Investment evaluation with respect to commercial uncertainty
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Chapter 6

Generalizations of the experiments

In Chapters 4 and 5 the commercial scope, the corresponding robustness and risk measures, and the induced constraints, introduced in Chapter 3, were applied to a robustness analysis and the evaluation of a few given investment alternatives, based on instances of the matchplus model which was described in Chapter 2. These applications are sufficiently relevant to conclude that the commercial scope and the derived characteristics of investment plans can give useful and practicable information in an investment planning context. Yet the example is rather limited, and based on a small and very aggregate model. It is interesting to know to what extent the applicability depends on the simplifications used. To do so, in this chapter some important assumptions at the basis of the model in Chapter 4 and the experiments will be successively generalized. In this way the impact of every assumption is tested separately.

An important aspect of the matchplus model was the quality restrictions. The restrictions used so far are only part of the story. In Chapter 2 it was already indicated that in practice there are more quality requirements concerning, for instance, the methane content. In the next section a lower bound on the methane content of the gas delivered to the L-gas market is introduced, and studied for its implications for the commercial scope and its derived investment criteria.

In Section 6.2 the demand is not, as before, specified per year and distributed over the quarters using a fixed profile, but it is specified per quarter. The number of uncertain variables increases considerably. It is not possible any more to picture the scope in three dimensions. These and other consequences are studied in Section 6.2.

In the next section, the consequences of the introduction of a smaller time scale are addressed. The matchplus model is rather aggregated in time. The smallest time scale considered in the annual matching model is a quarter. In this section this time scale is brought down to months. The demand profile proves to be of particular importance.

In Section 6.4 the matchplus model is extended to contain elements of multiyear planning. Carry forward concerns the opportunity for production
planners to deviate from the annual contracted quantities by 'carrying' a part of it 'forward' into earlier years, where there may be better opportunities to deal with it. In other words, the production planning of one year can be used to influence the production planning problem of the next year. In Section 6.4, this feature is studied for its implications with respect to scenario analysis, the scope, and the robustness and risk measures derived from it.

Finally, some remarks will be made on the generation of investment alternatives using stochastic programming. The objective then is to balance scope-derived robustness measures and investment cost, an aspect which until this point has remained out of range.

In this chapter no attention is paid to nonlinear extensions of the model. These would come up if, for instance, the number of markets is increased, such that different interconnected mixing points occur. In the MATCHPLUS model only two markets and one mixing point was contained. Both the observations in Section 3.7 and related studies by others, ref. Pistikopoulos and Groetschel (1989a) and (1989b), demonstrate that the concepts can as well be applied in nonlinear contexts. If the commercial scope is convex, linear induced constraints can be generated using the same boundary searching techniques, which approximate the scope from outside.

6.1 A methane content lower bound

Some L-gas consuming customers of Gasunie do not use the gas for heating purposes. They use methane as a raw material for some chemical process. Therefore they demand a minimum methane content of the gas delivered to them. L-gas from Slochteren has a high methane content (see Table 2.4). H-gas also has a high methane content, but if it is mixed with nitrogen to fit the L-gas Wobbe bounds, the methane content decreases drastically. The lower bound on the methane content therefore gives rise to an additional upper bound on the amount of H-gas that can be transferred per quarter. In Chapters 4 and 5 it showed that the annual overflow of H-gas into the L-gas system was already restricted from above by the induced constraints (A0), (A1) and (A2).

Other examples of quality restrictions emerging in practice are an oxygen or a carbon dioxide upper bound. All these additional quality indicators show a modelling advantage over the Wobbe index in that they mix linearly (ref. Section 2.2). As an example, the most important additional quality restriction in practice, the lower bound on the methane content of L-gas delivered, will be worked out in this section. The other quality indices can be included likewise.

The methane lower bound can be introduced into the MATCHPLUS model in different ways, depending on the assumptions with respect to the L-gas market. In this section it is assumed that each gas flow contains two types of gas: methane and non-methane. The percentages in all source gas flows are assumed to be known and are given in Tables 2.4 and 2.5. In the first subsection, the methane content lower bound will apply to the entire L-gas market. In the
second subsection, only a part of the L-gas market needs the minimum methane content.

6.1.1 Methane content lower bound for entire L-gas market

Let us assume that all L-gas output is required to meet a lower bound on its methane content. Mathematically, the following restrictions are needed in addition to the MATCHPLUS restrictions.

\[ M_{ph} f_t + M_{pl} p_t + 0n_t + M_{ug} u_t = M_v^* v_t \]
\[ ML \leq M_v \tag{6.1} \]

with:
- \( M_{ph} = \) methane content of H-gas
- \( M_{pl} = \) methane content of L-gas
- \( M_{ug} = \) methane content of gas in storage
- \( M_v = \) methane content of the gas offered on the market
- \( ML = \) lower bound on methane content of L-gas

All methane content parameters are in terms of volume percents. We assume the quality of the storage to be constant and equal to a "mean" output value, see Table 2.5. The output methane content \( m_v \) varies with the amounts of L-gas, H-gas, nitrogen and stored gas produced. The other methane contents are given in Table 2.4.

From (6.1) the variable \( m_v \) can be eliminated to get:

\[ M_{ph} f_t + M_{pl} p_t + 0n_t + M_{ug} u_t \geq ML v_t \]

for \( t = 0, 1, 2, 3, 4 \). These inequalities were added to the MATCHPLUS model restrictions (2.8) to (2.30). In this way no additional variables and only one additional inequality per quarter are introduced. With this model, for all previously considered investment alternatives the directional scope was analysed for the coordinate directions, leaving from \( s^0 \). The results are listed in Table 6.1. A comparison with Table 5.2 shows that, as was to be expected, the methane content lower bound does not affect the directional scopes in other directions than \( DH^- \) and \( ACQ^+ \). In these directions it reduces the scope marginally (\( \leq 1\% \)) under investment alternative \( x^4 \) and a little more under \( x^3 \). Note that these alternatives use underground storage. In case only the nitrogen capacity is increased (alternative \( x^2 \)) a marginal reduction (\( 0.5\% \)) of the scope in the direction \( DH^- \) is revealed.

Obviously, in 5 cases a new induced constraint was encountered. Further study reveals three different new induced constraints are involved. In the remaining part of this section symbolic representations of these induced constraints are described and interpreted. The following notation is used.

\[ \mu_g = W_g - W - \frac{M_g}{ML} (W_U - W) \]
Generalizations of the experiments

Table 6.1. Scope in coordinate directions under different alternatives with methane lower bound for all L-gas market (in 10^6 m^3 35.17)

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$x^0$</th>
<th>$x^1$</th>
<th>$x^2$</th>
<th>$x^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DH^+$</td>
<td>16.80</td>
<td>16.80</td>
<td>16.80</td>
<td>16.80</td>
</tr>
<tr>
<td>$DH^-$</td>
<td>1.18</td>
<td>3.23</td>
<td>8.95</td>
<td>12.52</td>
</tr>
<tr>
<td>$DL^+$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$DL^-$</td>
<td>5.50</td>
<td>15.31</td>
<td>17.40</td>
<td>17.40</td>
</tr>
<tr>
<td>$ACQ^+$</td>
<td>1.30</td>
<td>3.53</td>
<td>9.80</td>
<td>13.88</td>
</tr>
<tr>
<td>$ACQ^-$</td>
<td>24.89</td>
<td>24.89</td>
<td>24.89</td>
<td>24.89</td>
</tr>
</tbody>
</table>

The results differing from those in Table 5.2 are in boldface

for $g = ph, pl, ug$. These parameters have the value:

$\mu_{ph} = 0.255$

$\mu_{pl} = -1.374$

$\mu_{ug} = -0.159$

A direct interpretation of these quantities is not available. The symbols $V_g$, $g = ph, pl, ug$ and $PLL$ are defined on page 107.

1. Under investment alternative $x^1$, when going from scenario $s^0$ in the directions $DH^-$ and $ACQ^+$, in both cases the same induced constraints is encountered:

$$\gamma_{ACQ - DH} \leq$$

$$\frac{C_{ph}}{C_{pl} V_{ph}} \sum_{t=1,2,4} \{(WU - W_{pl}) FL_t : DL + (WU - W_{pl}) \cdot NU\}$$

$$+ \frac{C_{ph}}{C_{pl} (C_{ph} \mu_{pl} - C_{pl} \mu_{pl})} \{C_{pl} \mu_{pl} FL_3 : DL - (C_{ug} \mu_{ug} - C_{pl} \mu_{ug}) UGL\}$$

$$+ \frac{C_{ph}}{C_{pl} V_{ph}} V_{ug} UGL$$

In numerical format, this induced constraint can be described as:

$$-DH - 0.197 DL + 0.9 ACQ \leq 30.248 NU - 0.712 UGL$$

Recall that $UGL < 0$.

2. Under investment alternative $x^3$, when going from the basic scenario $s^0$ in the direction $DH^-$, the following facet-inducing constraint is encountered:
$$\gamma ACQ - DH \leq$$

\[
\frac{C_{ph}}{C_{pl}V_{ph}} \sum_{t=1,4} \{(WU - W_{pt})FL_t \cdot DL + (WU - W) \cdot NU\} \\
+ \frac{C_{ph}}{C_{pl}(C_{ph} \mu_{pl} - C_{pt} \mu_{ph})} \sum_{t=2,3} \{(C_{pt} \mu_{pl} FL_t \cdot DL \\
\quad - (C_{pl} \mu_{pt} - C_{pt} \mu_{pl}) UGL\} \\
+ 2 \cdot \frac{C_{ph}}{C_{pl}V_{ph}} V_{us} UGL
\]

In numerical format:

$$-DH - 0.332 \cdot DL + 0.9 \cdot ACQ \leq 20.166 \cdot NU - 1.425 \cdot UGL$$

3. Under investment alternative \(x^3\), when going from scenario \(s^0\) in the direction \(ACQ^+\) and under alternative \(x^2\) going in the direction \(DH^-\), the following induced constraint is encountered:

$$\gamma ACQ - DH \leq$$

\[
\frac{C_{ph}}{C_{pl}V_{ph}} \sum_{t=1,4} \{(WU - W_{pt})FL_t \cdot DL + (WU - W) \cdot NU\} \\
+ \frac{C_{ph}}{C_{pl}(C_{ph} \mu_{pl} - C_{pt} \mu_{ph})} \{C_{pt} \mu_{pl} FL \cdot DL - (C_{us} \mu_{pl} - C_{pl} \mu_{us}) UGL\} \\
+ FL_3 DL - PLL - C_{us} UGL + 2 \cdot \frac{C_{ph}}{C_{pl}V_{ph}} V_{us} UGL
\]

In numerical format:

$$-DH - 0.349 \cdot DL + 0.906 \cdot ACQ \leq$$

$$-PLL_3 + 20.166 \cdot NU - 1.668 \cdot UGL$$

These induced constraints may be interpreted in a similar way as the induced constraints \((A1)\) and \((A2)\) in Chapter 4, Section 4.3.

the annual overflow from the H-gas system to the L-gas system (the left-hand side) should not exceed an upper bound representing the processing capacity of the L-gas system (the right-hand side).

Like in Section 4.3, the upper bound is built up out of upper bounds on the per quarter H-gas overflows \(f_t\). The upper bounds on \(f_t\) that were calculated in Section 4.3 are still valid. But, because of the methane lower bound, new ones come in. Which one of the upper bounds on \(f_t\) is restrictive in any quarter depends on the circumstances in that quarter, which depend on the scenario
and the investment alternative. One of the new upper bounds is found to be restrictive in some cases in quarters with low demand. This new one depends on the methane content lower bound \((ML)\), the Wobbe index upper bound \((WU)\) and the underground storage inflow restriction \((UGL)\). The abovementioned induced constraints are different combinations of the old per quarter upper bounds and the new one.

It is natural to find the methane lower bound restrictive in quarters with low demand, since in such quarters, due to H-gas production obligations, most L-gas will stem from H-gas production. This must be suppleted with nitrogen, to adjust the Wobbe index, which also brings down the methane content. The H-gas that cannot be sold for this reason can to some extent be absorbed by the underground storage. Due to the annual storage balance \((2.20)\), this must be compensated for by a flow out of storage and therefore a lower H-gas overflow in another quarter, which accounts for the complicated terms involving \(UGL\).

### 6.1.2 Lower bound on methane content for a part of the L-gas market

As a generalization of the previous subsection, the methane lower bound is assumed to hold for only a part of the L-gas market. Assume that a fixed part \(\phi\) of the whole L-gas market must satisfy the methane lower bound. Note that if \(\phi = 1\), the lower bound holds for all L-gas demand and the situation of the previous subsection comes up, whereas if \(\phi = 0\), the model has no methane lower bound and is identical to \textsc{matchplus}. In words, the following restrictions are needed:

- for the corresponding submarket, the lower bound on methane must be satisfied,
- the amount of methane and the amount of other gases in the output of both submarkets must be nonnegative.

To model these restrictions, some new variables are introduced into the model of Chapter 4. The production of L-gas \((pl_t)\) and nitrogen \((n_t)\), the H-gas overflow to the L-gas system \((f_t)\), the net flow out of the storage \((ug_t)\) and the output volume \((v_t)\) are duplicated, and indexed by a \(j \in \{1, 2\}\). The variables indexed by \(j\) describe the input flows of the L-gas submarket \(j\). The production capacity restrictions on the input flows still hold, but now do apply to the total flow. For instance the nitrogen capacity restrictions \((2.16)\) become:

\[
0 \leq n_{t1} + n_{t2} \leq NU
\]

It is assumed that the input flows have the same quality properties for both markets since they stem from the same source. The output qualities may differ between the two markets. The contractual Wobbe index bounds \((2.11)\) and \((2.12)\) are duplicated to hold for both markets separately:

\[
W_{ph} f_{tj} + W_{pl} pl_{tj} + W_{n} n_{tj} + W_{ug} uq_{tj} - WU v_{tj} \leq 0 \quad j = 1, 2
\]
and

\[ W_{ph}f_{ij} + W_{pl}p_{ij} + W_{n}n_{ij} + W_{ug}u_{ij} - W_{L}v_{ij} \geq 0 \quad j = 1, 2 \]

Both markets need their own balances so we get two L-gas volume balances (2.9):

\[ f_{ij} + p_{ij} + n_{ij} + u_{ij} - v_{ij} = 0 \quad j = 1, 2 \]

and L-gas energy balances (2.10):

\[ C_{ph}f_{ij} + C_{pl}p_{ij} + C_{ug}u_{ij} = F_{L1j} \cdot C_{pl} \cdot DL \quad j = 1, 2 \]

If \( j = 1 \) is assumed to indicate the submarket with the methane content restriction, \( F_{L1} = \phi \cdot F_{L} \) and \( F_{L2} = (1 - \phi) \cdot F_{L} \).

The introduction of these new variables and the duplication of the restrictions does not alter the model: it is still equivalent to MATCHPLUS. The following restrictions are essentially new elements, which do actually change the model. Firstly, the nonnegativity constraints on the amounts of methane and non-methane gas in both markets are added:

\[
M_{ph}f_{ij} + (1 - M_{ph})f_{ij} + M_{pl}p_{ij} + (1 - M_{pl})p_{ij} + M_{ug}u_{ij} + (1 - M_{ug})u_{ij} \geq 0
\]

for \( j = 1, 2 \). These nonnegativity constraints are nontrivial as a result of the presence of the underground storage. The lower bounds on the other alternatives are all nonnegative.

Finally, for submarket \( j = 1 \) the methane restriction must also be satisfied:

\[
(M_{ph} - ML)f_{11} - MLn_{11} + (M_{pl} - ML)p_{11} + (M_{ug} - ML)u_{11} \geq 0
\]

This formulation was explained in Section 6.1.1.

The resulting model is analysed for the directional scope in the coordinate directions leaving from scenario \( s_0 \), for different values of \( \phi \). Only the third investment alternative was studied, because, in the previous subsection, that alternative showed the largest differences between the model with and the model without the methane lower bound. The results can be read from Table 6.2.

From Table 6.2 it can be seen that L-gas demand in the submarket with the methane restriction must be larger than 95% of the total L-gas demand to have any effect on the scope in the directions tried. Such large values of \( \phi \) are not realistic.

Conclusions

Both a total and a partial methane lower bound can easily be incorporated into the model, where particularly the partial methane lower bound goes at the cost
of a considerable increase in the problem size. The scope is only changed if the methane lower bound concerns a high percentage of the L-gas market, say for $\phi > 90\%$. Such values are not realistic.

At these high values some new induced constraints involving the methane lower bound get irredundant and dominate the constraints $(A0)$, $(A1)$ and $(A2)$ in the directions $ACQ^+$ and $DH^-$. The new induced constraints resemble $(A0)$, $(A1)$ and $(A2)$ and have an analogous interpretation. Apparently the methane lower bound in some cases sets a stronger upper bound to the H-gas overflow, both per quarter and per year. But then again, only at unrealistic values of $\phi$.

### 6.2 Uncoupling demands per quarter

To this point, in the experiments two types of demand were considered, namely for H-gas and L-gas. Both types of demand have been given per year. Demand per quarter, and maximum daily demand for the production capacity test, were derived as a fixed fraction of annual demand. Only in Chapter 4, Section 4.6.1, these fractions were varied in a sensitivity analysis. This analysis showed that the robustness results derived in Chapter 4 are very sensitive to the value of these fractions.

In this section, demands per quarter will be considered as separate uncertain variables. That is, the commercial scenario vector is extended with elements for the demand per quarter. The relation of per quarter demand to annual demand is not given any more by a fixed fraction. There are other ways to express the undeniable relationship. It can be expressed by considering special non-coordinate directions in the boundary searches described in Section 3.4. In stochastic computations the relationship is properly expressed.

### Table 6.2. Scope in coordinate directions under alternative $x^3$, with a varying fraction of the L-gas market satisfying the methane lower bound (in m$^3$ 35.17)

<table>
<thead>
<tr>
<th>Direction</th>
<th>Fraction of L-gas in submarket 1 ($\phi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$DH^+$</td>
<td>16.80</td>
</tr>
<tr>
<td>$DH^-$</td>
<td>13.19</td>
</tr>
<tr>
<td>$DL^+$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$DL^-$</td>
<td>17.40</td>
</tr>
<tr>
<td>$ACQ^-$</td>
<td>24.89</td>
</tr>
</tbody>
</table>
by a positive correlation between annual demand and demand per quarter or between demand in different quarters, either for each type of gas separately or also across the gas types. Namely both the demand for L-gas and the demand for H-gas are correlated with temperature and business cycle. We do not further go into this point, since in the sequel we shall only perform directional searches in the coordinate directions.

6.2.1 The production capacity test uncoupled

As an example, in this subsection assumptions for demand in period 0 (the production capacity test) will be uncoupled from the assumptions for demand in the other periods (the annual matching restrictions). Demand in periods 1 to 4 still is assumed to be a fixed fraction of annual demand. The commercial scenario vector then contains elements for annual demand of both types and of period 0 demand of both types:

\[
\begin{pmatrix}
DH_0 \\
DH \\
DL_0 \\
DL \\
ACQ
\end{pmatrix}
\]

The restrictions of the model are the same as described in Chapter 2, albeit that in the H-gas balance of period 0 (2.21) one should read \(DH_0\) instead of \(FH_0\), \(DH\), and in the L-gas energy balance in period 0 (2.23) \(DL_0\) instead of \(FL_0\). In Section 2.6 the MATCHPLUS model is described in matrix notation as:

\[
Cy_t \odot D_t s + Ex, \text{ for } t = 1, \ldots, 4
\]

\[
\sum_{t=1}^{4} F y_t = F \sum_{t=1}^{4} y_t = \odot_{agg} G s + Hx
\]

where the matrices were given in Table 2.7.

For our present purposes matrix \(G\) is extended with two zero columns. The matrices \(D_0\) and \(D_t, \ t = 1, 2, 3, 4\) are changed to be as follows:
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\[
D_0 = \begin{pmatrix}
C_{pl} / C_{ph} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{pl} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

and, for \( t = 1, 2, 3, 4 \):

\[
D_t = \begin{pmatrix}
0 & (C_{pl} / C_{ph})FH_t & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C_{pl}FL_t & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

The other matrices remain unchanged.

For the basic scenario \( s^0 \), we use the data of Chapter 2, with this extension that in the basic scenario \( DH_0 = FH_0 \), \( DH = 9.24 \) and \( DL_0 = FL_0 \), \( DL = 42 \). The directional scopes in the coordinate directions are given in Table 6.3. The directions \( DH-, DL+, DL-, ACQ+ \) and \( ACQ- \) have not been included,
6.2 Uncoupling demands per quarter

Table 6.3. Directional scopes and induced constraints in the model with period 0 uncoupled, as far as they differ from the original model.

<table>
<thead>
<tr>
<th>Direction</th>
<th>$x^0$</th>
<th>$x^1$</th>
<th>$x^2$</th>
<th>$x^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DH^+$</td>
<td>29.40 (E)</td>
<td>29.40 (E)</td>
<td>29.40 (E)</td>
<td>29.40 (E)</td>
</tr>
<tr>
<td>$DH_0^+$</td>
<td>7.39 (B)</td>
<td>7.39 (B)</td>
<td>7.39 (B)</td>
<td>7.39 (B)</td>
</tr>
<tr>
<td>$DH_0^-$</td>
<td>16.87 (F)</td>
<td>16.99 (F)</td>
<td>20.30 (F)</td>
<td>20.42 (F)</td>
</tr>
<tr>
<td>$DL_0^+$</td>
<td>$+\infty$</td>
<td>$+\infty$</td>
<td>$+\infty$</td>
<td>$+\infty$</td>
</tr>
<tr>
<td>$DL_0^-$</td>
<td>40.71 (G)</td>
<td>42.01 (G)</td>
<td>40.71 (G)</td>
<td>42.01 (H)</td>
</tr>
</tbody>
</table>

The boundary searches underlying Table 6.3 revealed four new induced constraints. They are named (E), (F), (G) and (H). In Table 6.3 it is indicated at which directional searches they were encountered. Before describing them, we like to remark that it is not surprising that new induced constraints come up. As has been argued in Appendix 3B, there are many extremal rays of the dual feasible cone, all leading to induced constraints, but only a few will be irredundant because of the low dimension of $s$ and $x$. In the current model, the dimension of $s$ is increased, so that higher values are not very likely.

The induced constraints (E), (F), (G) and (H) are described below.

1. $DH \leq \gamma ACQ$  

   Annual H-gas demand must not exceed annual H-gas production capacity. Clearly this is a natural constraint. It cannot be influenced by
Generalizations of the experiments

investments and thus constructs a part of the boundary of the ‘super
scope’. The constraint is not very restrictive. Compared to the basic
scenario $s^0$, the situation has to be dramatically different before it will
be violated.

2. \[
\frac{C_p}{C_{ph}} V_{ph} (\alpha \gamma ACQ - DH_0) \leq \\
(WU - W_{pl}) DL_0 + (WU - W) NU + V_{ug} UGL
\]  \hspace{1cm} (F)
Parameters $V_{ph}$ and $V_{ug}$ are given on page 107. The left-hand
side of this constraint is a lower bound and the right-hand side an upper
bound on $f_0$, as was derived in Section 4.3. It forbids the situation
where the minimum H-gas overflow of period 0 alone cannot be pro-
cessed any more.

3. \[
C_p DL_0 \geq \\
C_p PLLS + C_p (1 - \gamma) ACQ + C_{ug} UGL
\]  \hspace{1cm} (G)

The demand for energy by the L-gas market in period 0, expressed by
the left-hand side of this constraint, must not get below the minimum
energy production level, as expressed in the right-hand side. Note that
$UGL \leq 0$ so that if $UGL$ is low enough, the right-hand side gets neg-
ative and the induced constraint is trivial.

4. \[
C_p (W_{ph} - W_{pl}) DL_0 \geq \\
(C_p W_{ph} - C_{ph} W_{pl} - C_{ug} (W_{ph} - W_{pl})) \\
+ W_{ug} (C_{ph} - C_{pl}) UGL
\]  \hspace{1cm} (H)

This induced constraint is made redundant by the natural constraint
$DL_0 \geq 0$ (the coefficients of $DL$ and $UGL$ are positive, whereas $UGL$
itslf is nonnegative). Notice that the directional scope in the direction
$DL_0$—is greater than the value of $DL_0$ under the basic scenario.

Of course, $(B)$ is now to be interpreted as:

\[
\beta_0 \gamma ACQ \geq DH_0
\]

instead of the expression on page 111.

If in the new induced constraints $DH_0$ is replaced by $FH_0 \cdot DH$ and $DL_0$
by $FL_0 \cdot DL$, the resulting constraints are indeed induced constraints on the
original commercial scope of dimension three. In the original model, however,
they are redundant since they are dominated by the induced constraints that
were found in preceding chapters.
Table 6.4. Central scenario values in the uncoupled model

<table>
<thead>
<tr>
<th></th>
<th>( t = 0 )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DH_t )</td>
<td>9.24</td>
<td>6.51</td>
<td>4.83</td>
<td>3.78</td>
<td>5.88</td>
</tr>
<tr>
<td>( DL_t )</td>
<td>42</td>
<td>21.84</td>
<td>10.08</td>
<td>6.73</td>
<td>17.36</td>
</tr>
<tr>
<td>( ACQ )</td>
<td>56</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.2 Uncoupling demands per quarter

6.2.2 All quarterly demands uncoupled

In this subsection all quarterly demands are uncoupled. As a result, the annual demand disappears from the model. The commercial scenario vector then contains elements for both types of demand for all five periods:

\[
\begin{pmatrix}
DH_0 \\
DH_1 \\
DH_2 \\
DH_3 \\
DH_4 \\
DL_0 \\
DL_1 \\
DL_2 \\
DL_3 \\
ACQ
\end{pmatrix}
\]

The matrices \( G \) and \( D_t, \ t = 0, 1, \ldots, 4 \) are extended accordingly. The basic scenario \( s^0 \) is given by taking \( DH_t = FH_t \cdot DH \) and \( DL_t = FL_t \cdot DL \) where \( DH \) and \( DL \) are the central values used in the preceding chapters. For the ease of survey we present the central scenario \( s^0 \) in Table 6.4.

Again, some directional scopes are computed. The directional scopes are only given for the investment alternative \( x^1 \). The results of the experiments are given in Table 6.5. No essentially new facets determining induced constraints are encountered. The abovementioned constraint \((B)\) is now called \((B_0)\) to contrast it with analogous constraints for the other periods: \((B_1), (B_2), (B_3)\) and \((B_4)\). For instance, \((B_2)\) is given by:

\[
\beta_\gamma ACQ \geq DH_2
\]

The other yet unknown induced constraint, which is denoted by \((A2')\), very much resembles \((A2)\). Both give an upper bound on the annual overflow of H-gas to the L-gas system, constructed on the same two quarterly upper bounds. For both induced constraints, both quarterly upper bounds are active.
Table 6.5. Directional scopes and induced constraints in the model with all periods uncoupled, under alternative $x^0$.

<table>
<thead>
<tr>
<th>Direction</th>
<th>$t = 0$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DH_f^+$</td>
<td>7.39 ($B_9$)</td>
<td>14.66 ($B_1$)</td>
<td>16.34 ($B_2$)</td>
<td>17.39 ($B_3$)</td>
<td>15.29 ($B_4$)</td>
</tr>
<tr>
<td>$DH_f^-$</td>
<td>16.87 ($F$)</td>
<td>1.18 ($A1$)</td>
<td>1.18 ($A1$)</td>
<td>1.18 ($A1$)</td>
<td>1.18 ($A1$)</td>
</tr>
<tr>
<td>$DL_t^+$</td>
<td>$+\infty$</td>
<td>$+\infty$</td>
<td>$+\infty$</td>
<td>$+\infty$</td>
<td>$+\infty$</td>
</tr>
<tr>
<td>$DL_t^-$</td>
<td>40.71 ($G$)</td>
<td>11.00 ($A1$)</td>
<td>2.13 ($A2$)</td>
<td>1.18 ($A2$)</td>
<td>8.63 ($A2'$)</td>
</tr>
<tr>
<td>$ACQ^+$</td>
<td>1.30 ($A1$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ACQ^-$</td>
<td>24.89 ($B_0$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

in two quarters. Only, where for ($A2$) the upper bound involving $PLL_s$ is most restrictive in quarters 2 and 3, for ($A2'$) it is in quarters 3 and 4. Apparently, L-gas demand in quarter 4 can be less than the basic scenario L-gas demand in quarter 2 before the boundary of the scope is reached.

It is somewhat surprising to see that variation of the demands in the quarters 1 to 4 does not give rise to new (types of) induced constraints. Other striking facts are that the uncoupling does not have effect on the directional scopes that result from varying $ACQ$. Due to the fact that the restrictions in period 0 and the other periods are completely independent, uncoupling the demands in the quarters does not have any effect on the directional scopes that result from varying the demand in period 0. Remark that for increasing values of H-gas demand in any of the quarters, the directional scope is exactly equal. Apparently the upper bound on the annual H-gas demand per year following from induced constraint ($A1$) dominates eventual upper bounds per quarter. In fact, all induced constraints in the ($A$)-series are a function of $DH$ rather than $FH, DH$. It also holds for the quarterly L-gas demands that the ($A$)-series induced constraints impose upper bounds on the possible demand increases. However, only ($A0$) is a direct function of annual L-gas demand $DL$, and all other constraints have different coefficients for the demands in the different quarters. This is the reason why the L-gas demand directional scopes do not work out so uniform as the H-gas demand directional scopes do.

6.2.3 Conclusions on increasing numbers of uncertain variables
In general, if the number of uncertain variables $n_s$ increases, it is less attractive to perform a thorough robustness analysis based on the commercial scope, as was done in the preceding chapters.

It will not be possible to depict the scope if the number of uncertain variables is more than three. And even in three variables, a picture of the scope is not easily accessible.
To explore the boundary of a more-dimensional scope more boundary searches are necessary. There will be more induced constraints, and there will be more redundant, ref. Table 6.5: for the same alternative $x^0$ there are ten induced constraints instead of three. Furthermore, it is not so easy any more to check whether these ten sufficiently describe that scope. In Chapter 4 this was done by ad hoc directional boundary searches, and by checking for some samples of scenarios whether they satisfy the given induced constraints and yet are infeasible. Such checks are respectively impossible and too toilsome for higher dimensions.

If the extreme scenarios for the completely uncoupled model would be constructed in the same way as in Chapter 4, there will be $2^{11} = 2048$ of them in the completely uncoupled model. The quintessence is that there are horribly many directions in $\mathbb{R}^{11}$, there are $2^{11}$ orthants, each having dimension 11. Through the extreme scenarios we still would have only one representative in each orthant. So it is impossible to achieve a complete description of the scope through directional boundary searches. It is impossible to check enough directions, so that a strategic selection of directions will be necessary. For instance one could make use of dependencies, that is, bring the greater number of uncertain variables back to a smaller number of underlying basic uncertainties.

To conclude, one might say that the analysis of induced constraints (be it in a symbolic form or not) is particularly informative if the number of uncertain variables is low. That is, if one is willing to bring the uncertainty of the commercial future down to a few major indicators.

If the number of uncertain variables increases, the heuristic boundary searching techniques are still useful. However, it is practically impossible to aim at a complete description of the commercial robustness. For increasing numbers of uncertain variables, one should be satisfied with a less complete description, where one can still use the same robustness indicators. In the end, one may end up with using one or two of them, for instance the reliability and the infeasibility risk.

### 6.3 Smaller subperiods

To this point, the smallest time scale considered in this part of the model was a quarter. In this section this is brought down to a month. That is, in this section one planning year is considered to have twelve subperiods $t = 1, \ldots, 12$. Like in the foregoing, the production capacity test is described by an artificial period 0. First we describe how the model MATCHPLUS should be adapted to represent the finer time scale. Then we describe some results of the monthly model.
6.3.1 Adaptation of the model

If we use a smaller time scale, the model of the production planning program still has the same structure: find a vector $y$ such that the linear restrictions $Ky \leq Ls + Mx$ are satisfied. To adapt the model such that it represents the finer time scale, the production decision vector $y$ and the number of restrictions are extended, and the value of some elements of the coefficient matrices is changed. The commercial scenario $s$ and the investment decision vector $x$ remain unaltered. The production decision vector $y$ is extended in the following way:

$$y := \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{12} \end{pmatrix} \quad \text{with} \quad y_t := \begin{pmatrix} ph_t \\ f_t \\ p1_t \\ n_t \\ u_g_t \\ v_l_t \end{pmatrix}$$

Consequently, $y \in \mathbb{R}^{78}$. Note that the production decisions per month are the same as those per quarter. The structure of the restrictions per month is essentially equal to that of the restrictions per month. Only the values of some matrix coefficients concerning production capacity and demand fractions have to be adjusted. The MATCHPLUS model of the production planning problem is built up out of two parts: the annual matching restrictions (periods 1 to 12) and the production capacity test (period 0).

The annual matching problem

In the annual matching part of the MATCHPLUS model, represented by restrictions (2.8) up to (2.20), one can distinguish restrictions per subperiod $t$, described by (2.8) through (2.17), and the aggregating restrictions, described by (2.18), (2.19) and (2.20).

In the twelve period model the restrictions per subperiod are as described in Chapter 2, although the index $t$ now reflects months instead of quarters. The aggregating restrictions are also as described in that chapter, albeit that the summations run over $t \in \{1, \ldots, 12\}$. As in Chapter 2 (see (2.32) and (2.33)) the restrictions per period can be written as the following system of linear equalities and inequalities:

$$Cy_t \otimes Djs + Ex, \quad \text{for } t = 1, \ldots, 12$$

and the aggregating restrictions as:

$$F \sum_t y_t \otimes agg Gs + Hx$$
where all matrices but $\tilde{E}$ are given in Table 2.7, along with the vectors $\diamond$ and $\diamond_{agg}$. Notice that, as in the four period model, all matrices but $D_t$ are independent of the subperiod $t$.\footnote{The twelve period model now has $12 \times 14 + 4 = 172$ restrictions and $12 \times 6 = 72$ variables. (This reduces to $12 \times 11 + 2 = 134$ resp. $12 \times 3 - 2 = 34$ if variables are eliminated as much as possible using the equalities). This is considerably larger than the original model. Yet it was no problem to run our Turbo Pascal routines on it, using an MS-DOS machine with 4 MByte active memory, albeit that it was necessary to use the ‘32-bits extension’ of Turbo Pascal.}

The production decision vector $x$ describes some production capacities per quarter. In the twelve period model all these elements could be divided by three to give the capacities per month. However we choose to leave the investment alternatives untouched, still describing production capacities per quarter, and instead to adapt the coefficients of matrix $E$ (together with matrix $H$ generating matrix $M$). Matrix $H$ remains unchanged, since it describes annual production capacities. Matrix $\tilde{E}$ then is given as $\frac{1}{3} E$, with $E$ given in Table 2.7.

The values of some parameters of the matrices $C$ and $D_t$, given in Table 2.7, change according to the reduction of the subperiods. All quality parameters remain the same. The parameters $\alpha$ and $\beta$, relating minimum and maximum non-Slochteren production per subperiod to the annual production obligations, should be divided by three. The new values of these parameters are given in Table 6.6. The demand profiles of H- and L-gas have to be adapted as well to give the percentages of annual demand per month. Different demand profiles were tried. Details are given in the next subsection.

The production capacity test

As before, period zero represents a day with extremely high demand. The test for feasibility of the production planning in such a day we called the production capacity test. The production capacity test part of the model MATCHPLUS is given in the restrictions (2.21) through (2.30). The parameters of these restrictions were given so as to describe a day with extremely high demand and a certain non-Slochteren gas production capacity unavailability. In the quarterly model this extreme day was ‘blown up’ to the level of a quarter, the artificial period 0. To this end, all demands and capacities on this extreme day were multiplied by 91. To incorporate the production capacity test in the monthly model, the extreme day is ‘blown up’ to the level of a month by dividing all appropriate parameters of the period 0, as they were given in Chapter 2, by three. Using the matrix notation of Section 2.6, the restrictions of period 0 still can be written as the following system of equalities and inequalities:

$$C y_0 \diamond D_0 s + \tilde{E} x,$$

where the matrices $C$ and $D_0$ and the vector $\diamond$ are given in Table 2.7, albeit that the value of some parameters in $D_0$ change. These are given in Table 6.6.
Table 6.6. Values of some parameters in the twelve period model differing from the four period model

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.0292</td>
</tr>
<tr>
<td>$\beta_1 = \ldots = \beta_4$</td>
<td>0.1413</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>0.11</td>
</tr>
<tr>
<td>$FH_0$</td>
<td>0.15</td>
</tr>
<tr>
<td>$FL_0$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

As in the annual matching part of the model, matrix $\tilde{E} = \frac{1}{4}E$, for $E$ specified in Table 2.7.

6.3.2 Results

More than the quarterly demand profile the monthly profile is subject to uncertainty and variations. Different profiles should therefore be considered. We analysed two different profiles, to be described below. From the calculations it appeared that the results strongly depend on the profile used. Remark that the scenario variables are unaltered.

The flat profile

To relate the monthly model to the quarterly model, we first study the ‘flat’ profile: the demand fraction in every month is equal to one third of the fraction of the corresponding quarter, according to the quarterly profile used in the preceding.

$$FH_1 = FH_2 = FH_3 = 1/3 \cdot 0.31 = 0.103$$
$$FH_4 = FH_5 = FH_6 = 1/3 \cdot 0.23 = 0.077$$
$$FH_7 = FH_8 = FH_9 = 1/3 \cdot 0.18 = 0.066$$
$$FH_{10} = FH_{11} = FH_{12} = 1/3 \cdot 0.28 = 0.093$$

$$FL_1 = FL_2 = FL_3 = 1/3 \cdot 0.39 = 0.130$$
$$FL_4 = FL_5 = FL_6 = 1/3 \cdot 0.18 = 0.060$$
$$FL_7 = FL_8 = FL_9 = 1/3 \cdot 0.12 = 0.040$$
$$FL_{10} = FL_{11} = FL_{12} = 1/3 \cdot 0.31 = 0.103$$

It appears that under this profile, the scope, in terms of annual demand and annual contracted production quantity, remains exactly the same. Even the induced constraints do. The directional scopes are given in Table 5.2. Induced constraints were encountered as given in Table 5.5. Induced constraints (A0), (A1), (A2), (B) and (D) are as given in Chapters 4 and 5, albeit that the quarterly fractions should be read as the sum of the corresponding monthly
fractions. For instance, instead of quarterly fraction $FL_3$ one should read the sum of the monthly fractions $FL_7 + FL_8 + FL_9$ etc.

Apparently, a monthly model of which all monthly fractions within each quarter separately are equal, gives exactly the same scope as the quarterly model of which the quarterly fractions are just the sums of the monthly fractions per quarter.

A more realistic profile

A more realistic monthly demand profile is given in Table 6.7 and Figure 6.1. It has a smoother path and it shows stronger fluctuations. Still, the sums per quarter of the monthly fractions are equal to the previously used quarterly fractions. In other words, this profile can be seen, like the ‘flat’ profile, as a special case of the quarterly profile previously used.

The results of directional boundary searches in the coordinate directions are given in Table 6.8. The directional scopes that differ from their counterparts in the quarterly model, given in Table 5.2, are boldface typed.

Clearly, under this profile, in the ‘risky’ directions $ACQ^+, DH^-$ and $DL^-$ the scope is even reduced by some 85% ! The directional scopes are very small: the central scenario $s^0$ lies practically on the boundary of the scope $S(x^0)$. A small deviation from the profile as given in Table 6.7 (for instance: $FL_3 = 0.12$ and $FL_9 = 0.05$) makes the central scenario $s^0$ infeasible. Under $x^1$ in these directions the directional scopes are reduced by 25% up to 30%, so that the effectiveness of this investment alternative is rather reduced. In absolute terms,

Table 6.7. Demand profiles per month

<table>
<thead>
<tr>
<th>period t</th>
<th>$FH_t$</th>
<th>$FL_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>5</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>6</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>7</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>9</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>10</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>11</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>12</td>
<td>0.11</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Generalizations of the experiments

Figure 6.1. Monthly demand fractions

the decrease of scope in direction $DL-$ under the alternatives $x^0$ and $x^1$ is alarming.

In Table 6.8, beside the directional scopes in the coordinate directions, the induced constraints that were encountered during the boundary searches have been given as well. In comparison to the quarterly model, see Table 5.5, it can be seen that no new results have been found in the directions $ACQ-$, $DH+$ and $DL+$. The same holds in the direction $DL-$ under alternatives $x^2$ and $x^3$. One ends up (if at all) on facet $(B)$ or $(D)$, which were already known before. Since both $(B)$ and $(D)$ bear no relation to the subperiod structure of the annual gas flow matching part of the MATCHPLUS model, this indifference

Table 6.8. Directional scopes and facets under different alternatives with twelveperiod model

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$x^0$</th>
<th>$x^1$</th>
<th>$x^2$</th>
<th>$x^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DH+$</td>
<td>16.80</td>
<td>(B)</td>
<td>16.80</td>
<td>(B)</td>
</tr>
<tr>
<td>$DH-$</td>
<td>0.18</td>
<td>(Aiv)</td>
<td>2.24</td>
<td>(A)</td>
</tr>
<tr>
<td>$DL+$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$DL-$</td>
<td>0.92</td>
<td>(Aiv)</td>
<td>11.61</td>
<td>(Aiii)</td>
</tr>
<tr>
<td>$ACQ+$</td>
<td>0.20</td>
<td>(Aiv)</td>
<td>2.48</td>
<td>(A)</td>
</tr>
<tr>
<td>$ACQ-$</td>
<td>24.89</td>
<td>(B)</td>
<td>24.89</td>
<td>(B)</td>
</tr>
</tbody>
</table>


result is not surprising. However, the other directional searches in Table 6.8, previously ending up at the faces (A0), (A1) or (A2), give rise to other induced constraints. We found five new constraints, to be denoted by (Ai), (Aiii), (Aiv), (Av) and (I).

The first four constraints describe upper bounds on the annual H-gas overflow, just as (A0), (A1) and (A2) did in the quarterly model. For those constraints, the relevant summations of upper bounds on the H-gas overflow per quarter, given by (4.6) and (4.7), resulted in the upper bound on the annual H-gas overflow. Now, analogously, the annual upper bounds are the sum of upper bounds of the type (4.6) and (4.7), this time however holding per month. The constraints are described by

\[ \gamma ACQ - DH \leq \]
\[ \frac{C_{ph}}{C_{pl} V_{ph}} \sum_{t \in T} \left\{ (WU - W_{pl})FL_t \cdot DL + (WU - W) \cdot NU \right\} + \sum_{t \in T} \left\{ FL_t \cdot DL - PLL - \left( \frac{C_{ug}}{C_{pl}} - \frac{C_{ph} V_{ug}}{C_{pl} V_{ph}} \right) UGL \right\} \]

where

\[ T = \begin{cases} 
\{7\} & \text{for (Ai),} \\
\{6, 7, 8\} & \text{for (Aiii),} \\
\{6, 7, 8, 9\} & \text{for (Aiv),} \\
\{5, 6, 7, 8, 9\} & \text{for (Av).} 
\end{cases} \]

For the \( (A)\)-series, like as in the quarterly model, which one of the monthly upper bounds is most restrictive in a month depends on the conditions in that month, especially on the demand for L-gas. And, as in the quarterly model, the use of the underground storage in some month is only possible if compensated with a reverse flow in the other months, inflicting the upper bounds in these months so that, in the formulation of the induced constraint, a correction term is necessary. Upper bound (4.7), involving \( PLL_s \) and \( UGL \), is most restrictive in those months with low L-gas demand \( (t \in T) \), whereas the other upper bound is most restrictive in months with higher L-gas demand \( (t \notin T) \).

Constraint (I) is described by:

\[ C_{pl}(DH_t + DL_t) \geq C_{pl}(PLL_s + \alpha ACQ) + C_{ug} UGL \]

This constraint concerns the month with minimum demand. It states that total energy demand in that month must not get below minimum monthly production obligations.
6.3.3 Conclusions on smaller subperiods

A monthly model is easily constructed, using the building blocks that were previously used. The model of course is larger, but not essentially more complex. The induced constraints under the monthly model are not essentially more difficult than under the quarterly model. Only one really new induced constraint, named (I), was found.

The results of the experiments with a monthly model put the results of the previous chapters in a different light. Apparently, the aggregation over time into quarters makes results overly optimistic. The quarterly model corresponds to the monthly model with the examined flat profile, where all fractions are ‘averaged out’ over the quarters. The quarterly model insufficiently captures the usual seasonal fluctuation of demand, which is an important determinant of the annual matching problem. If a realistic monthly profile is applied, the annual matching problem is strongly affected. Apparently, the annual matching problem is worsened as the variation in especially the L-gas demand profile increases.

Another question is whether the monthly model yet has enough detail to give sufficiently realistic results. Demand fluctuations are still ‘averaged’ over each month. Profiles on a lower aggregation level could show even more dramatic demand fluctuations and therefore an even worse picture.

6.4 Towards a multiyear consideration: carry forward

Investment decisions will never be based on the analysis of one single future year. Most ‘traditional’ investment criteria, like the internal rate of return or the payback period, explicitly address the fact that the investment is supposed to be effective for some period.

The commercial scope does not change in time, unless investments are carried out (or existing capacity wears out or breaks down). Hence, if the evolution of possible future commercial circumstances is described by one or more multiyear scenarios, the scope is still a useful concept to describe robustness in a multiyear setting. A useful criterion would be for instance the first year when infeasibility occurs, according to the multiyear scenario. Such an analysis can for instance be helpful in discussing the question: is it worthwhile to postpone an investment action? We do not further work out these ideas.

Instead, we discuss the possible use of multiyear models of the same type as the MATCHPLUS model. In such models, the feasibility of production planning problems of a number of consecutive years is considered simultaneously, and that gives the opportunity to model the interdependency between these years in more detail. For instance, in multiyear models it becomes possible to relax the annual underground storage balance (2.20) in the sense that it is allowed to accumulate (resp. deplete) gas in the underground storage and deplete (resp. replenish) it in a later year. A second example of interdependency between successive years is called carry forward. This means that in some year more gas
6.4 Towards a multiyear consideration: carry forward

is taken in than the Annual Contracted Quantity ACQ, giving Gasunie the right to take less in a future year. The contracts with non-Slochteren producers allow the build up of carry forward to a certain extent, but the use of it costs Gasunie money. In fact, for various non-Slochteren sources a 'stock' of carry forward is built up providing Gasunie with flexibility in satisfying current commitments by depleting such stocks. The depletion of the stocks is restricted by the use that can be made of these sources through the year. Sometimes this flexibility is only virtual, since the source for which there exists a formal carry forward stock is already empty, so that the stock is a paper tiger that cannot be used any more.

Despite of the fact that some aspects such as control of the underground storage and the use of carry forward fit nicely into multiyear models, we do not support the use of such models in the study of robustness of investment decisions. Recall that basically we adopt a two-stage information structure with respect to the variables (ref. Figure 2.9): the investment variables \( x \) are chosen without knowledge of the actual commercial scenario \( s \) that will occur, whereas the production variables are chosen with complete knowledge of the actual commercial scenario. This approach is already rather optimistic, in the sense that in practice there is no complete knowledge available on the volumes of demand in the rest of the year, at the moment the daily production decisions have to be made. Applying this to a multiyear model would imply, that in the model the production plans for the coming five years (say) are determined under complete knowledge of the commercial scenarios of these years. This is not realistic, and will give a too optimistic view of the commercial robustness.

Of course, in principle it is possible to adopt a multistage information structure. That is: assume that in any year the commercial scenario of that year is completely known, whereas the commercial scenarios of future years are unknown. But this is not very attractive from a practical point of view: the commercial scope in year \( t \) will depend on the value of the scenario variables in preceding years, so that actually one has to consider as many scopes for year \( t \) as there are 'histories' of the scenarios up till year \( t \). As a result, it is to be expected that the scope will lose its attractiveness in a multistage model due to conceptual and computational difficulties.

This does not imply that it is impossible to study the effects of underground storage control and of carry forward by means of a one-year model. This will be illustrated in the next subsection, where as an example, the carry forward instrument is analysed.

6.4.1 One year with carry forward

In modelling the interdependency between the production planning problems of different years, we concentrate on the introduction of \( H \)-gas carry forward. That is, the restriction that the annual net outflow of the underground storage is zero is maintained, see (2.20), and the restriction that total annual production of non-Slochteren \( H \)-gas equals its annual contracted quantity \( (\gamma ACQ) \) is
relaxed, see (2.19). Actually, the production of non-Slochteren L-gas can be carried forward as well, but that is left out of consideration. Carry forward does not influence the contracted volume. Annual H-gas production may differ from ACQ, but ACQ remains the same, and so does the production capacity of all gas types. Carry forward is an infeasibility absolving strategy, related to but different from the strategy to decrease infeasibly high ACQ, which is described in Section 3.5 and implied for instance in Section 4.2.3.

Let us consider the influence of the presence of a carry forward stock at the beginning of the year on the production planning problem. To introduce carry forward in the model restrictions of MATCHPLUS, the following symbols are needed. The total ‘stock’ of H-gas carried forward, built up until the year under consideration, is denoted by $SCF_0$ and is considered to be completely known at the moment the production decision have to be made. The stock of H-gas carried forward at the end of the year, which should be decided upon together with the production decision variables, is denoted by $scf$. The amount of H-gas carry forward deployed in the year under consideration is therefore equal to $SCF_0 - scf$. The carry forward deployed can be negative (so that stock is built up). We assume that the carry forward stock $SCF_0$, present at the beginning of the year, can wholly be depleted in the course of the year. This is a gross simplification since the stock will be divided over different non-Slochteren sources. The depletion of the stock is restricted by the use that can be made of these sources through the year.

Replace in model MATCHPLUS the aggregating restriction on the production of H-gas (2.19) by

$$\sum_{t=1}^{4} ph_t = \frac{C_{ph}}{C_{ph}} \cdot (\gamma ACQ - SCF_0 + scf)$$

We are interested in the influence of the invocation of the carry forward instrument on the scope of a specific year. Therefore we assume that the initial carry forward stock $SCF_0$ is completely used in the course of the year, and therefore $scf = 0$, and analyse the scope for different values of $SCF_0$.

The commercial scope induced by this model still is a subset of $\mathbb{R}^n$. Since the feasibility of the production plan is influenced by the value of $SCF_0$, the scope also depends on it and is denoted by $S(x; SCF_0)$. Note that $S(x; 0) = S(x)$ where $S(x)$ denotes the commercial scope induced by MATCHPLUS. Using the same data as in Chapter 4, Section 4.1, by means of boundary search experiments some facets of the scope $S(x; SCF_0)$ of this new model have been found. Different levels of $SCF_0$ have been tried. At each level of $SCF_0$ all coordinate directions were tried, leaving from $x^0$. The results are given in Table 6.9 for the zero investment alternative $x^0$ and in Table 6.10 for investment alternative $x^3$. In the tables, the codes between brackets refer to the names of the induced constraints, making up the facets that were encountered in these directions.
6.4 Towards a multiyear consideration: carry forward

Table 6.9. Directional scope and induced constraints in coordinate directions at different levels of \( SCF_0 \) \((s = s^0, \ x = x^0)\)

<table>
<thead>
<tr>
<th>Direction</th>
<th>( SCF_0 = 0 )</th>
<th>( SCF_0 = 3 )</th>
<th>( SCF_0 = 6 )</th>
<th>( SCF_0 = 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DH^+ )</td>
<td>16.80 (B)</td>
<td>16.80 (B)</td>
<td>16.80 (B)</td>
<td>16.80 (B)</td>
</tr>
<tr>
<td>( DH^- )</td>
<td>1.18 (A1')</td>
<td>4.18 (A1')</td>
<td>7.18 (A1')</td>
<td>10.18 (A1')</td>
</tr>
<tr>
<td>( DL^+ )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( DL^- )</td>
<td>5.50 (A1')</td>
<td>13.69 (A2')</td>
<td>21.69 (A2')</td>
<td>26.40 (A2')</td>
</tr>
<tr>
<td>( ACQ^+ )</td>
<td>1.30 (A1')</td>
<td>4.60 (A1')</td>
<td>7.90 (A1')</td>
<td>11.20 (A1')</td>
</tr>
<tr>
<td>( ACQ^- )</td>
<td>24.89 (B)</td>
<td>24.89 (B)</td>
<td>24.89 (B)</td>
<td>22.67 (B)</td>
</tr>
</tbody>
</table>

The columns in both Tables giving the results for \( SCF_0 = 0 \), can be compared to Tables 5.2 and 5.5. Obviously, the results under \( SCF_0 = 0 \) are equal to those in Table 5.2.

It appears that the availability of a positive carry forward stock significantly relaxes the scope in some directions. Since in this case these directions are exactly the ‘dangerous’ directions \( DH^-, DL^- \) and \( ACQ^+ \), where the boundary of the scope is very close to \( s^0 \), it seems that carry forward is an effective instrument to solve the lack of commercial robustness under \( x^0 \). In the direction \( DH^+ \) the directional scope is directly enlarged by the amount of carry forward stock. This is a result of the fact that in the induced constraints involved (see below) \( DH \) and \( SCF_0 \) only occur as their sum \( DH + SCF_0 \). The relaxation in the directions \( DL^- \) and \( ACQ^- \) is even more than proportional.

Some of the induced constraints encountered are new. It was to be expected that the induced constraint (B) in the original scope, which had a zero dual multiplier on the position of restriction on the annual production of H-gas

Table 6.10. Directional scope and induced constraints in coordinate directions at different levels of \( SCF_0 \) \((s = s^0, \ x = x^0)\)

<table>
<thead>
<tr>
<th>Direction</th>
<th>( SCF_0 = 0 )</th>
<th>( SCF_0 = 3 )</th>
<th>( SCF_0 = 6 )</th>
<th>( SCF_0 = 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( DH^+ )</td>
<td>16.80 (B)</td>
<td>16.80 (B)</td>
<td>16.80 (B)</td>
<td>16.80 (B)</td>
</tr>
<tr>
<td>( DH^- )</td>
<td>13.19 (A2')</td>
<td>16.19 (A2')</td>
<td>19.19 (A2')</td>
<td>22.19 (A2')</td>
</tr>
<tr>
<td>( DL^+ )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( DL^- )</td>
<td>17.40 (D')</td>
<td>20.40 (D')</td>
<td>23.40 (D')</td>
<td>26.40 (D')</td>
</tr>
<tr>
<td>( ACQ^+ )</td>
<td>14.38 (A2')</td>
<td>17.65 (A2')</td>
<td>20.92 (A2')</td>
<td>24.19 (A2')</td>
</tr>
<tr>
<td>( ACQ^- )</td>
<td>24.89 (B)</td>
<td>24.89 (B)</td>
<td>24.89 (B)</td>
<td>22.67 (J)</td>
</tr>
</tbody>
</table>
Generalizations of the experiments

(2.19), still would be valid for this relaxed model. After all, it only concerns production capacity in the maximum demand period (period 0). However, in the direction $ACQ$, for high values of $SCF_0$, $(B)$ is overruled by a new induced constraint:

$$\gamma ACQ \geq DH + SCF_0$$

For even higher values of $SCF_0$ than those given in the tables, $(J)$ starts to dominate $(B)$ in the direction $DH+$ as well. This induced constraint gets tighter for higher values of $SCF_0$. This illustrates the fact that invoking carry forward to solve one problem may introduce other problems.

The other induced constraints are relaxed by the existence of a positive carry forward stock $SCF_0$. The corresponding dual multiplier vectors have a nonzero multiplier on the position of restriction (6.2). Their structure however stays the same for the major part.

As to induced constraints $(A0)$, $(A1)$ and $(A2)$: the left-hand side of all these constraints, as they were given on pages 106 and 123, was interpreted as the annual overflow from the H-gas to the L-gas system. As a result of restrictions (6.2) and (2.8) the annual H-gas overflow is not fixed any more, but bounded from below by an amount depending on $SCF_0$:

$$\sum_{t=1}^{4} f_t \geq \frac{C_{ph}}{C_{pl}} \cdot (\gamma ACQ - SCF_0 - DH)$$

The right-hand sides of the induced constraints $(A0)$, $(A1)$ and $(A2)$ give upper bounds on the annual overflow, which are still valid. Clearly, to have a feasible annual overflow, the lower bound (6.3) should not exceed these upper bounds. This is exactly the interpretation of the new induced constraints $(A0')$, $(A1')$ and $(A2')$. As an example, the new induced constraint $(A1')$ is shown, the other two follow analogously.

$$\gamma ACQ - SCF_0 - DH \leq$$

$$\frac{C_{ph}}{C_{pl}} \cdot \sum_{t=1,2,4} \left\{ (WU - W_p)FL_t \cdot DL + (WU - W) \cdot NU \right\}$$

+ $FL_3 \cdot DL - PLL - \frac{C_{ug2}U GL}{C_{pl}} + \frac{C_{ph}V_{ug2}U GL}{C_{pl}C_{ph}}$ $(A1')$

where $V_g = \frac{C_{ug}}{C_{pl}}(WU - W_p) - (WU - W)$ for $g = ph, ug$ and $PLL = PLL + \alpha(1 - \gamma)ACQ$.

As to induced constraint $(D)$: as a result of the presence of carry forward stock, the minimum annual energy production is diminished by an amount of
Clearly, if the minimum total energy production in this way gets below zero, the induced constraint is redundant.

\[ 4 \cdot PLL_s + ACQ - SCF_0 \leq DH + DL \]  \hspace{1cm} (D')

Given the formulation of the induced constraints, the relationship between the directional scopes and \( SCF_0 \) can easily be derived.

### 6.5 Selecting investment alternatives using stochastic programming

The applications given in this thesis up to this point all use one or more given investment alternatives \( x^i \) as a starting point. For a given set of investment alternatives \( x^i \), their robustness and risk characteristics based on their commercial scopes \( S(x^i) \) have been studied. Standard multicriteria decision-making techniques can be applied to select the best alternative based on a balancing of both robustness and financial criteria. For instance, if one is willing and able to specify weights for the different criteria, the weighted sum of the scores to different criteria can be used to generate a ranking of the alternatives.

For a robustness analysis of the existing situation, in Chapter 4 denoted by the zero-investment alternative \( x^0 \), it is only natural to start from a given alternative. But if investments are necessary, a good alternative may not be available from the start. In the next subsections, it is described how a good investment alternative can be generated using stochastic programming.

#### 6.5.1 A two-stage stochastic programming model

As has been observed earlier, the construction of the commercial scope is based on a two-stage information structure with respect to the decision variables. The investment decision on the value of \( x \) is a so-called here-and-now decision, since the commercial future is uncertain at the moment the investment decisions have to be made. On the other hand, the production decision on the value of \( y \) is a so-called wait-and-see decision, since at this stage the commercial variables are supposed to be known with certainty. This distinction refers to two-stage stochastic programming. An excellent state-of-the-art overview of stochastic programming models, algorithms and applications can be found in Ermoliev and Wets (1988).

In fact, if it is possible to describe the set of feasible investment alternatives \( X \) explicitly and if one is willing to interpret the uncertain commercial scenario as a random vector \( s(\omega) \) and to specify its probability distribution, then it is only natural to formulate the investment planning problem as a two-stage stochastic programming model. Typically, such models are aimed at balancing between a cost function, connected to the first stage decision, and ‘risks’.

Since at Gasunie it is felt that the customer reliability, which in (3.9) was defined as the probability of a feasible scenario \( \Pr \{ s \in S(x) \} \), is very important,
Generalizations of the experiments

it is obvious that it should be kept above a prescribed high level $\alpha$. Within the alternatives left by this restriction, it is reasonable to try and find investments that minimize investment cost and infeasibility risk. This attitude is reflected in the following stochastic programming model:²

$$\inf_{x \in X} \{ c(x) + \chi E[f(s(\omega), x)] \mid \Pr\{s(\omega) \in S(x)\} \geq \alpha \}$$

(SP)

where: $X =$ the set of feasible investment alternatives
$c =$ investment cost function
$\chi =$ some weight factor
$\alpha =$ required level of reliability

and $f(s, x)$ represents the penalty cost that is incurred if, under investment alternative $x$, the commercial scenario $s$ happens to occur. For instance, $f$ may be given by (3.13):

$$f(s, x) = \inf_{z^+ \geq 0, z^- \geq 0} \{ q z^+ + r z^- \mid s - z^+ + z^- \in S(x) \}$$

for positive per unit penalty cost vectors $q$ and $r$. Obviously, $f(s, x) = 0$ if $s \in S(x)$. The weight factor is induced by the fact that the penalty cost is not necessarily expressed in a real currency, or by the fact that the investment cost contains a contingency surcharge, to correct for unavailability (see Section 4.6.1).

The model $(SP)$ resembles very much standard stochastic linear programs with recourse including a chance constraint. Although there is no software available that can directly solve a problem of this form, theoretically it is well solvable. See for instance Prékopa and Szántai (1978). For those models, $X$ is a polyhedral set, $c$ is linear and $f$ is the minimal recourse cost, defined as a linear programming problem in the second stage decision variables. The difference is, that in $(SP)$ the recourse cost as well as the chance constraint are defined in terms of the scope $S(x)$ rather than by linear (in)equalities as is the case in standard stochastic programming models. But this difference is not essential, as is demonstrated in the next subsection.

6.5.2 The use of induced constraints on the scenario vector

Since the commercial scope is defined as

$$S(x) = \{ s \in \mathbb{R}^n \mid \exists y \in \mathbb{R}^n, Ky \leq Ls + Mx \}$$

the minimal recourse cost function can be written as:

$$f(s, x) = \inf_{y, \ z^+ \geq 0, \ z^- \geq 0} \{ q z^+ + r z^- \mid Ky + Lz^+ - Lz^- \leq Ls + Mx \}$$

２. As an alternative to this model, Pistikopoulos and Grossmann (1988a, 1988b) study the optimization of investment cost, given a certain minimum level for the flexibility index (see Section 3.4). In other studies however, Pistikopoulos and Grossmann (1988b, 1988b) use the flexibility index level as a first stage decision variable, like the investment variables. In a hybrid, both stochastic and non-parametric way, they perform two-stage stochastic optimization to find the flexibility level that optimizes expected profit.
and this is a special case of standard linear recourse. With respect to the chance constraint the transformation to a standard form is more complicated, since

$$\Pr\{3y \in \mathbb{R}^n, Ky \leq Ls(\omega) + Mx \} \geq \alpha$$

is not easily dealt with. An explicit representation of the scope, using induced constraints, is possible according to Corollary 3.1:

$$S(x) = \{s \in \mathbb{R}^n : Bs \geq Ax\}$$

such that the chance constraint can be written as:

$$\Pr\{Ax \leq Bs(\omega)\} \geq \alpha$$

Such a chance constraint fits into the standard format, and can be dealt with in algorithms (especially if $s(\omega)$ is normally distributed). However, in Appendix 3B it was shown that, in problems where the vector $y$ has a nontrivial size, the available direct elimination algorithms might end up with an unnecessary complex system of restrictions, and therefore prove not to be beneficial at all.

On the other hand, in Appendix 3B we conjectured, confirmed by our experiments, that the number of irredundant induced constraints is relatively low, so that the chance constraint is approximated by

$$\Pr\{A_Ix \leq B_Is(\omega)\} \geq \alpha$$

where $(A_I, B_I)$ denotes a relatively small selection of rows of $(A, B)$. Of course, if this selection is small indeed, one may prefer to use this explicit representation of $S(x)$ also in the description of the recourse decision part in $(SP)$, in order to reduce the size of the problem:

$$f(s, x) = \inf_{z \geq 0, z^+ \geq 0} \{qz^+ + rz^- \mid B_1z^+ - B_1z^- \leq B_Is - A_Ix\}$$

We conclude that the prototype investment planning model $(SP)$ can be solved as a standard stochastic programming problem if the commercial scope can be sufficiently described using only a few induced constraints.

In our experiments it appeared that this is indeed the case for the MATCH PLUS model, at least for a few given values for $x$. In general, the induced constraints collected in $B_1s \geq A_Ix$ may arise for instance from directional boundary searches for a given alternative $x = x^j$, as in our experiments. By changing $x$, other irredundant induced constraints may come up, that were redundant under $x^j$. So it is quite possible that for the optimal solution $x^*$ of $(SP)$ some actually irredundant induced constraint are violated since they were not included in the initial selection used. However, using the techniques presented in Chapter 3, it is possible to perform a posteriori additional boundary searches under $x^*$. If new irredundant induced constraints are encountered, they can be implemented in $(SP)$, and then $(SP)$ can be solved again. It is to be
expected that after a few iterations of this type a good investment alternative will be found.

In any two-stage stochastic programming model, if there are second stage decision variables which do not have any influence on the objective, like in our case the decision variables \( y \), the calculation of the optimum may be facilitated by the elimination of these variables. Pistikopoulos and Grossmann (1989a), (1989b) use such elimination algorithms as an integrated part of optimizations involving their (scope-derived) ‘flexibility index’. Unfortunately, they use a direct extensive algorithm to eliminate \( y \), of which we already remarked that for non-trivial problem sizes the complexity is unnecessarily high.

In a footnote to Definition 3.5 we described the use of induced constraints in stochastic programming. As said before, the definition of induced constraints used by stochastic programming deviates from ours, since there they are defined as constraints on the investment decision vector \( x \), representing those vectors \( x \) for which the production planning restrictions \( Ky \leq Ls + Mx \) are feasible for all realistic \( s \). This type of induced constraints is explicitly used in so-called \( L \)-shaped algorithms used for solving stochastic programs with recourse. For an overview, see Ermoliev and Wets (1988, Chapter 3). Basically, these methods iteratively solve optimizations involving only the first stage decision \( x \), leaving out the restrictions involving the second stage decision \( y \). If after an iteration it follows that the restrictions on \( y \) are infeasible for an \( s \) in the support of \( s(\omega) \), an induced constraint is generated, ruling out the ‘infeasible’ value for the first stage decision \( x \), after which the optimization is solved another time, until the solution is feasible for all realizations \( s \) in the support of \( s(\omega) \).

### 6.5.3 A simplified example

The application in this section is simplified in the sense that the reliability constraint is not included. The example is based on the MATCHPLUS model, and on the data as given in Sections 4.1. The set of feasible investment alternatives \( X \) is specified in such a way that it includes the zero investment alternative \( x^0 \) and the alternatives \( x^1, x^2, x^3 \) given in Section 5.1, which have been studied extensively before. That is, the capacities of the Slochteren field are kept fixed throughout: \( PLU_S = +\infty \) and \( PLL_S = 0.910^6 m^3 \), but the nitrogen production capacity \( NU \) as well as the underground storage in- and outflow capacities are allowed to vary, be it that \( UGL \) and \( UGU \) have the same proportion as in \( x^1 \) and \( x^3 \).

Denoting the deviation from \( x^0 \) by

\[
\Delta x := \begin{pmatrix}
0 \\
0 \\
\Delta NU \\
\Delta UGU \\
\Delta UGL
\end{pmatrix}
\]

we have therefore the following set of feasible investment alternatives:
\[ X = \{ x \in \mathbb{R}^5 \mid x = x^0 + \Delta x, \Delta NU \geq 0, \]
\[ \Delta UGU \geq 0, \quad 2.20 \cdot \Delta UGU + 3.25 \cdot \Delta UGL = 0 \} \]

With respect to the recourse cost, we assume just as in Chapters 4 and 5, that shortages are forbidden \((r = (\infty, \infty, \infty, \infty, \infty))\) where the unit penalty cost for surpluses are given by \(q = (10, 10, 0.25)\). Moreover, it is assumed that the cost of one \(m^3\) per quarter increase of nitrogen production capacity is equal to \(\psi\) times the cost of one \(m^3\) per quarter extra underground storage inflow or outflow capacity and to \(\chi\) times one unit penalty cost. Finally, it is assumed that for all relevant investment alternatives the scope is well described by the induced constraints \((A0), (A1), (A2), (B)\) and \((D)\). Both symbolic descriptions and the numerical values of the coefficients of these induced constraints are given in Section 4.3 and 5.3. Together they construct a system of induced constraints on \(S(x)\) denoted by \(A x \leq B s\), being a row selection out of the explicit formulation \(A x \leq B s\). Thus we have the following simplified example of \((SP)\):

\[
\inf_{\Delta NU, \Delta UGU, \Delta UGL} \Delta NU + \psi(\Delta UGU - \Delta UGL) + \chi f(s(\omega), x)
\]

subject to:

\[
x = x^0 + \Delta x
\]
\[
\Delta x = (0, 0, \Delta NU, \Delta UGU, \Delta UGL),
\]
\[
2.20 \cdot \Delta UGU + 3.25 \cdot \Delta UGL = 0,
\]
\[
\Delta NU \geq 0,
\]
\[
\Delta UGU \geq 0
\]

where

\[
f(s, x) = \inf_{z \geq 0} \{ (10, 10, 0.25) z^+ \mid B_I z^+ \leq B_I s - A_I x \}
\]

This optimization problem has been solved using the SLP-JOR software, developed at the Institute of Operations Research at Zürich. In fact, we solved the problem for a variety of combinations of the cost ratios \(\psi\) and \(\chi\). In some cases we performed additional scope boundary searches under the new found alternative, but no new induced constraints were found. In Figure 6.2 it is denoted for which combinations of \(\psi\) and \(\chi\) which one of the following types of investment actions is optimal:

- do nothing,
- build additional nitrogen capacity,
- build an underground storage facility,
- build both additional nitrogen capacity and an underground storage facility

Figure 6.2 shows, that investments in nitrogen production capacity are only made if it is not too expensive in relation to surplus costs \((\chi \geq 0.20)\).
Investments in underground storage inflow and outflow capacity are made only if $\psi$ is not too large. The upper bound depends on the value of $\chi$, and seems to be constant (about 0.04) for $\chi \geq 0.40$. In order to relate the optimal solutions with the alternatives $x^0$, $x^1$, $x^2$ and $x^3$ we select four combinations of $\psi$ and $\chi$.

a. If $\chi = 0$ and $\psi$ is arbitrary, the zero investment alternative is optimal in (SP). This is not surprising since if $\chi = 0$, the cost of infeasibility gets weight 0.

b. If $\psi = 0$ and $\chi = 0.15$ then under the given data example it is optimal to only invest in storage inflow and outflow capacity. The optimal solution then is: $\Delta NU = 0$, $\Delta UGU = 4.1139$ and $\Delta UGL = -2.7848$.

c. If $\psi = 0.05$ and $\chi = 0.35$, under the conditions assumed the optimal investment policy is to only invest in nitrogen production capacity: $\Delta NU = 0.0072$ and the storage capacities are zero. Note that this gives only a very small increase of nitrogen production capacity.

d. If $\psi = 0.01$ and $\chi = 0.5$, under the given assumptions it is best to invest both in storage and nitrogen production capacity. The optimum in this case was: $\Delta NU = 0.0881$, $\Delta UGU = 4.2473$ and $\Delta UGL = -2.875$.

For the cost ratios selected, the optimal investment decisions gave only small nitrogen production capacity increases, if any. The optimal investment decisions involving investments in underground storage show storage capacity investments that are significantly greater than the alternatives we considered.
Of course, the above observations do only hold for the specific example problem. Since this example is based on fictitious data, nothing is said about any real world decision problem. Furthermore, no attention is paid to fixed investment costs. The introduction of fixed cost for any investment project (which would not depend on the capacity added) can change the picture drastically. Nevertheless, these computational exercises show that it is quite possible to use stochastic programming to generate interesting investment alternatives, and that the use of induced constraints can facilitate the solution.

6.6 Conclusions

The commercial scope and related concepts are very useful, not only in the context of simulations using the MATCHPLUS model, but in a more general setting as well.

The generalizations of this chapter showed that the MATCHPLUS model is easily extended with different features. Additional quality constraints are easily incorporated, but the specific examples considered did not significantly affect the commercial scope. The uncoupling of the demand per quarter did not introduce serious problems or generate new insights, apart from the fact that the commercial scope gets increasingly difficult, involving more facet-inducing induced constraints (in the example we had at least eleven instead of the original three for $S(x^0)$). The introduction of more detail in the form of a monthly model revealed that the robustness results strongly depend on the variation of demand. The more the model is disaggregated in time, the stronger the demand fluctuations emerge and the more difficult the production planning turns out to be. In the previous section it was shown that the scope instruments are useful in searching a balance between investment cost or other financial criteria on the one hand, and the commercial robustness on the other.

The use of induced constraints to describe the facets of the scope proved to be very useful in the applications on the relatively small and simple MATCHPLUS model. If this linear model gets more complicated in the sense that more constraints and/or more production decision variables are added, the commercial scope can still be usefully analysed using induced constraints that follow from a heuristic boundary search. If the number of uncertain variables increases, it gets increasingly difficult to perform such a broad analysis, aiming at an extensive description of all aspects of the scope. More efforts are needed and more data are generated, which are increasingly less surveyable. For greater numbers of uncertain variables, one should to concentrate on either a smaller number of 'key uncertainties', or a smaller number of global scope measures like risk, reliability and perhaps the directional scope in one or two important directions. To find out which directions are most important, in the sense that the boundary of the scope is closest in those directions, some ‘preprocessing’ on a smaller example problem could be useful.