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Bergshoeff, E.; Sezgin, E.; Nishino, H.

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HETEROTIC σ -MODELS AND CONFORMAL SUPERGRAVITY IN TWO DIMENSIONS

E. BERGSHOEFF, E. SEZGIN

International Centre for Theoretical Physics, I-34100 Trieste, Italy

and

H. NISHINO

Department of Physics, University of Maryland, College Park, MD 20742, USA

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The $(1, 0)$ and $(2, 0)$ type heterotic σ -models with Wess–Zumino term are coupled to conformal supergravity in two dimensions. There are no new restrictions on the σ -model manifolds in addition to those which arise in the globally supersymmetric cases. In the $(1, 0)$ case possible isometries of the scalar manifold are gauged. A derivation of $d = 2$ conformal supergravity based on the super Lie algebra $\text{OSp}(2, N) \oplus \text{OSp}(2, N)$ ($N = 1, 2$) is given.

1. Introduction. Conformally invariant σ -models in two dimensions are potential candidates for string theories. For example, the Neveu–Schwarz–Ramond string theory [1] is described by an $N = 1$ locally supersymmetric σ -model parametrizing the lorentzian plane $\text{ISO}(9, 1)/\text{SO}(9, 1)$ in two dimensions [2]. Another example is the Green–Schwarz superstring theory [3] described by a SUSY ($N = 2$)/ $\text{SO}(9, 1)$ locally supersymmetric σ -model with Wess–Zumino term in $d = 2$ [4].

Recently, an $N = 1$ locally supersymmetric σ -model with Wess–Zumino term based on an arbitrary riemannian manifold, M , has been constructed [5]. In the case $M = M_d \times G$, where M_d is a d -dimensional Minkowski space and G is a group manifold, this model has led to a lowering of the critical dimension [6,5]. More recently, Witten has constructed [7] the Green–Schwarz superstring action in curved background. The question of which backgrounds other than $d = 10$ Minkowski spacetime are consistent at the quantum level is not known at present.

A natural extension of the results mentioned above is to consider $d = 2$ σ -models with extended supersymmetry. In this context we recall that an NSR type string moving in Minkowski background with two dimensional $N = 2$ supersymmetry was constructed in refs. [8,9]. In this model the critical dimension turned out to be two [8,10]. In analogy with the case of $N = 1$, it is expected that the critical dimension will change for an $N = 2$ superstring moving in curved background.

In this paper, we extend the results of refs. [8,9] by constructing a locally supersymmetric σ -model with Wess–Zumino term describing an $N = 2$ superstring moving in curved space. We extensively make use of the recent work of Hull and Witten [11] which describes globally supersymmetric $(1,0)$ and $(2,0)$ heterotic σ -models in $d = 2$, where the notation (p, q) refers to p left-handed and q right-handed spinorial supercharges. These models are heterotic in the sense that they describe couplings of right-handed fermions with no bosonic partners to ordinary scalar supermultiplets containing left-handed fermions with bosonic partners. In fact, it is such a model which describes the heterotic string [12].

We first consider the coupling of a $(1,0)$ heterotic σ -model to supergravity. As in the globally supersymmetric case [13], the scalar manifold is an arbitrary riemannian manifold. We then gauge the isometries (if any) of this manifold.

The $N = 2$ globally supersymmetric σ -models [13] with Wess–Zumino term are hermitian manifolds with torsion [14,11]. For vanishing torsion these manifolds are Kähler. We find that no new restrictions arise on the scalar manifolds as we couple them to $N = 2$ conformal supergravity. We leave to the future the investigation of the conformal invariance at the quantum level, along the lines of ref. [5], and thus the strong application of this model. In this context see ref. [15].

In this note we also give a derivation of the transformation rules of $N = 1, 2$ conformal supergravity in $d = 2$, based on the Lie superalgebra $\text{OSp}(2, N) \oplus \text{OSp}(2, n)$, ($N = 1, 2$).

2. The $(1, 0)$ heterotic σ -model and its gauging. The ungauged $(1, 0)$ σ -model with Wess–Zumino term coupled to $N = 1$ conformal supergravity can be obtained from the $(1, 1)$ model of ref. [5]. The fields of the $(1, 1)$ model are (ϕ^i, χ^i) and (e_μ^a, ψ_μ) where ϕ^i are the coordinates of an arbitrary Riemann manifold, χ^i are their fermionic partners, e_μ^a ($\mu, a = 1, 2$) is the zweibein and ψ_μ is the gravitino field. The $(1, 0)$ model is obtained by the following chiral truncation:

$$(1 - \gamma_5)\psi_\mu = 0, \quad (1 + \gamma_5)\chi^i = 0, \quad (1 - \gamma_5)\epsilon = 0. \quad (1)$$

The resulting $(1, 0)$ lagrangian is ^{†1}

$$e^{-1}\mathcal{L}_1 = -\frac{1}{2}(g^{\mu\nu}g_{ij} + \alpha\epsilon^{\mu\nu}b_{ij})\partial_\mu\phi^i\partial_\nu\phi^j - \frac{1}{2}i\bar{\chi}_i\gamma^\mu(\partial_\mu\chi^i + \Gamma_{jk}^i\partial_\mu\phi^j\chi^k) - i\bar{\psi}_\mu\gamma^\nu\gamma^\mu\chi_i\partial_\nu\phi^i \\ - \frac{1}{8}\alpha T_{ijk}\bar{\psi}_\mu\chi^i\bar{\chi}^j\gamma^\mu\chi^k, \quad (2)$$

where α is an arbitrary constant,

$$b_{ij}(\phi) = -b_{ji}(\phi), \quad T_{ijk} = \frac{1}{2}(\partial_i b_{jk} + \partial_k b_{ij} + \partial_j b_{ki}), \quad (3)$$

and the connection Γ_{jk}^i which contains torsion is given by

$$\Gamma_{jk}^i = \{_{jk}^i\} - \alpha T_{jk}^i. \quad (4)$$

The action of $\mathcal{L}_1$ is invariant under

$$\delta e_\mu^a = 2i\bar{\epsilon}\gamma^a\psi_\mu - \Lambda_D e_\mu^a + \Lambda_M \epsilon^{ab} e_\mu^b, \quad \delta\psi_\mu = [\partial_\mu + \frac{1}{2}\omega_\mu(e, \psi)]\epsilon + \gamma_\mu\eta - \frac{1}{2}\Lambda_D\psi_\mu - \frac{1}{2}\Lambda_M\psi_\mu, \\ \delta\phi^i = -i\bar{\epsilon}\chi^i, \quad \delta\chi^i = -\gamma^\mu(\partial_\mu\phi^i + i\bar{\psi}_\mu\chi^i)\epsilon + \frac{1}{2}\Lambda_D\chi^i + \frac{1}{2}\Lambda_M\chi^i, \quad (5)$$

where $(\epsilon, \eta, \Lambda_D, \Lambda_M)$ are the parameters of supersymmetry, special supersymmetry, dilatation and Lorentz transformations, respectively, and

$$\omega_\mu(e, \psi) = \epsilon^{\rho\sigma}(-e_\mu^a\partial_\rho e_{\sigma a} + i\bar{\psi}_\rho\gamma_\mu\psi_\sigma).$$

Following Hull and Witten [11] we now add to the above lagrangian a new invariant describing a set of right-handed fermions, ψ^A , with no bosonic partners, given by

$$e^{-1}\mathcal{L}_2 = -\frac{1}{2}i\bar{\psi}^A e_\mu^a\gamma^a(\partial_\mu\psi^B + A_i^B{}_C\partial_\mu\phi^i)\psi^C G_{AB}(\phi) - \frac{1}{8}\hat{F}_{ijAB}\bar{\chi}^i\gamma^\mu\chi^j\bar{\psi}^A\gamma_\mu\psi^B. \quad (6)$$

Here, ψ^A ($A = 1, \dots, n$) takes value in an n -dimensional vector bundle, V , over the scalar manifold (which need not be the tangent bundle), $G_{AB}(\phi)$ is the metric on the fibers and $A_i^B{}_C(\phi)$ is the connection on V . The structure group of V is $G \subseteq \text{SO}(n)$, and the G -valued curvature $\hat{F}_{ijAB}$ is defined by

$$\hat{F}_{ijAB} = \partial_i\hat{A}_{jAB} + \hat{A}_{iAC}\hat{A}_{jB}{}^C - (i \leftrightarrow j), \quad (7)$$

^{†1} Conventions: $\eta_{ab} = \text{diag}(-1, +1)$; $\gamma^0 = -i\sigma^2$, $\gamma^1 = \sigma^1$, $\gamma^5 = \sigma^3$, $\epsilon_{01} = +1$; $\gamma_\mu\gamma_\nu = \eta_{\mu\nu} + \epsilon_{\mu\nu}\gamma_5$, $\epsilon^{\mu\nu}\gamma_\nu = -\gamma^\mu\gamma_5$; Fierz formula for Majorana fermions: $\psi\bar{\chi} = -\frac{1}{2}(\bar{\chi}\psi) - \frac{1}{2}(\bar{\chi}\gamma_5\psi)\gamma_5 - \frac{1}{2}(\bar{\chi}\gamma^\lambda\psi)\gamma_\lambda$.

where

$$\hat{A}_i^A{}_B = A_i^A{}_B + \frac{1}{2} G^A{}_C \partial_i G_{CB} . \quad (8)$$

We find that the action of $\mathcal{L}_1 + \mathcal{L}_2$, without any further modification, is invariant under the local superconformal transformations (5), with ψ^A transforming as follows [11]:

$$\delta \psi^A = i\bar{\epsilon} \chi^i \hat{A}_i^A{}_B \psi^B + \frac{1}{2} \Lambda_D \psi^A - \frac{1}{2} \Lambda_M \psi^A . \quad (9)$$

The fact that the second term in (8) is needed can be seen as follows: The variation of $\partial_\mu \phi^i$ and G_{AB} in the ψ^A kinetic term is easily seen to yield

$$-(D_\mu \bar{\psi}^A) \gamma^\mu (\bar{\epsilon} \chi^i) (A_{iAB} + \frac{1}{2} G_{AB,i}) \psi^B . \quad (10)$$

This is cancelled by defining $A_{iA}{}^B$ as in (8) and varying ψ^A as in (9) in the ψ^A -kinetic term. Regarding the local supersymmetry of (6) the structures $\epsilon \psi_\mu \psi^\mu$, $\epsilon \chi \psi^2 \psi_\mu^2$ and $\epsilon \psi_\mu \chi^2 \psi^2 F$ occur, and they are easily shown to cancel.

Suppose that the scalar manifold admits isometries characterized by a set of Killing vectors $\xi^{i(r)}$, $r = 1, \dots, \dim H$, where H is the isometry group

$$\delta \phi^i = \Lambda^{(r)} \xi^{i(r)} . \quad (11)$$

Here $\Lambda^{(r)}$ is a constant gauge parameter. One can gauge H , and thus take $\Lambda^{(r)}$ to be a function of x , by introducing an off-shell vector multiplet consisting of left-handed fermions $\lambda^{(r)}$ and vector fields $A^{(r)}$ transforming as

$$\delta A_\mu^{(r)} = 2i\bar{\epsilon} \gamma_\mu \lambda^{(r)}, \quad \delta \lambda^{(r)} = -\frac{1}{4} \epsilon^{\mu\nu} F_{\mu\nu} \epsilon + i\bar{\lambda}^{(r)} \gamma^\mu \psi_\mu . \quad (12)$$

The derivatives of ϕ^i and χ^i must now be gauge covariantized as follows [16]:

$$D_\mu \phi^i = \partial_\mu \phi^i - g A_\mu^{(r)} \xi^{i(r)}, \quad D_\mu \chi^i = \partial_\mu \chi^i - \frac{1}{2} \omega_\mu(e, \psi) \chi^i + \Gamma_{jk}^i D_\mu \phi^j \chi^k - g A_\mu^{(r)} \partial_j \xi^{i(r)} \chi^j , \quad (13)$$

where g is the gauge coupling constant. These covariantizations break supersymmetry which can be restored by adding to $\mathcal{L}_1 + \mathcal{L}_2$ the following term

$$e^{-1} \mathcal{L}_3 = -2ig \bar{\chi}^i \lambda^{(r)} \xi^{j(r)} (g_{ij} - \alpha b_{ij}) . \quad (14)$$

It should be emphasized that both the gauge invariance of the action as well as the cancellation of the $g b_{ij} \chi^2 \lambda \epsilon$ terms arising in the supersymmetric variation of the action require that b_{ij} has the vanishing Lie derivative

$$\mathcal{L}_{\xi^{(r)}} b_{ij} = 0 . \quad (15)$$

Note also that, due to the presence of the gauge coupling dependent terms, the action is no longer invariant under the composite tensor gauge transformation: $\delta b_{ij} = \partial_i \Lambda_j - \partial_j \Lambda_i$. Gauged Wess–Zumino actions for group manifolds in $d = 2$ flat superspace have been discussed in ref. [17].

3. The $(2, 0)$ heterotic σ -model coupled to $(2, 0)$ conformal supergravity. The globally supersymmetric $(2, 0)$ heterotic σ -model with Wess–Zumino term has been constructed in refs. [11,14]. The model contains n complex scalars, ϕ^α ($\alpha = 1, \dots, n$), and n Weyl spinors χ^α . It has been shown that the scalars parametrize a hermitian manifold with torsion [11,14]. In the case of vanishing torsion, there is no Wess–Zumino term and the manifold becomes Kähler. We wish to couple the $(2, 0)$ model with Wess–Zumino term to $(2, 0)$ conformal supergravity [8,9] containing the fields: the zweibein e_μ^m , the gravitino ψ_μ , and a real vector field A_μ .

We find that there are no new restrictions on the scalar manifold in addition to those which arise in the globally supersymmetric case. The properties of the scalar manifold [11,14], which play an important role in providing the local supersymmetry of the action are worth recalling.

The scalar manifold is hermitian. Thus, $g\alpha\beta = 0$. Moreover, it is endowed with a connection, Γ , with a totally antisymmetric torsion, T , with respect to which the complex structure of the manifold is covariantly constant. This implies that $\Gamma_{\beta\gamma}^\alpha = 0$, and $\Gamma_{\beta\gamma}^\alpha = 0$. Hence, writing $\Gamma = \{ \} + T$ schematically, one finds [11]

$$T_{\alpha\beta\bar{\gamma}} = \frac{1}{2}(g_{\alpha\bar{\gamma},\beta} - g_{\beta\bar{\gamma},\alpha}), \quad T_{\alpha\beta\gamma} = 0. \quad (16)$$

It is required for the construction of the Wess–Zumino term, and consistent with the above, to write the torsion as the curl of an antisymmetric potential $b_{\alpha\bar{\beta}} = -b_{\beta\bar{\alpha}}$, as follows:

$$T_{\alpha\beta\bar{\gamma}} = \frac{1}{2}(b_{\alpha\bar{\gamma},\beta} - b_{\beta\bar{\gamma},\alpha}). \quad (17)$$

Using (16) and (17) repeatedly, the usual Noether procedure yields the following result for the (2, 0) heterotic σ -model coupled to (2, 0) conformal supergravity:

$$e^{-1}\mathcal{L}_1 = \frac{1}{2}(g^{\mu\nu}g_{\alpha\bar{\beta}} - \epsilon^{\mu\nu}b_{\alpha\bar{\beta}})\partial_\mu\phi^\alpha\partial_\nu\phi^{\bar{\beta}} + (\frac{1}{2}i\bar{\chi}_\beta\gamma^\mu D_\mu\chi^\beta + \bar{\chi}_\beta\gamma^\nu\gamma^\mu\psi_\nu\partial_\mu\phi^\beta + \bar{\chi}_\beta\gamma^\nu\gamma^\mu\bar{\psi}_\mu\chi^\beta + 2iT_{\alpha\beta\bar{\gamma}}\bar{\psi}_\mu\chi^\alpha\bar{\chi}^{\bar{\gamma}}\gamma^\mu\chi^\beta + \text{h.c.}), \quad (18)$$

where

$$D_\mu\chi^\beta = [\partial_\mu\chi^\beta - \frac{1}{2}\omega_\mu(e, \psi)\chi^\beta + \Gamma_{\gamma\delta}^\beta\partial_\mu\phi^\gamma\chi^\delta + \Gamma_{\gamma\delta}^\beta\partial_\mu\phi^{\bar{\gamma}}\chi^{\bar{\delta}} - iA_\mu\chi^\beta]. \quad (19)$$

The action $I_1 = \int d\sigma d\tau \mathcal{L}_1$ has the following invariances

$$\begin{aligned} \delta e_\mu^a &= -2i\bar{\epsilon}\gamma^a\psi_\mu + \text{h.c.} - \Lambda_D e_\mu^a + \Lambda_M \epsilon^{ab} e_\mu^b, \\ \delta\psi_\mu &= [\partial_\mu + \frac{1}{2}\omega_\mu(e, \psi) + iA_\mu]\epsilon + \gamma_\mu\eta - \frac{1}{2}\Lambda_D\psi_\mu - \frac{1}{2}\Lambda_M\psi_\mu + i\Lambda\psi_\mu, \\ \delta A_\mu &= -e^{-1}\epsilon^{\rho\sigma}\bar{\epsilon}\gamma_\mu D_\rho\psi_\sigma + \bar{\psi}_\nu\gamma_\mu\gamma^\nu\eta + \text{h.c.} + \frac{1}{2}(g_{\mu\nu} - \epsilon_{\mu\nu})\partial^\nu\Lambda + \frac{1}{2}(g_{\mu\nu} + \epsilon_{\mu\nu})\partial^\nu\Lambda', \\ \delta\phi^\alpha &= -2\bar{\epsilon}\chi^\alpha, \delta\chi^\alpha = i(\partial_\mu\phi^\alpha + 2\bar{\psi}_\mu\chi^\alpha)\gamma^\mu\epsilon + \frac{1}{2}\Lambda_D\chi^\alpha + \frac{1}{2}\Lambda_M\chi^\alpha + i\Lambda'. \end{aligned} \quad (20)$$

Here, $(\epsilon, \eta, \Lambda_D, \Lambda_M, \Lambda, \Lambda')$ are the parameters of local supersymmetry, conformal supersymmetry, dilatation, Lorentz U(1) and U(1)' transformations, respectively. Note that the coefficient of the Wess–Zumino term, unlike in the (1, 0) case, is fixed. Apart from the presence of A_μ , note also the similarity of this action with the one for (1, 0). The presence of the unconventional two U(1) transformations can be best understood from the study of the underlying superconformal Lie algebra. This we do in the next section.

We now consider the coupling of a set of right-handed Weyl fermions, ψ^A , to the model, as was done in the earlier section for the (1, 0) case. To this end, we covariantize the result of Hull and Witten [11],

$$e^{-1}\mathcal{L}_2 = [\frac{1}{2}iG_{AB}(\phi)\bar{\psi}^A e_a^\mu\gamma^a(\partial_\mu\psi^B + A_\alpha^B{}_C\partial_\mu\phi^\alpha + A_{\bar{\alpha}}^A{}_C\partial_\mu\phi^{\bar{\alpha}})\psi^C + \text{h.c.}] + 2\hat{F}_{\bar{\alpha}\beta AB}\bar{\chi}^{\bar{\alpha}}\gamma_\mu\chi^\beta\bar{\psi}^A\gamma^\mu\psi^B, \quad (21)$$

with ψ^A transforming as

$$\delta\psi^A = 2\bar{\epsilon}\chi^\alpha\hat{A}_\alpha^A{}_B\psi^B + 2\bar{\epsilon}\chi^{\bar{\alpha}}\hat{A}_{\bar{\alpha}}^A{}_B\psi^B + \frac{1}{2}\Lambda_D\psi^A - \frac{1}{2}\Lambda_M\psi^A, \quad (22)$$

where

$$\begin{aligned} \hat{A}_\alpha^A{}_B &= A_\alpha^A{}_B + \frac{1}{2}G_{BC,\alpha}G^AC, \\ \hat{F}_{\bar{\alpha}\beta AB} &= \partial_\alpha\hat{A}_{\bar{\beta}AB} + \hat{A}_{\alpha AC}\hat{A}_{\bar{\beta}}^C{}_B - (\alpha \leftrightarrow \bar{\beta}). \end{aligned} \quad (23)$$

The interpretation of ψ^A , and $G^{AB}(\phi)$ is as discussed below eq. (6). The only additional requirement coming from the (global) supersymmetry of the action is that $F_{\alpha\beta AB} = 0$ [11].

What we find is that $I = \int d\sigma d\tau (\mathcal{L}_1 + \mathcal{L}_2)$ is invariant under the transformations given in (20) and (22). Note that ψ^A is inert under Λ' -transformations. This is consistent with the fact that in the $[\delta_\epsilon, \delta_\eta]$ commutator on ψ^A , the Λ_D and Λ_M transformations cancel. Since this commutator must yield Λ_D , Λ_M and Λ' transformations on ψ^A , it follows that the U(1)' charge is zero.

We close this section with some remarks on the possibility of gauging isometries (if any) of the scalar manifold.

The off-shell (2, 0) Yang-Mills multiplet consists of the vectors $A_\mu^{(r)}$, Weyl spinors $\lambda^{(r)}$ and real scalars $D^{(r)}$. Provided that there exists r functions of the scalar fields, $E^{(r)}(\phi)$, satisfying $g_{\alpha\beta}\xi^\beta(r) = 2i\partial_\alpha E^{(r)}(\phi)$, one can gauge an isometry group H generated by the Killing vectors $\xi^\alpha(r)$. The trouble is that one ends up with a term of the form $\sqrt{-g}E^{(r)}(\phi)D^{(r)}$ in the lagrangian. This term, through the $D^{(r)}$ field equation implies that $E^{(r)}(\phi) = 0$, and hence $\xi^\alpha(r)(\phi) = 0$. To bypass this problem, probably more general invariants are needed.

4. A Lie superalgebra derivation of the $N = 1, 2$ Weyl multiplet. We now derive the transformation rules of $N = 1, 2, d = 2$ conformal supergravity based on the Lie superalgebra $\text{OSp}(2, N) \oplus \text{OSp}(2, N)$ ($N = 1, 2$)^{*2}. The non-zero commutation relations between the different generators of this algebra are given in table 1. The case of $N = 1$ has already been given in ref. [18]. To each generator of the superalgebra we assign a gauge field h_μ^A in the following way:

$$h_\mu^A T_A = e_\mu^a P_a + \omega_\mu M + b_\mu D + f_\mu^a K_a + \bar{\psi}_\mu^i Q_i + \bar{\phi}_\mu^i S_i + V_\mu^{ij} V_{ij} + A_\mu^{ij} A_{ij}. \quad (24)$$

Here ψ_μ^i and ϕ_μ^i are Majorana spinors. Furthermore, the gauge fields V_μ^{ij}, A_μ^{ij} satisfy

$$V_\mu^{ij} = -V_\mu^{ji}, \quad A_\mu^{ij} = -A_\mu^{ji}. \quad (25)$$

Using the structure constants of the $\text{OSp}(2, N) \oplus \text{OSp}(2, N)$ algebra

$$[T_A, T_B] \equiv T_A T_B \pm T_B T_A = f_{AB}^C T_C \quad (26)$$

(+ sign if A and B are fermionic) and the basic rules

$$\begin{aligned} \delta h_\mu^A &= \partial_\mu \epsilon^A + \epsilon^C h_\mu^B f_{BC}^A, \\ R_{\mu\nu}^A &= 2\partial_{[\mu} h_{\nu]}^A + h_\nu^B h_\mu^C f_{BC}^A, \end{aligned} \quad (27)$$

One can immediately determine the gauge transformations of the superconformal gauge fields h_μ^A and the curvature tensors $R_{\mu\nu}^A$. Since the results are quite lengthy we will not present them here.

To achieve a maximal irreducibility of the superconformal gauge field configuration we impose a maximal set of so-called conventional constraints on the superconformal curvatures. More explicitly, each curvature that con-

^{*2} In this letter we do not consider the gauging of the infinite-dimensional super-Virasoro algebra [containing $\text{OSp}(2, N) \oplus \text{OSp}(2, N)$ as a finite-dimensional subalgebra].

Table 1

Non-zero commutators of the Lie superalgebra $\text{OSp}(2, N) \oplus \text{OSp}(2, N)$. The generators (M, D, P, K, Q, S, V, A) correspond to Lorentz rotations, dilatations, translations, conformal boosts, Q and S supersymmetry and $\text{SO}(N) \oplus \text{SO}(N)$ rotations, respectively. $(\gamma_a)_{\alpha\beta} \equiv [\frac{1}{2}(1 + \gamma_5)\gamma_a]_{\alpha\beta}$, $(\gamma_a)_{\dot{\alpha}\dot{\beta}} \equiv [\frac{1}{2}(1 - \gamma_5)\gamma_a]_{\dot{\alpha}\dot{\beta}}$ and the same for all other matrices. The spinor generators Q_α^i, S_α^i ($i = 1, \dots, N$; $\alpha = 1, 2$) are Majorana: $Q_\alpha^i = C_{\alpha\beta} Q^{\beta i}$.

$$\begin{aligned} [M, P_a] &= \epsilon_{ab} P_b, \quad [M, K_a] = \epsilon_{ab} K_b, \quad [P_a, K_b] = -\frac{1}{2}(\delta_{ab} D + \epsilon_{ab} M), \quad [D, P_a] = P_a, \quad [D, K_a] = -K_a, \\ \{Q_\alpha^i, Q_\beta^j\} &= -2i(\gamma_a)_{\alpha\beta} P^a \delta^{ij}, \quad \{S_\alpha^i, S_\beta^j\} = +2i(\gamma_a)_{\alpha\beta} K^a \delta^{ij}, \quad \{Q_\alpha^i, S_\beta^j\} = -iC_{\alpha\beta} D \delta^{ij} + i(\gamma_5)_{\alpha\beta} M \delta^{ij} - 2iC_{\alpha\beta} V^{ij} - 2iC_{\alpha\beta} A^{ij}, \\ [M, Q_\alpha^i] &= -\frac{1}{2}(\gamma_5 Q)_\alpha^i, \quad [M, S_\alpha^i] = -\frac{1}{2}(\gamma_5 S)_\alpha^i, \quad [D, Q_\alpha^i] = \frac{1}{2}Q_\alpha^i, \quad [D, S_\alpha^i] = -\frac{1}{2}S_\alpha^i, \\ [K_a, Q_\alpha^i] &= \frac{1}{2}(\gamma_a S)_\alpha^i, \quad [K_a, S_\alpha^i] = -\frac{1}{2}(\gamma_a Q)_\alpha^i, \quad [V^{ij}, Q_\alpha^k] = \delta^{k[i} Q_\alpha^{j]}, \quad [V^{ij}, S_\alpha^k] = \delta^{k[i} S_\alpha^{j]}, \\ [A^{ij}, Q_\alpha^k] &= \delta^{k[i} Q_\alpha^{j]}, \quad [A^{ij}, S_\alpha^k] = \delta^{k[i} S_\alpha^{j]}, \quad [V^{ij}, V_{kl}] = -2\delta_{[k}^{[i} V_{l]}^{j]}, \quad [A^{ij}, A_{kl}] = -2\delta_{[k}^{[i} A_{l]}^{j]} \end{aligned}$$

tains a term proportional to a connection field multiplied by a zweibein field is set equal to zero. This gives the following set of constraints:

$$R_{\mu\nu}^a(P) = 0, \quad R_{\mu\nu}(M) = 0, \quad R_{\mu\nu}(D) = 0, \quad R_{\mu\nu}(Q) = 0. \quad (28)$$

An unusual feature in $d = 2$ is that the constraints $R_{\mu\nu}(M) = 0$ and $R_{\mu\nu}(Q) = 0$ only solve part of the gauge fields f_μ^a and ϕ_μ^i , respectively. More explicitly, they only solve for $e_\mu^a f_\mu^a$, $e_\mu^a e_\mu^b f_\mu^b$ and $\gamma^\mu \phi_\mu^i$. Recall that for $d > 2$, the solutions for f_μ^a and ϕ_μ^i contain the factor $(d-2)^{-1}$.

A straightforward calculation shows that in the presence of the constraints (28) one can never realize a superconformal commutator algebra, containing a general coordinate transformation, on the non-solvable parts of f_μ^a and ϕ_μ^i . Therefore our strategy will be to eliminate the nonsolvable parts of ϕ_μ^i and f_μ^a from the multiplet containing the zweibein ^{†3}. It appears that this can be achieved by performing appropriate field dependent K and S transformations. To clarify this we give two examples. First consider the transformation of ω_μ under Q and K :

$$\delta \omega_\mu = 2i\bar{\epsilon}^i \gamma_5 \phi_\mu^i + \frac{1}{2} \epsilon_{\mu\lambda} \Lambda_K^\lambda = 2i\bar{\epsilon}^i \gamma_5 \gamma_\mu \gamma^\lambda \phi_\lambda^i + \frac{1}{2} \epsilon_{\mu\lambda} (\Lambda_K^\lambda - 4i\bar{\epsilon}^i \phi_\lambda^i). \quad (29)$$

It is clear that the nonsolvable part of ϕ_μ^i can be eliminated from the transformation rule of ω_μ by performing a K transformation with parameter

$$\Lambda_\mu^K = +4i\bar{\epsilon}^i \phi_\mu^i. \quad (30)$$

Next consider the transformation rule of ψ_μ^i under Q and S :

$$\begin{aligned} \delta \psi_\mu^i &= \partial_\mu \epsilon^i + \frac{1}{2} \omega_\mu \gamma_5 \epsilon^i - \frac{1}{2} (1 + \gamma_5) V_\mu^{ij} \epsilon^j - \frac{1}{2} (1 - \gamma_5) A_\mu^{ij} \epsilon^j + \frac{1}{2} b_\mu \epsilon^i + \gamma_\mu \eta^i \\ &= \partial_\mu \epsilon^i + \frac{1}{2} \omega_\mu \gamma_5 \epsilon^i + \frac{1}{2} b_\mu \epsilon^i - \frac{1}{2} X_\mu^{ij} \epsilon^j + \gamma_\mu (\eta^i - \frac{1}{2} \gamma^\lambda \gamma_5 Y_\lambda^{ij} \epsilon^j), \end{aligned} \quad (31)$$

with

$$X_\mu^{ij} = V_\mu^{ij} + A_\mu^{ij} - \epsilon_{\mu\nu} (V_\nu^{ij} - A_\nu^{ij}), \quad Y_\mu^{ij} = V_\mu^{ij} - A_\mu^{ij}. \quad (32)$$

Note that the gauge field combination Y_μ^{ij} occurs as a field dependent S transformation in the Q transformation of ψ_μ^i . Now under Q the gauge field combinations X_μ^{ij} and Y_μ^{ij} transform as

$$\delta X_\mu^{ij} = -4i\bar{\epsilon}^i [\gamma_\mu \gamma^\lambda \phi_\lambda^j], \quad \delta Y_\mu^{ij} = 4i\bar{\epsilon}^i [\gamma_5 \phi_\mu^j]. \quad (33)$$

Since Y_μ^{ij} transforms into the unsolvable part of ϕ_μ^i , it has to be eliminated from the transformation rule of ψ_μ^i . From (30) we see that by performing an S transformation with parameter

$$\eta^i = \frac{1}{2} \gamma^\lambda \gamma_5 Y_\lambda^{ij} \epsilon^j, \quad (34)$$

we can eliminate the gauge field combination Y_μ^{ij} from the Q variation of ψ_μ^i and we are left with the combination X_μ^{ij} which only transforms to the solvable part of ϕ_μ^i .

One can show that by performing K and Q transformations with parameters given in (30) and (34) one can consistently eliminate the gauge field Y_μ^{ij} and the nonsolvable parts of the gauge fields f_μ^a and ϕ_μ^i from the transformation rules of all the remaining gauge fields. We now present the final result. In the K gauge $b_\mu = 0$ the variation of the gauge fields (e_μ^a , ψ_μ^i , ω_μ , X_μ^{ij}) are given by

$$\begin{aligned} \delta e_\mu^a &= 2i\bar{\epsilon}^i \gamma^a \psi_\mu^i + \Lambda_M \epsilon^{ab} e_\mu^b - \Lambda_D e_\mu^a, \\ \delta \psi_\mu^i &= \partial_\mu \epsilon^i + \frac{1}{2} \omega_\mu (e, \psi) \gamma_5 \epsilon^i - \frac{1}{2} X_\mu^{ij} \epsilon^j + \gamma_\mu \eta^i - \frac{1}{2} \Lambda_M \gamma_5 \psi_\mu^i - \frac{1}{2} \Lambda_D \psi_\mu^i + \frac{1}{2} \Lambda_1^{ij} \psi_\mu^j + \frac{1}{2} \Lambda_2^{ij} \gamma_5 \psi_\mu^j, \end{aligned} \quad (35)$$

^{†3} Another strategy suggested by dimensional reduction of $d = 3$ conformal supergravity [19] to $d = 2$, might be to equate the nonsolvable parts of ϕ_μ^i and f_μ^a to some matter fields.

$$\begin{aligned}\delta\omega_\mu &= 2i\bar{\epsilon}^i\gamma_5\gamma_\mu\gamma^\lambda\phi_\lambda^i(e, \psi, X) + 2i\bar{\eta}^i\gamma^\lambda\gamma_\mu\gamma_5\psi_\lambda^i + \partial_\mu\Lambda_M - \epsilon_{\mu\nu}\partial^\nu\Lambda_D, \\ \delta X_\mu^{ij} &= -4i\bar{\epsilon}^i[\gamma_\mu\gamma^\lambda\phi_\lambda^j](e, \psi, X) - 4i\bar{\eta}^j[\gamma^\lambda\gamma_\mu\psi_\lambda^i] + \partial_\mu\Lambda_1^{ij} - \epsilon_{\mu\nu}\partial^\nu\Lambda_2^{ij},\end{aligned}\quad (35 \text{ cont'd})$$

where

$$\omega_\mu(e, \psi) = (-e_{\mu a}\epsilon^{\rho\sigma}\partial_\rho e_\sigma^a + i\epsilon^{\rho\sigma}\bar{\psi}_\rho^i\gamma_\mu\psi_\sigma^i), \quad \gamma^\mu\phi_\mu^i(e, \psi, X) = \gamma_5\epsilon^{\rho\sigma}[\partial_\rho\psi_\sigma^i + \frac{1}{2}\omega_\rho(e, \psi)\psi_\sigma^i - \frac{1}{2}X_\rho^{ij}\psi_\sigma^j]. \quad (36)$$

We have checked that for any N the following commutator algebra is realized in the gauge fields e_μ^a , ψ_μ^i and ω_μ :

$$\begin{aligned}[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] &= y_{\text{gct}}(\xi^\lambda = 2i\bar{\epsilon}_2^j\gamma^\lambda\epsilon_1^j) + \delta_M(\Lambda_M = -\xi^\lambda\omega_\lambda(e, \psi)) + \delta_Q(\epsilon^i = -\xi^\lambda\psi_\lambda^i) \\ &+ \delta_S(\eta^i = 2i\bar{\epsilon}_1^j\gamma_5\epsilon_2^j\gamma_5\gamma^\lambda\phi_\lambda^i - i\bar{\epsilon}_1^j\gamma_5\epsilon_2^j\gamma_5\gamma^\lambda\phi_\lambda^i - \frac{1}{2}\xi^\mu\gamma_\mu\gamma^\lambda\phi_\lambda^i) + \delta_{\text{SO}(N)} \oplus \text{SO}(N)(\Lambda_1^{ij} - \xi^\lambda X_\lambda^{ij}), \\ [\delta_S(\eta), \delta_Q(\epsilon)] &= \delta_D(\Lambda_D = -2i\bar{\epsilon}^i\eta^i) + \delta_M(\Lambda_M = 2i\bar{\epsilon}^i\gamma_5\eta^i) + \delta_{\text{SO}(N)} \oplus \text{SO}(N)(\Lambda_1^{ij} = -4i\bar{\epsilon}^i[\gamma^\lambda\eta^j], \Lambda_2^{ij} = +4i\bar{\epsilon}^i[\gamma_5\eta^j]) \\ &+ \delta_S(i\bar{\epsilon}^i(\gamma^\lambda\eta^j)\gamma^\lambda\psi_\lambda^j - \frac{1}{2}i\bar{\epsilon}^j\eta^j\gamma^\lambda\psi_\lambda^i + i\bar{\epsilon}^i(\gamma_5\eta^j)\gamma_5\gamma^\lambda\psi_\lambda^j - \frac{1}{2}i\bar{\epsilon}^j\gamma_5\eta^j\gamma_5\gamma^\lambda\psi_\lambda^i \\ &- i\bar{\epsilon}^i[\gamma_\rho\eta^j]\gamma^\lambda\gamma^\rho\psi_\lambda^j - \frac{1}{2}i\bar{\epsilon}^j\gamma_\rho\eta^j\gamma^\lambda\gamma^\rho\psi_\lambda^i). \quad (37)\end{aligned}$$

Furthermore we find that the same commutator algebra is realized on X_μ^{ij} only for $N = 1, 2$ (note that $X_\mu^{ij} = 0$ for $N = 1$). It would be interesting to see what happens for $N > 2$.

Thus we have found the full transformation rules and the commutator algebra of the $N = 1$ Weyl multiplet (e_μ^a, ψ_μ) and the $N = 2$ Weyl multiplet $(e_\mu^a, \psi_\mu^i, X_\mu^{ij})$. The chiral cases $(1, 0)$ and $(2, 0)$ are easily obtained by the chiral truncations $\psi_\mu^i = \gamma_5\psi_\mu^i$, $\eta^i = -\gamma_5\eta^i$ and $\epsilon^i = \gamma_5\epsilon^i$.

Note added. After the completion of the present paper we learnt that in ref. [20] a super Lie algebra derivation of $N = 1, 2$ conformal supergravities in $d = 2$ is given. In that paper the unsolvable parts of f_μ^a and ϕ_μ are solved by the imposition of two constraints in addition to the ones given in eq. (28), as opposed to the performing of the field redefinitions given in eqs. (30) and (34). We also received a preprint (ref. [21]), in which $N = 4$ conformal supergravity in $d = 2$ is constructed and is coupled to a non-linear σ -model.

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