Chapter 3

Input-output analysis

The previous chapter presented two methods for the determination of the energy requirements of households. This chapter details one of these methods based on economic input-output analysis. Economic input-output analysis, developed by W.W. Leontief, is used to study the relations between economic sectors. Since the 30s, Leontief has applied the input-output model on the U.S. economy (Leontief, 1941; Leontief, 1951; Leontief, 1986). The pioneering work of Leontief, for which he received the Nobel Prize in 1973, has stimulated many researchers to enter the field of input-output analysis. Since Leontief’s first publications, hundreds of books and articles on input-output analysis have been published. A standard textbook on input-output analysis, which is often used in introductory courses, is written by Miller and Blair (1985). A state-of-the-art overview is given by e.g. Sohn (1986), and Rose and Miernyk (1989). The journal of the International Input-Output Association, Economic Systems Research, is a specialized journal on input-output analysis.

Two main input-output techniques are impact analysis and the projection (or imputation) of primary inputs (Oosterhaven, 1981; Konijn, 1994). Impact analysis, which is the most traditional input-output technique, studies the impact of a changing final demand on the production of economic sectors. The second technique concerns the projection of primary inputs on final demand. Although traditionally, only the inputs of labour and capital were analysed, later other types of inputs were also taken into account. Since the early 70s, projection has been applied in the energy field (input-output energy analysis). Both input-output techniques mentioned require the calculation of so-called Leontief multipliers.

This chapter starts with a theoretical introduction on economic input-output analysis in which the derivation of the Leontief multipliers is given. The main part of the chapter considers the calculation of energy intensities of economic production sectors by using input-output energy analysis. Further, the input-output model for the calculation of household energy requirements is given.

---

1 This technique is a form of impact analysis which goes the other way around.
2 Some other interesting applications of input-output analysis concern the analysis of ecosystems (Hannon, 1973; Hannon, 1995) or water pollution (CBS, 1977).
3.1 ECONOMIC INPUT-OUTPUT ANALYSIS

Section 3.1.1 describes some principles of input-output analysis. These principles are illustrated in section 3.1.2 by using an example. After that, section 3.1.3 describes how imports and capital goods are taken into account in input-output analysis.

### 3.1.1 Model Leontief multipliers

Starting point for an input-output analysis is an input-output table. An input-output table is a description of the flows of goods and services through an economy in financial terms. These flows concern a particular period, usually a one-year period. Table 3.1 shows a representation of an input-output table.

**Table 3.1** Schematic representation input-output table.

<table>
<thead>
<tr>
<th>econ. sectors</th>
<th>final demand</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>Y</td>
<td>x</td>
</tr>
<tr>
<td>W</td>
<td>V</td>
<td>u</td>
</tr>
<tr>
<td>x'</td>
<td>t'</td>
<td></td>
</tr>
</tbody>
</table>

Each row of an input-output table contains the revenues of an economic sector. These revenues correspond to the deliveries of that sector to other production sectors, the so-called intermediate deliveries (Z), and the deliveries to final demand (Y). A column of an input-output table contains all outlays of the corresponding economic sector. These outlays concern the purchases of intermediate deliveries and the primary inputs (W).

So, apart from the totals, an input-output table consists of four quadrants. The main quadrant is the matrix of intermediate deliveries between the production sectors. These intermediate deliveries are necessary for the production of the final deliveries, which form the second quadrant. The final deliveries concern the final consumption categories, i.e. exports, private and government consumption, investments, and stock changes. From the perspective of the final user, the final deliveries are called final consumption or final demand. Furthermore, an input-output table contains primary inputs. The primary inputs consist of imports of goods and services and gross value added. The gross value added comprises depreciation, indirect taxes and subsidies, gross wages and salaries, social charges and profits. The third quadrant contains the primary inputs of the intermediate sectors. Finally, the fourth quadrant concerns the primary inputs of final demand (V).

For each sector, the value of total production (x) is the sum of the...
intermediate deliveries and the final deliveries. In matrix notation:

\[ x = Z_i + Y_i \]  \hspace{1cm} (3.1)

For reasons of convenience, the final deliveries are represented by one vector, \( y \):

\[ x = Z_i + y \]  \hspace{1cm} (3.2)

In the same way, the value of total production of each sector equals total outlays of that sector which consist of intermediate consumption and primary inputs. In matrix notation:

\[ x^T = i^T Z + i^T W \]  \hspace{1cm} (3.3)

In input-output analysis, assumptions are made about the economic relations on which the input-output table is based. The first assumption is that, in short run analysis, these economic relations are time-invariant. The second assumption is that the inputs of each sector are proportional with the output. This is based on the expectation that with increasing total production of a sector, the consumption of intermediate and primary inputs of that sector will increase proportionally. So, the intermediate matrix can be written as a function of total production:

\[ Z = A x^D \]  \hspace{1cm} (3.4)

Since matrix \( A \) reflects the production process with the present technology, this matrix is called \textit{technological matrix}. Another name is \textit{direct requirements matrix} (James et al., 1978). For every sector, the matrix reflects the direct inputs that are required for the production of one financial unit of output. Matrix \( A \) can be derived from (3.4):

\[ A = Z (x^D)^{-1} \]  \hspace{1cm} (3.5)

Thus, the direct, first order, requirement of each sector has been derived. However, for the production of these direct inputs, the sectors that deliver these inputs, in turn, require inputs themselves as well. These new inputs also require inputs, and so forth. Summarizing, there appears to be a complex connecting network, i.e. a set of connections, between the different economic sectors. The

\(^3\) See Appendix A for notation and definition of matrices and vectors.
second order requirement of the economic sectors is $A^2$, the third order requirement $A^3$, et cetera. The total requirements needed for one financial unit of total production are:

$$A + A^2 + A^3 + \ldots$$  \hspace{1cm} (3.6)

One of the objectives of economic input-output analysis is the determination of the total production that is needed for a specific final demand. Formula 3.6 gives the production that each sector has to deliver for one unit total production. In order that every sector delivers one unit final demand, every sector has to produce not only its own final demand, but also the direct and indirect requirements needed for its own and the other final demand. Matrix $B$ gives the total production that is needed, so that each sector produces one financial unit of final demand. Matrix $B$ is:

$$B = I + A + A^2 + A^3 + \ldots$$  \hspace{1cm} (3.7)

A sufficient condition for the convergence of the series in formula (3.6) is that $iA < i$ (Nikaido, 1968). Then:

$$B = (I - A)^{-1}$$  \hspace{1cm} (3.8)

Matrix $B$ is the so-called Leontief inverse matrix. It is also called matrix of Leontief multipliers or total requirements matrix. Each row of $B$ contains the production of the corresponding sector needed, so that each sector is able to produce one unit of final demand. For each sector, the corresponding column of $B$ contains the production of all sectors, so that the sector is able to produce one unit of final demand.

Another way to obtain the Leontief inverse matrix is by substituting equation (3.4) in equation (3.2) and by solving the new equation:

$$x = Ax + y$$  \hspace{1cm} (3.9)

$$x = (I - A)^{-1} y$$  \hspace{1cm} (3.10)

So, the Leontief inverse matrix determines the total production of each sector for a specific final demand under the assumption that technology, represented by matrix $A$, remains the same.
3.1.2 Example

For illustration, this section gives an example of an input-output table for a fictive economy consisting of only two producing sectors (table 3.2). The final demand and primary inputs are each summarized in one vector. A row contains the output of a sector, i.e. the value of the deliveries of a sector to the different destinations. E.g., sector 1 delivers products with a value of 25 guilders (Dfl) to sector 1, products with a value of 40 guilders to sector 2 and an 85 guilders equivalent of final demand. Value of total production of sector 1 is 150 guilders. A column contains the value of the inputs of a sector, i.e. the costs of the deliveries from other sectors and the primary costs. For a production of 150 Dfl, sector 1 spends 25 Dfl in sector 1, 30 Dfl in sector 2 and the primary costs are 95 Dfl.

Table 3.2  Example of an input-output table with 2 sectors (in Dfl).

<table>
<thead>
<tr>
<th>sector 1</th>
<th>sector 2</th>
<th>final demand</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>sector 1</td>
<td>25</td>
<td>40</td>
<td>85</td>
</tr>
<tr>
<td>sector 2</td>
<td>30</td>
<td>80</td>
<td>50</td>
</tr>
<tr>
<td>primary inputs</td>
<td>95</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>150</td>
<td>160</td>
<td>135</td>
</tr>
</tbody>
</table>

The technological matrix is derived from the intermediate matrix by dividing every outlay, i.e. input, of a sector by the total production of that sector. So, in accordance with (3.5):

$$A_{ij} = \frac{Z_{ij}}{x_j} \quad (3.11)$$

Table 3.3 shows the technological matrix of the fictitious economy. For every sector, the matrix gives the inputs that are needed for a production of 1 Dfl. E.g. for a production of 1 Dfl in sector 1, sector 1 has to spend 1/6 Dfl in sector 1 and 1/5 Dfl in sector 2. The matrix gives the direct requirements for each sector. But to produce these direct requirements, the delivering sectors in their turn need inputs as well. And these inputs also require inputs, and so on, and so on. So, the determination of the required deliveries for the 1/6 Dfl from sector 1 needs the technological matrix once more. The deliveries required are:

Table 3.3  Technological matrix of a fictitious economy (in Dfl/Dfl).

<table>
<thead>
<tr>
<th>sector 1</th>
<th>sector 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>sector 1</td>
<td>1/6</td>
</tr>
<tr>
<td>sector 2</td>
<td>1/5</td>
</tr>
</tbody>
</table>
\[
\frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \text{ Dfl from sector 1 and} \\
\frac{1}{6} \times \frac{1}{5} = \frac{1}{30} \text{ Dfl from sector 2.}
\]

Similarly, the \(\frac{1}{5}\) Dfl from sector 2 requires:

\[
\frac{1}{5} \times \frac{1}{4} = \frac{1}{20} \text{ Dfl from sector 1} \\
\frac{1}{5} \times \frac{1}{2} = \frac{1}{10} \text{ Dfl from sector 2.}
\]

In total, for a total production of 1 Dfl from sector 1, the second order requirements are:

\[
\frac{1}{36} + \frac{1}{20} = \frac{7}{90} \text{ Dfl from sector 1 and} \\
\frac{1}{30} + \frac{1}{10} = \frac{2}{15} \text{ Dfl from sector 2.}
\]

Continuing in this way, the third and higher order requirements for the production of 1 Dfl are determined. Finally, the requirements approach zero. The sum of all higher order requirements gives the total production of a sector needed for 1 Dfl output of sector 1. This production is:

Production of sector 1: \(\frac{1}{6} + \frac{7}{90} + \frac{5}{108} + \ldots = \frac{4}{11}\) Dfl

A similar calculation gives the total production of sector 2 needed for 1 Dfl output of sector 1:

Production of sector 2: \(\frac{1}{5} + \frac{2}{15} + \frac{37}{450} + \ldots = \frac{6}{11}\) Dfl

To determine the total production for 1 Dfl final demand, the production of this final demand has to be added to the total requirements. The production of 1 Dfl final demand of sector 1 requires a total production of \(1 + \frac{4}{11} = \frac{15}{11}\) Dfl of sector 1 and a total production of \(\frac{6}{11}\) Dfl of sector 2.

Formula (3.8) already showed that the Leontief inverse matrix is the mathematical formulation for the determination of total production for one unit final demand. Table 3.4 shows this inverse matrix calculated for the fictitious economy. Each row of the Leontief inverse matrix contains the production of the corresponding sector needed so that all sectors can deliver one unit of final demand. Each column contains the production of all sectors needed for the production of one unit final demand of the sector that corresponds with the column.

Multiplication of the Leontief inverse matrix with the final demand vector (see table 3.2) gives the total production vector (according to formula (3.10)). For sector 1, total production needed for the production of all final
deliveries is $15/11 \times 85 + 15/22 \times 50 = 150$ Dfl. Total production of sector 2 needed for all final deliveries is $6/11 \times 85 + 25/11 \times 50 = 160$. These values equal the values in table 3.2. So, for a given technological structure and a certain final demand, the Leontief inverse matrix determines total production for the separate economic sectors.

3.1.3 Imports and capital goods
According to energy analysis, the calculation of the energy requirements should include all goods and services consumed in the production process, so imports and capital goods too. In input-output tables published by the Netherlands Central Bureau of Statistics (CBS), imports of goods and services are part of the primary inputs. Therefore, the calculation of the Leontief multipliers does not include imports and so, final demand requires more production than appears from the Leontief inverse matrix.

Imports can be divided in competitive and non-competitive imports. Competitive imports are imports of commodities that are also produced domestically. The Netherlands Central Bureau of Statistics publishes the competitive imports in a matrix with a similar classification as the matrix of the intermediate deliveries. So, the destinations for the competitive imports are known. Non-competitive imports are goods and services which are not produced domestically. These imports are described in an additional row in the table of competitive imports. Table 3.5 shows an example of an import table for the fictitious economy described in section 3.1.2. The competitive imports are divided in competitive sectors and destinations. No imports are supposed to go to final demand directly. Sector 1 receives deliveries with a value of 25 Dfl from competitive sector 1 and with a value of 35 Dfl from competitive sector 2.

<table>
<thead>
<tr>
<th>Table 3.4 Leontief inverse matrix of fictitious economy (in Dfl/Dfl).</th>
</tr>
</thead>
<tbody>
<tr>
<td>sector 1</td>
</tr>
<tr>
<td>sector 1</td>
</tr>
<tr>
<td>sector 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3.5 Table of competitive and non-competitive imports of the fictitious economy (in Dfl).</th>
</tr>
</thead>
<tbody>
<tr>
<td>sector 1</td>
</tr>
<tr>
<td>sector 1</td>
</tr>
<tr>
<td>sector 2</td>
</tr>
<tr>
<td>total competitive imports</td>
</tr>
<tr>
<td>non-competitive imports</td>
</tr>
<tr>
<td>total imports</td>
</tr>
</tbody>
</table>
2. For the total production of 150 Dfl, sector 1 also requires 5 Dfl of non-competitive imports.

In input-output analysis, one assumes that competitive imports are produced in a similar way as domestic production. For the non-competitive imports, such assumptions cannot be made, since the direct inputs needed for the production of these goods and services are not known. In the calculation of the Leontief inverse matrix, the matrix of competitive imports is combined with the domestic intermediate matrix. Table 3.6 shows this for the fictitious economy. Now, total production also contains competitive imports.

<table>
<thead>
<tr>
<th>sector 1</th>
<th>sector 2</th>
<th>final demand</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>sector 1</td>
<td>50</td>
<td>50</td>
<td>85</td>
</tr>
<tr>
<td>sector 2</td>
<td>65</td>
<td>90</td>
<td>50</td>
</tr>
<tr>
<td>non-comp. imports</td>
<td>5</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>value added</td>
<td>30</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>150</td>
<td>160</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.7 shows the technological matrix and the Leontief inverse matrix calculated for the fictitious economy. The comparison of these matrices with the matrices in tables 3.3 and 3.4 shows the effect of including imports. Chapter 5 carries out such a comparison for the input-output tables of the Netherlands.

Multiplication of the Leontief inverse matrix with the vector of final demand gives total production of all sectors needed for final demand. The production of sector 1 becomes 338 Dfl and the production of sector 2 becomes 449.1 Dfl. These values are considerably higher than the values calculated without considering imports.

For some countries, e.g. the U.S. and France, the input-output table is published in a form like table 3.6. The technological matrix can be derived directly from the intermediate matrix. To determine the domestic total production, the competitive imports of each sector have to be subtracted from total production (Bullard and Herendeen, 1975).

The second order inputs in energy analysis are capital goods, which actually are the investments made in the past. Capital goods are goods with a lifespan of more than one year, e.g. means of transport, machines, office furnish and buildings. In input-output tables, investments are not part of the intermediate matrix, but belong to final demand. The best way to deal with capital goods requires a dynamic input-output analysis, since only such an analysis allows for the annual changes in capital goods and energy requirements.
Table 3.7 Technological matrix A and Leontief inverse matrix B (including imports).

<table>
<thead>
<tr>
<th></th>
<th>sector 1</th>
<th>sector 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>sector 1</td>
<td>1/3</td>
<td>5/16</td>
</tr>
<tr>
<td>sector 2</td>
<td>13/30</td>
<td>9/16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>sector 1</th>
<th>sector 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>sector 1</td>
<td>42/15</td>
<td>2</td>
</tr>
<tr>
<td>sector 2</td>
<td>416/150</td>
<td>64/15</td>
</tr>
</tbody>
</table>

of capital goods. For practical reasons, in this study, the following static solution is used.

Capital goods are involved in the calculations by using the depreciations of the capital goods purchased in the past. In input-output tables, depreciations belong to primary inputs. Since input-output tables do not specify depreciations, past investments are supposed to have the same composition as new investments. By adding a row, depreciation, and column, investments, in the intermediate matrix, capital goods are involved in the calculation of the Leontief inverse matrix. In case of investments from abroad, these also have to be added to the column of investments to ensure that all investments are accounted for.

3.2 INPUT-OUTPUT ENERGY ANALYSIS

Input-output energy analysis is a specific application of economic input-output analysis. Input-output energy analysis origins from the 1970s with pioneering work of Wright (1974), and Bullard and Herendeen (1975). A recent overview of input-output energy analysis is given by Peet (1993).

The objective of input-output energy analysis is the calculation of energy intensities. The energy intensity of an economic sector gives the total amount of energy, both direct and indirect, that is needed for one financial unit of production of that sector. The direct energy use of an economic sector comprises the energy directly used in the production process of that sector. The indirect energy use of an economic sector comprises all the energy that is needed for the production and delivery of the goods and services that are used in the production process. These goods and services include both the goods and services that are purchased from other economic sectors. The energy intensities are typically expressed in units such as kilowatthours, or joules per unit of output. A disadvantage of adding a row and column to the intermediate matrix is that the structure of the input-output table is changed. The method of Oosterhaven (1995), which gives the same results, does not have this drawback. The following formula determines the elements of the technological matrix (A):

\[ A_{ij} = \frac{Z_{ij}}{x_i} + \frac{w_i}{w^t} \left( \frac{v_j}{x_j} \right) \]

with \( w \) the vector of investments, \( v \) the vector of depreciations and \( w^t = w^t i \).
services from domestic and foreign origin and the capital goods.

In this study, input-output energy analysis is used for the determination of the total primary energy needed for the production of final demand. The primary energy requirements of final demand are also called the cumulative energy requirements, total energy requirements, or the embodied energy of final demand. The total amount of primary energy that is required for producing final demand is allocated to this final demand. In principle, primary energy is used in a restricted number of energy sectors and distributed, in the form of goods and services, over all final deliveries.

First, section 3.2.1 gives the derivation of the model for calculating energy intensities followed by a discussion of the model on the basis of an example in section 3.2.2. After that, section 3.2.3 presents the model for the calculation of CO₂ intensities which is closely related to the energy intensity model.

### 3.2.1 Energy intensity model

One of the assumptions in the determination of energy intensities, \( e \), is that, for all economic sectors, energy input and energy output are in balance. The energy flowing out of a sector embodied in the production of that sector, has entered the sector via the intermediate deliveries and the direct primary energy use, \( c \) (figure 3.1). The equation for the energy balance is:

\[
 c^T + e^T Z = e^T x^D \tag{3.12}
\]

Further, it is assumed that, for all sectors, direct energy use is proportional with total production, in formula:

\[
 c^T = d^T x^D \tag{3.13}
\]

In (3.13), \( d \) is the vector of direct energy intensities. The direct energy intensity

![Figure 3.1](image)  
**Figure 3.1** Energy balance of economic sector i.
of an economic sector is the direct energy use of that sector per unit production of that sector. Substitution of equation (3.13) in (3.12) and using equation (3.4) gives:

\[ d^T x^D + e^T A x^D = e^T x^D \]  

(3.14)

Elimination of \( x^D \) and solving of \( e \) from (3.14) gives:

\[ e^T = d^T (I - A)^{-1} \]  

(3.15)

So, the energy intensity vector is the matrix product of the direct energy intensity vector and the Leontief inverse matrix. Multiplication of the energy intensity vector with the final demand vector gives the energy requirements of final demand, \( \varepsilon_{out} \). By using formulas (3.15) and (3.10), formula (3.16) demonstrates that \( \varepsilon_{out} \) equals total energy use of the production sectors, \( \varepsilon_{in} \):

\[ \varepsilon_{out} = e^T y = d^T (I - A)^{-1} y = d^T x = \varepsilon_{in} \]  

(3.16)

Formula (3.16) corresponds to the assumption that all energy entering the production sectors also flows out of these production sectors. The direct primary energy use of all production sectors is allocated to final demand.

Until now, in the model, the direct primary energy use of each sector is presented in one value. In some cases, e.g. if substitution between energy carriers is studied, it is necessary to differentiate between energy carriers. Another distinction concerns the difference in energy usage, i.e. as fuel or as feedstock. By using a matrix, \( C \), for the energy use data, the distinction is made. Analogous to formula (3.13), formula (3.17) gives the relation between \( C \) and the direct energy intensity matrix \( D \):

\[ C^T = D^T x^D \]  

(3.17)

The energy intensity matrix, \( E \), becomes analogous to formula (3.15):

\[ E^T = D^T (I - A)^{-1} \]  

(3.18)

Summing up the energy intensities in \( E \) for a sector gives the total primary energy intensity of that sector.
3.2.2 Example
This section discusses the model on the basis of a fictitious economy with 2 sectors (table 3.8). The first sector is an energy sector that converts primary energy into secondary energy. The second sector produces materials. It is assumed that the first sector converts 80 MJ primary energy. In accordance with the monetary deliveries, this primary energy is distributed over the two sectors and the final demand: the energy sector receives 20 MJ, the materials sector 20 MJ and final demand 40 MJ.

| Table 3.8 Input-output table of a fictitious economy (Dfl). |
|------------------|------------------|------------------|------------------|
|                  | energy           | materials        | final demand     | total            |
| energy           | 10               | 10               | 20               | 40               |
| materials        | 10               | 0                | 10               | 20               |
| primary inputs   | 20               | 10               |                  | 30               |
| total            | 40               | 20               | 30               |

Only sector 1 uses primary energy (80 MJ). The energy intensities of both sectors are determined by using the energy balance of both sectors. For each sector, total energy input, i.e. energy carriers and embodied energy in goods and services, must equal total energy output. With \( \mathbf{e} \) the vector of energy intensities, the energy balance of sector 1 is:

\[
80 \text{ MJ} + 10 \times e_1 + 10 \times e_2 = 40 \times e_1
\]

The energy balance for sector 2 is:

\[
0 \text{ MJ} + 10 \times e_1 = 20 \times e_2
\]

The solution of these equations is:

\[
e_1 = 16/5 \text{ MJ/Dfl}; \ e_2 = 8/5 \text{ MJ/Dfl}.
\]

Now, the vector of energy intensities is: \((16/5 \ 8/5)^T\) MJ/Dfl. Obviously, calculating the energy intensities with formula (3.15) gives the same results. This calculation requires the direct energy intensities. The direct energy intensity of sector 1 is 2 MJ/Dfl and the direct energy intensity of sector 2 is 0 MJ/Dfl. Moreover, table 3.9 shows the technological matrix \( \mathbf{A} \) and the Leontief inverse matrix \( \mathbf{B} \) of the fictitious economy.
The energy intensities are:

\[(2 \ 0) \ (I-A)^{-1} = (16/5 \ 8/5) \text{ MJ/Dfl.}\]

The energy intensity gives the energy required for one unit final demand. The total energy requirements for final demand are:

Final demand sector 1: 20 Dfl x 16/5 MJ/Dfl = 64 MJ.
Final demand sector 2: 10 Dfl x 8/5 MJ/Dfl = 16 MJ.

So, the 80 MJ primary energy that is used in the production sectors is attributed to final demand.

### 3.2.3 CO₂ intensities

The calculation of CO₂ intensities takes place analogous to the calculation of energy intensities\(^5\). The CO₂ intensity of an economic sector is the total CO₂ emission per unit production of that sector. Since CO₂ emissions are mainly resulting from the use of fossil fuels, a direct relationship between energy use and CO₂ emission exists.

The CO₂ emission that is allocated to the total production of an economic sector is the direct emission of that sector plus the indirect emission of that sector resulting from the production of the goods and services required by that sector. Formula (3.19) gives, analogous to formula (3.12), an emission balance. In the formula is \(k\) the direct emission vector and \(m\) the emission intensity vector.

\[k^T + m^T Z = m^T x^D\]  \hspace{1cm} (3.19)

Analogous to formula (3.13), it is assumed that the direct emission, \(k\), of a sector is proportional with the total production, \(x\), of that sector.

\[k^T = 1^T x^D\]  \hspace{1cm} (3.20)

\(^5\) This concerns not only CO₂, but also other emissions (see e.g. CBS, 1977 and 1979; Hordijk et al., 1979).

---

**Table 3.9** Technological matrix A and Leontief inverse matrix B of the fictitious economy.

<table>
<thead>
<tr>
<th></th>
<th>energy</th>
<th>materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy</td>
<td>1/4</td>
<td>1/2</td>
</tr>
<tr>
<td>materials</td>
<td>1/4</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>energy</th>
<th>materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy</td>
<td>8/5</td>
<td>4/5</td>
</tr>
<tr>
<td>materials</td>
<td>2/5</td>
<td>6/5</td>
</tr>
</tbody>
</table>
In formula (3.20), $l$ is the vector of direct emission intensities. The direct emission intensity is the emission per unit production. By using formula (3.4), formulas (3.19) and (3.20) are solved. The vector of emission intensities, $m$, is:

$$m^T = l^T (I - A)^{-1} \quad (3.21)$$

These formulas are valid for the emission of various substances. E.g., CBS (1977) investigated water pollution. In the case of $CO_2$ emissions, the direct emissions are related to the energy use per energy carrier. Each energy carrier has a specific emission coefficient which gives the amount of emission per unit energy by using that energy carrier. The formula for $l$ is, with $D$ the direct energy intensity matrix per energy carrier and $p$ the direct emission coefficient vector:

$$l^T = p^T D^T \quad (3.22)$$

The substitution of (3.22) in (3.21) gives:

$$m^T = p^T D^T (I - A)^{-1} \quad (3.23)$$

By using (3.18) this gives:

$$m^T = p^T E^T \quad (3.24)$$

In this formula, $E$ is the energy intensity matrix per energy carrier.

Energy sources are both used as fuel and as feedstock. By using energy as feedstock, no immediate CO$_2$ emission occurs. Possibly this emission takes place later on, e.g. with waste combustion. Assuming that, ultimately, all carbon stored in energy is emitted into the atmosphere, no distinction has to be made between fuel and feedstock. All carbon stored in energy sources, fuel and feedstock, will be allocated to final demand.

By using the example in 3.2.2, the calculation of the CO$_2$ intensities is illustrated. Table 3.8 shows the input-output table. The total primary energy use is 80 MJ for which it is assumed that it consists of two different energy carriers.

<table>
<thead>
<tr>
<th>Energy Carrier</th>
<th>MJ</th>
<th>kg/MJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy carrier 1</td>
<td>60</td>
<td>0.10</td>
</tr>
<tr>
<td>energy carrier 2</td>
<td>20</td>
<td>0.05</td>
</tr>
</tbody>
</table>
both with their own specific emission coefficient. Table 3.10 shows the energy use and the direct emission coefficients for both energy carriers.

The vector of CO₂ intensities, \( \mathbf{m} \), is calculated by using formula (3.23) and the Leontief inverse matrix (table 3.9):

\[
\mathbf{m}^T = \begin{bmatrix}
1/10 & 1/20 \\
1/2 & 0 \\
0 & 2/5 \\
3/2 & 0 \\
8/5 & 4/5 \\
2/5 & 6/5 \\
1/2 & 0 \\
14/50 & 7/50
\end{bmatrix}
\]

The vector of CO₂ intensities is: \((14/50 \ 7/50)^T\). So, for each Dutch guilder final demand of sector 1, the total CO₂ emission is twice as high as for one guilder final demand from sector 2.

3.3 INPUT-OUTPUT ENERGY ANALYSIS IN PRACTICE

According to formula (3.15), the energy intensities are calculated from the Leontief inverse matrix and the primary energy use of the sectors. Only the energy sectors that extract primary energy use energy. The model assumes that the energy via the intermediate deliveries is attributed to final demand (figure 3.2). One of the basic assumptions of input-output analysis is price uniformity. This means that all production sectors and final demand sectors pay the same price for all deliveries from a sector. Since, in practice, this is not the case for the energy sector, the deliveries from the energy sectors, in monetary terms, do not correspond to the physical deliveries. This problem was first recognized by Bullard and Herendeen (1975). As a solution for the problem, they introduced the physical units method\(^6\) (section 3.3.1), which represents the deliveries of the energy sectors in the input-output table in physical units. Herewith, the problem of energy prices is solved adequately.

Other solutions have been suggested for the price problem, since the physical units method is not applicable in all cases\(^7\). These solutions are based on the energy use data of economic sectors. Van Engelenburg et al. (1991) proposed the ERE conversion method that uses the energy use data of the non-energy sectors (section 3.3.2). The energy use of the energy sectors is attributed to the non-energy sectors by means of ERE values. Section 3.3.3 proposes the secondary energy method which does not make the difference between energy sectors and non-energy sectors. This method uses the energy use data of all sectors.

\(^6\) Bullard and Herendeen called this method the hybrid method.

\(^7\) E.g. in case a sector delivers both energy and non-energy products.
The next sections discuss the three methods indicated above. In case the energy price is the same for each sector, all methods lead to the same energy intensities for the non-energy sectors. The energy intensities of the energy sectors depend upon the chosen method. For all methods, the relation between the calculated energy intensities and those calculated with the primary energy method (section 3.2.1) is given.

3.3.1 Physical units method
The physical method uses hybrid input-output tables in which the deliveries of the energy sectors are given in physical units and the deliveries of the non-energy sectors in monetary units (Bullard and Herendeen, 1975; Miller and Blair, 1985). Since the energy deliveries are in physical units, the energy prices play no role anymore in the calculation of the energy intensities. For the calculation of the energy intensities, the technological matrix, $A^*$, and the Leontief inverse matrix, $(I-A^*)^{-1}$, are determined in turn. Each row of the Leontief inverse matrix contains the deliveries of the corresponding sector, in physical or monetary units, so that each sector is able to deliver one unit of final demand. The rows in the Leontief inverse matrix corresponding with the energy sector contain the energy requirements needed for one unit final demand for the different sectors. These energy requirements per unit production are the energy intensities. By premultiplying the Leontief inverse matrix with a vector, $d^*$, which contains an '1' for the energy sector and a '0' for the other sectors, the energy intensities are filtered out. The formula for the calculation of the energy intensities is:

\[ (I-A^*)^{-1} \]

For reasons of convenience, the formulas assume only one energy sector. The possible extension to more energy sectors according to formulas (3.17) and (3.18) is self-evident.
The hybrid energy intensity, $e^*$, is calculated easily from the energy intensity, $e$, which is calculated in 3.2.1. The relationship is:

$$(e^*)^T = (e^T)^T (I - A^*)^{-1}$$  

(3.25)

$P$ is a diagonal matrix with the diagonal elements of the energy sectors equal to the energy prices and the other diagonal elements equal to unity. Appendix 3.A.1 contains a derivation for formula (3.26).

Table 3.11 shows the hybrid version of the input-output table of the example in 3.2.2. The 80 MJ primary energy used in the energy sector is divided over the sectors in accordance with the monetary deliveries assuming an energy price of 0.5 Dfl/MJ.

<table>
<thead>
<tr>
<th>materials</th>
<th>final demand</th>
<th>total prod.</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy (MJ)</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>materials (Dfl)</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3.12 shows the Leontief inverse matrix for the hybrid input-output table of the fictitious economy. Now, the first row of the Leontief inverse matrix contains the energy intensities: $(8/5 8/5)^T$ (MJ/MJ MJ/Dfl). The distribution over the final deliveries is:

- Final deliveries sector 1: 40 MJ x $8/5$ MJ/MJ = 64 MJ.
- Final deliveries sector 2: 10 Dfl x $8/5$ MJ/Dfl = 16 MJ.

Using the energy price of 0.5 Dfl/MJ, the energy intensities calculated with formula (3.26) are:

$$(e^*)^T = e^T P = \begin{bmatrix} 16/5 & 8/5 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8/5 & 8/5 \end{bmatrix}$$
3.3.2 ERE conversion method

Engelenburg et al. (1991) proposed another method for treating the price problem. This method uses the direct energy use data, both primary and secondary, of the non-energy sectors. The model in formula (3.15) calculates the energy intensities. The energy use in each non-energy sector is expressed in primary energy by allocating the losses at conversion in the energy sectors to the non-energy sectors. This allocation is made by using ERE values of energy carriers (see 2.3). To avoid double counting, the energy use of the energy sectors should be zero (figure 3.3).

The energy use of the final demand sectors is allocated directly to these sectors. Therefore this energy use and the corresponding conversion losses in the energy sectors are not included in the energy use of the production system. Since the total production remains unchanged, the energy intensities of the energy sectors are lower than those calculated with the primary energy method (3.2.1). The relationship between the energy intensity vector calculated in 3.2.1, \( e \), and the energy intensity vector calculated in this section, \( e' \), is:

\[
(e') = e - Qd
\]

In formula (3.27), \( d \) is the direct energy intensity vector in 3.2.1 and \( Q \) is a diagonal matrix with the diagonal elements of the energy sectors equal to the ERE values of the energy carriers and the other diagonal elements equal to unity. Since in \( d \) only the element of the energy sector is non-zero, in the new energy intensity vector only the value of the energy sector is changed.

The example only considers the energy use of sector 2, which is the non-energy sector. The energy use is 20 MJ (see table 3.11). The energy use of sector 1, 20 MJ, is distributed over sector 2 and the final demand. Actually, the energy use of sector 2 and final demand, 60 MJ, require 80 MJ of primary energy.

Figure 3.3 Calculation of energy intensities with the ERE conversion method (\( d \) = direct energy intensity vector; \( e \) = total energy intensity vector).
energy. So, the ERE value is 4/3 MJ per MJ. The primary energy use of sector 2 becomes 80/3 MJ and the primary energy use of final demand becomes 160/3 MJ. The direct energy intensity vector is: (0 4/3) MJ/Dfl. The energy intensity is (using table 3.9):

\[(0 4/3) (I-A)^{-1} = (8/15 8/5) \text{ MJ/Dfl.}\]

The energy requirements of final demand are calculated as follows:

\[
(8/15 8/5) (20 10)^T + (8/3 0) (20 10)^T = 80/3 + 160/3 = 80 \text{ MJ.}
\]

The energy requirements of the final deliveries of sector 1 are 64 MJ and those of sector 2 are 16 MJ. Calculating the energy intensities using formula (3.27) gives the same result:

\[
(e^T - e - Qd = \begin{bmatrix} 16/5 \\ 8/5 \end{bmatrix} - \begin{bmatrix} 4/3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 8/15 \\ 8/5 \end{bmatrix})
\]

The energy intensity of sector 2 calculated with the ERE conversion method is the same as calculated with the physical units method. The energy intensity of sector 1 is lower, since the new energy intensity does not contain the direct energy use of final demand. The energy intensity of the energy sector only concerns the indirect energy use of this sector, i.e. the energy embodied in the goods and services delivered to the energy sector. In case of different energy prices, errors only arise in this indirect part.

### 3.3.3 Secondary energy method

The third method for solving the price problem uses the energy use data of all sectors. This method does not require a conversion from secondary energy to primary energy with ERE values. The energy needed for conversion in the energy sectors is attributed to the intermediate sectors and final demand through the deliveries in the input-output table (figure 3.4).
This method also calculates the energy intensities using formula (3.15). The difference with the ERE conversion method concerns the direct energy intensity vector. The energy intensity of the energy sector calculated in this way does not contain the energy use of final demand, but it contains the conversion losses in the energy sector in favour of this energy use. The relationship between the energy intensities from the secondary energy method, $e^2$, and those from the primary energy method (section 3.2.1) is:

$$(e^2) = e - d$$

(3.28)

The energy intensities calculated with the secondary energy method have a lower energy intensity for the energy sector. The energy intensities of the non-energy sectors are the same. Appendix 3.A.2 contains the derivation of formula (3.28).

Table 3.11 shows that the energy use of both production sectors is 20 MJ. The direct energy intensity vector is $(1/2\ 1)$ MJ/Dfl. The energy intensity vector becomes:

$$(1/2\ 1) (I-A)^{-1} = (6/5\ 8/5) \text{ (MJ/Dfl)}$$

The energy requirements of final demand are calculated as follows:

$$(6/5\ 8/5) (20\ 10) + (2\ 0) (20\ 10)^T = 40 + 40 = 80 \text{ MJ}.$$ 

The energy requirements of the final deliveries of sector 1 are 64 MJ and those of sector 2 are 16 MJ. Calculating the energy intensities using formula (3.28) gives the same result:

$$(e^2)^T = e^T - d^T = \left[ 16/5\ 8/5 \right] - \left[ 2\ 0 \right] = \left[ 6/5\ 8/5 \right]$$

This method results in the same energy intensity for sector 2 as the previous methods. The energy intensity of sector 1 lies between the values of the primary energy method and the ERE conversion method, since this intensity does not contain the direct energy use, but contains the losses in the energy sector for the production of these final energy deliveries. Since the intermediate deliveries pass on the conversion losses, the errors in the energy intensities that occur in the case of different energy prices will be higher than with the ERE conversion method. This can be corrected by changing the deliveries of the energy sectors in the input-output table on the basis of the energy prices (Kazemier and Zijlmans, 1991). In fact, this is a variant of the physical units method.
3.4 HOUSEHOLD ENERGY REQUIREMENTS

The energy intensity of an economic sector gives the required amount of energy per unit deliveries of that sector. According to formula (3.16), the energy requirements of specific deliveries, e.g. final demand \( y \), are determined as follows:

\[
\varepsilon = d^T (I-A)^{-1} y = e^T y = \sum_i e_i y_i \tag{3.29}
\]

In input-output tables, household consumption is part of final demand. So, by multiplying the energy intensity of each sector with the deliveries to households and summing over all sectors, household energy requirements are calculated. Similarly, the energy requirements of other final demand categories, e.g. exports or investments, or of total production are determined. An alternative expression for (3.29) is:

\[
\varepsilon = d^T (I-A)^{-1} y = d^T x_h \tag{3.30}
\]

In this model, \( x_h \) is the household production vector which contains for each production sector the total production needed for household consumption, direct and indirect. E.g., for the steel industry the household production includes both the steel for the appliances of the households and the production of the steel for the machines in the electrotechnical industry to produce these appliances. Chapter 7 uses expression (3.30) in the calculations concerning future household energy requirements.

Chapters 5, 6 and 7, determine the energy requirements of households for the base year, the past and the future, respectively. For the calculations of the energy intensities, the ERE conversion method (section 3.3.2) is used. Since the energy intensities calculated with this method do not include the direct energy deliveries to households, formula (3.29) only concerns the indirect energy requirements of households. The addition of direct energy requirements of households and the corresponding energy use in the energy sectors to the indirect energy requirements gives total energy requirements of households. Energy statistics provide the direct energy use data of households. The energy required by the energy sectors for conversion is determined by ERE values.
3.5 UNCERTAINTIES IN INPUT-OUTPUT ANALYSIS

The previous section presented the input-output model for the determination of the energy requirements of households. An analysis of the model is needed for getting an impression of the plausibility and reliability of the outcomes of the method. Such an analysis consists of an uncertainty analysis and a sensitivity analysis. The uncertainty analysis investigates the uncertain aspects of the method and the effects of these uncertainties on the outcomes of the method. The sensitivity analysis investigates the influence of variations in the input parameters of the method on the outcomes (Janssen et al., 1990). This chapter only concerns uncertainties. A sensitivity analysis is part of chapter 7 in which prospective changes in production structure and consumption patterns are investigated.

3.5.1 Uncertainties in input-output tables

The input-output method uses aggregated data of economic sectors from input-output tables. By using aggregated data of economic sectors, the following problems arise (a.o. Casper et al., 1974; Bullard et al., 1978; Boustead and Hancock, 1979):

- One economic sector produces a large amount of products which all need their own inputs. The outcomes for each sector are valid only for an average product of that sector.
- The data of an economic sector are based on the data of a large amount of companies which all have their own efficiency. Differences in efficiency between companies remains hidden in the aggregated sectoral data.
- Each company is attributed to a single economic sector, often on the basis of the main product. Other products possibly belong to other sectors.

Furthermore, the processes of collecting and editing input-output data result in errors. Moreover, the balancing of input-output tables may also introduce errors.

3.5.2 Uncertainties in energy intensities

The uncertainty analysis concerns the investigation of the effects of uncertainties in the technological matrix and in the direct energy use of the production sectors on the energy intensities. Starting point is the model in formula (3.15). Assuming a certain reliability of the model parameters, $A$ and $d$, the reliability of the energy intensities is determined. First, the reliability of the Leontief inverse matrix, $B = (I-A)^{-1}$ is examined, assuming a given reliability of the technological matrix. Sebald (1974), and Bullard and Sebald (1977) determined analytically upper and lower bounds for the unreliabilities in the Leontief
inverse matrix. These bounds occur when all elements of the technological matrix have their extreme uncertainties in the same direction.

Sebald assumes that each element of the technological matrix $A_{ij}$ lies in a specific interval $[\alpha_{ij}, \beta_{ij}]$ around the nominal value (with $\alpha_{ij} \leq 0$, $\beta_{ij} \geq 0$). This leads to an unlimited number of new technological matrices $A^n$. For each matrix $A^n$, the Leontief inverse matrix, $B^n = (I - A^n)^{-1}$, is calculated. For each element of the original Leontief inverse matrix, $B_{ij}$, there exist an interval $[\gamma_{ij}, \delta_{ij}]$, $\gamma_{ij} \leq 0$, $\delta_{ij} \geq 0$, which contains all possible values of the corresponding element of $B^n$. Now, the question is what is the interval for each element of $B$, so that for all possible matrices $B^n$, all elements lie in these intervals. Sebald (1974) called this problem the tolerance problem.

Sebald demonstrated that for each $A$ there exists a specific $A^n$, so that for each element of $B$ the difference with the corresponding element of $B^n$ has its maximum value. This 'bad' case arises if all elements of $A^n$ have their maximum deviation. The elements of $B^n$ have their maximum value if for all elements of $A^n$ the value $\beta_{ij}$ is chosen. The elements of $B^n$ have their minimum value if for all elements of $A^n$ the negative value $\alpha_{ij}$ is chosen.

A prerequisite for this method is that all deviations have to lie in the same direction, thus all negative or all positive. A stochastic approach which assumes random uncertainties around the elements does not have this restriction. A disadvantage of a stochastic approach is the large number of matrix inversions that has to be calculated. Nevertheless, Sebald recommends investigations concerning the stochastic approach. He expects much lower uncertainties in the Leontief inverse matrix. Bullard en Sebald (1988) confirmed this expectancy.

After calculation of the Leontief inverse matrices with the maximum deviations, the effect of the changes in these matrices on the total energy intensities can be calculated. The effect of uncertainties in the direct energy intensities can be considered too.

### 3.5.3 Example

The uncertainty analysis is illustrated using example 3.2.2. Given the technological matrix and Leontief inverse matrix (table 3.9), and direct and total energy intensities:

\[
A = \begin{bmatrix} 1/4 & 1/2 \\ 1/4 & 0 \end{bmatrix}, \quad d = \begin{bmatrix} 2 \\ 0 \end{bmatrix}
\]
Around each element of the technological matrix, a positive and negative deviation of 10% is assumed. The maximum deviations in the Leontief inverse matrix arise with the maximum deviations in the technological matrix. The new technological matrices are:

$$A_n = \begin{bmatrix} 11/40 & 0 \\ 11/40 & 0 \end{bmatrix}, \quad A_n^e = \begin{bmatrix} 9/40 & 0 \\ 9/40 & 0 \end{bmatrix}$$

The Leontief inverse matrices, $B^n$, and the percentage deviations compared with matrix $B$ are:

$$B_n^e = \begin{bmatrix} 40 & 22 \\ 11 & 29 \end{bmatrix}, \quad \text{Percentage deviations: } \begin{bmatrix} 8,9\% & 19,8\% \\ 19,8\% & 5,3\% \end{bmatrix}$$

$$B_n^e = \begin{bmatrix} 40 & 18 \\ 9 & 31 \end{bmatrix}, \quad \text{Percentage deviations: } \begin{bmatrix} -7,2\% & -16,5\% \\ -16,5\% & -4,1\% \end{bmatrix}$$

The example shows that the elements of the Leontief inverse matrix are not equally influenced by the uncertainties of the elements in the technological matrix. By using graphical techniques, this can be made clear for large matrices (Sebald, 1974). Furthermore, the negative deviations are smaller than the positive deviations.

Assuming an uncertainty of 10% in the direct energy intensities, the direct energy intensities are:

$$d^n = \begin{bmatrix} 11/5 \\ 0 \end{bmatrix}, \quad d_n^e = \begin{bmatrix} 9/5 \\ 0 \end{bmatrix}$$

Combining the uncertainties in the direct energy intensities and in the Leontief inverse matrix, the energy intensities and their percentage deviations compared with the original intensities are:
Above, upper and lower bounds concerning the uncertainties in the energy intensities are determined. It is emphasized that these maximum uncertainties occur in the most unfavourable case, namely the case in which all elements have a maximum deviation. In chapter 5, the uncertainties for the energy intensities for the Netherlands are determined. These maximum uncertainties are compared with uncertainties derived from a stochastic analysis.

### 3.6 DISCUSSION AND CONCLUSIONS

This chapter presented the input-output model for the determination of household energy requirements. A main part of the model concerns the calculation of energy intensities (formula 3.15). The original method for calculating the energy intensities, which uses the primary energy use of the energy sectors, turns out to produce errors when the energy prices for the sectors are different. Three solutions for this problem are discussed. The physical units method, in 3.3.1, uses physical units for the energy deliveries and the energy price problem is fully solved. The ERE conversion method in 3.3.2, and the secondary energy method in 3.3.3 use a monetary input-output table. These methods use the energy use data of the economic production sectors. Since the direct energy use of the final consumption sectors is excluded from the calculation, the energy intensities of the energy sectors are lower than the intensities calculated with the primary energy method. The errors that occur due to different energy prices are lower with the ERE conversion method than with the secondary energy method, since the first method directly allocates the conversion losses in the energy sectors to the sectors by means of ERE values. On the other hand, the secondary energy method allocates the conversion losses to the energy sectors. Since the losses actually take place in these sectors, this method seems more accurate.

The ERE conversion method allocates the energy use in the energy sectors to the non-energy sectors by using ERE values which are derived from other studies. If these studies define other system boundaries, errors can occur. Applying the method to CO₂ emission, the emission in the energy sectors has to be allocated to the non-energy sectors by using emission coefficients. The
emission coefficient of an energy carrier contains the emissions that arise with the production of the energy carrier on a per unit base.

All goods and services used in the production process should be involved in the calculation of the energy intensities. Section 3.1 discussed how to take into account the competitive imports and the capital goods in the calculation of the Leontief inverse matrix. The non-competitive imports have to be treated in a different way. Since a major part of non-competitive imports concerns services[^10], the embodied energy of the non-competitive imports is determined by using an average energy intensity of services. This energy intensity is also used in order to determine the energy requirements of the goods and services for which the producing sector is not known.

Chapter 5 calculates household energy requirements for the base year 1990 by using the input-output model. The next chapter also contains a comparison of energy intensities calculated by three different methods. Theoretically, the three methods should produce the same results for the energy intensities. Further, chapter 5 studies the influence of imports and capital goods on the energy intensities.

APPENDIX 3.A: DERIVATIONS

This appendix contains the derivations of formulas (3.26) and (3.28).

3.A.1 Derivation formula (3.26)
Assuming the same energy price for all sectors, there is a simple relation between the energy intensities calculated with the physical units method (3.3.1) and the energy intensities calculated with the primary energy method (3.2.1). Formula (3.26) shows this relationship. This section gives the derivation of this relationship.

P is a diagonal matrix with the diagonal elements of the energy sectors equal to the energy prices and the other diagonal elements equal to unity. By using P, the hybrid matrices are expressed in monetary matrices using the following equations:

\[ Z^* = P^{-1} Z \quad \text{and} \quad x^* = P^{-1} x \]

The hybrid technological matrix is:

\[ \Lambda^* = Z^* ((x^*)^D)^{-1} = P^{-1} Z ((P^{-1} x)^D)^{-1} = P^{-1} Z (P^{-1} x^D)^{-1} \]

[^10]: Services are non-competitive by definition (Kazemier and Zijlmans, 1991).
\[ = P^{-1} Z (x^D)^{-1} P = P^{-1} A P \]

The hybrid direct energy intensity vector is:

\[
(d^*)^T = c^T ((x^D)^{-1}) = c^T (P^{-1} x)^{-1} = c^T (P^{-1} x D)^{-1} = c^T (x D)^{-1} P = d^T P
\]

Substitution of both equations in formula (3.25) gives:

\[
(e^*)^T = (d^*)^T (I - A)^{-1} = d^T P (I - P^{-1} A P)^{-1} = d^T ((I - P^{-1} A P)^{-1}) = d^T (P^{-1} - P^{-1} A)^{-1} = d^T (P^{-1} (I - A))^{-1} = d^T (I - A)^{-1} P = e^T P
\]

### 3.A.2 Derivation formula (3.28)

Formula (3.28) shows the relation between the energy intensities calculated with the secondary energy method and those calculated with the primary energy method. The secondary energy method uses a different direct energy intensity vector \((d^2)\):

\[
(d^2)^T x^D = d^T Z
\]

This gives:

\[
(d^2)^T = d^T Z (x^D)^{-1} = d^T A
\]

The energy intensities are calculated with formula (3.15). Substituting (3.A.2) in this formula gives:

\[
(e^2)^T = (d^2)^T (I - A)^{-1} = d^T A (I - A)^{-1} = d^T ((I - A)^{-1} - I) = d^T (I - A)^{-1} - d^T = e^T - d^T
\]