one will not be able to identify a projectile- and a target-like fragment.)

### 2.1.2 Reaction mechanisms

The BUU transport equation consists of two terms, a mean field term (left of equation 2.1) and a collision term (2.2). In the following we will refer to the mechanisms described by these terms as one- and two-body dissipation, respectively. Figure 2.2 gives a schematic representation of both mechanisms. In the top (left) figure a test particle moves from the projectile to the target after having collided once with a target test particle. This type of mechanism we refer to as two-body dissipation. In the top (right) figure a...
projectile test particle is captured by the target without suffering a collision. The reaction mechanism associated with this will be referred to as one-body dissipation. Both dissipation mechanisms can be distinguished by measuring nuclear bremsstrahlung (see the previous chapter) which is known to originate from nucleon-nucleon collisions and therefore is a probe for the two-body dissipation mechanism (the collision term in the BUU equation). The lower two pictures in figure 2.2 show both dissipation mechanisms but then for test particles going from target to projectile.

Figure 2.2: Trajectories of transferred nucleons. The upper two figures show a transfer from projectile to target via two-(left) and one-body (right) dissipation. The lower two figures show a transfer from target to projectile via two-(left) and one-body (right) dissipation. The different dashed circles represent the position of the projectiles at times 25, 50, 75 and 100 fm/c.
2.2 Abrasion-ablation model

The abrasion-ablation model (Abrabla) was used to generate a distribution of projectile-like fragments with their momentum and excitation energy. The ablation (evaporation) part of Abrabla was not used since it does not keep track of the kinematics of the evaporated light particles. Instead the statistical-decay code GEMINI was used which will be explained in the next section.

The fraction \( F \) abraded from the projectile for a given impact parameter is calculated by considering the volume of intersection of a sphere and a cylinder (a cylindrical fraction of the projectile is removed). The expression for \( F \) is shown in detail in the appendix of an article by Gosset \textit{et al.} [Gos77]. The impact parameter is randomly sampled according to the cross section. The mass of the projectile-like fragment is taken as the product of the removed fraction \( F \) and the projectile mass \( A_p \), thus assuming a constant nuclear density.

\[
A_{PLF} = F \cdot A_p
\]  

(2.7)

The next step is to determine the \( A/Z \) ratio of the projectile-like fragment. For this the hypergeometric model is used [Mor78]. Essentially, this model calculates the dispersion in the number of neutrons and protons removed from the projectile as equivalent to the number of ways the nucleons can be distributed in the projectile.

\[
V(Z_{PLF}) = \left( \begin{array}{c} Z_P \\ Z_{PLF} \end{array} \right) / \left( \begin{array}{c} N_P \\ N_{PLF} \end{array} \right) / \left( \begin{array}{c} A_P \\ A_{PLF} \end{array} \right)
\]

(2.8)

\[
V = \sum_{Z_{PLF}} V(Z_{PLF})
\]

(2.9)

The charge of the projectile-like fragment is calculated by taking a fraction of \( V \) (determined in a random way) and then summing over \( V(Z) \) until the fraction of \( V \) is reached. The \( Z \) for which this is the case is taken as the charge of the projectile-like fragment.

After the mass and charge of the projectile-like fragment have been calculated the excitation energy is calculated according to the statistical hole model proposed by Gaimard and Schmidt [Gai91]. In this model the excitation of the projectile-like fragment originates from the creation of a hole. The excitation energy is proportional to the single-particle level density. When more nucleons are removed the excitation energy is calculated as a convolution of these single-particle energies (figure 2.3).
Figure 2.3: Excitation energy distributions as calculated with the statistical hole model.

The distribution of the momentum width for the projectile-like fragment is calculated using the Goldhaber approach [Gol74]. Here, the width of the momentum distribution is a function of the projectile mass and the mass of the projectile-like fragment.

\[ \sigma_p = \sigma_0 \left( \frac{A_{PLF} (A_P - A_{PLF})}{(A_P - 1)} \right)^{1/2} \]  

(2.10)

\( \sigma_0 \) is a constant related to the Fermi momentum and is taken as 118 MeV/c in the model.

At this point the mass, charge, excitation energy and momenta are known for a given projectile-like fragment and these data are inserted in the statistical-decay code GEMINI.
2.3 GEMINI

To calculate the light-charged-particle evaporation of the excited primary projectile-like fragment the statistical decay code GEMINI [Cha88] was used. GEMINI calculates the sequential decay of a compound nucleus via a Monte-Carlo technique. The advantage of GEMINI compared to most statistical decay codes is that it keeps track of the kinematics of the evaporated particles. In figure 2.4 the structure of the code is schematically shown.

Figure 2.4: Schematic diagram of the statistical-decay code GEMINI.

step 1. All possible binary decays of the compound nucleus are calculated. GEMINI treats light-particle ($Z \leq 2$) emission and intermediate-mass-fragment production separately. In the present case only light-particle evaporation was considered. The partial decay widths for the evaporation are calculated using the Hauser-Feshbach formalism.
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step 2. A decay mode is randomly chosen from all the possible binary decays weighted according to the partial decay widths.

step 3. The energy of the light particle is randomly sampled according to a maxwellian distribution, the spin of the light particle is chosen in a Monte-Carlo fashion from the partial decay widths.

step 4. After this evaporation step a new compound nucleus is calculated, considering the decrease in excitation energy and angular momentum and the recoil due to the light-particle evaporation.

step 5. This procedure is repeated until the excitation energy of the compound nucleus is below the particle decay threshold.