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ON "SPINNING" MEMBRANE MODELS

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Several alternative actions for a bosonic membrane have recently been proposed. We show that a linearly realized *locally world-volume*-supersymmetric (*spinning* membrane) extension of any of these actions implies an analogous extension of the standard Dirac membrane action. We further show that a (Minkowski) spacetime Lorentz-covariant spinning membrane action (that reduces to the Dirac membrane action when all fermions vanish) cannot be constructed within the framework of the three-dimensional super-Poincaré tensor calculus.

1. Introduction

In 1962 Dirac proposed the action [1]

$$I_D = - \int d^3\xi [-\det(\partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu})]^{1/2} \quad (1)$$

for a (structureless) membrane (which we take to be propagating in a flat d -dimensional spacetime with Minkowski metric $\eta_{\mu\nu}$). There are two conceivable supersymmetric extensions of this action corresponding to either world-volume or spacetime supersymmetry. It is now known that spacetime supersymmetric extensions of (1), called supermembranes, exist provided $d=4,5,7$ or 11 [1-4]. It is natural to wonder whether there also exist world-volume supersymmetric extensions of (1), which we shall call spinning membranes. For the particle and string, supersymmetric extensions of both types exist. It is one of the miracles of string theory that under certain conditions the spinning string and the superstring are equivalent formulations of the same theory. This might suggest that there exists an alternative spinning membrane formulation of the supermembrane

theories. We shall show in this letter that subject to certain requirements, to be stated explicitly below, *no such spinning membrane action exists* (so the question of equivalence to the supermembrane will not arise).

An attempt to construct a spinning membrane action was made by Howe and Tucker [5]. They first rewrote the action (1) in the form

$$I_{HT} = -\frac{1}{2} \int d^3\xi (\sqrt{-g} g^{ij} \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu} - \sqrt{-g}) \quad (2)$$

in which the world-volume metric g_{ij} appears as an independent *auxiliary* variable. The action can now be considered as a three-dimensional field theory for scalar fields X^μ coupled to gravity. The world-volume supersymmetric extension will therefore be a three-dimensional supergravity/supermatter system, and can be constructed by standard techniques. In ref. [5] it was shown that the supersymmetric extension of (2) requires that the supergravity fields satisfy certain differential constraints, in which case the metric g_{ij} (in particular) is no longer an auxiliary variable. As a consequence, the action (2) becomes inequivalent to (1). An alternative statement of this problem is that if the supergravity fields are left un-

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constrained the supersymmetrization of (2) requires the inclusion of a $\sqrt{-g}R$ term. In this case the bosonic part of the action again becomes inequivalent to (1). Thus, the apparent conclusion to be drawn from the results of ref. [5] is that an acceptable spinning membrane action does not exist. In this letter we shall investigate the conditions under which this conclusion is valid.

The fact that one cannot supersymmetrize the action I_{HT} is not obviously fatal to the spinning membrane program because one can imagine that other actions exist that are also equivalent to I_{D} on elimination of auxiliary fields. In fact such actions have been recently proposed by Dolan and Tchrakian [6] and Inami and Yahikozawa [7] (and we shall present many more here), and it has been claimed [8] that the action of ref. [6] allows a spinning membrane extension. We have not been able to see how the results of ref. [8] justify this conclusion. In fact we shall establish here a theorem to the effect that a supersymmetrization of *any* bosonic membrane action *equivalent* to I_{D} implies the existence of a supersymmetrization of I_{HT} . Thus the issue of whether there exists a spinning membrane action can be reduced to whether the action I_{HT} allows a world-volume-supersymmetric extension. As just explained it does not, under assumptions to be made precise below, but which essentially amount to the supposition that all supersymmetric invariants can be constructed by means of a super-Poincaré tensor calculus.

2. Alternative actions

Following ref. [6] we define

$$A_j^i = g^{ik} \partial_k X^\mu \partial_j X^\nu \eta_{\mu\nu}. \quad (3)$$

The bosonic Howe–Tucker action can then be written as

$$I_{\text{HT}} = -\frac{1}{2} \int d^3 \xi \sqrt{-g} [\text{tr}(A) - 1]. \quad (4)$$

Dolan and Tchrakian proposed the action

$$I_{\text{DT}} = -\frac{1}{2} \int d^3 \xi \sqrt{-g} \left\{ \text{tr}(A) - \frac{1}{2} \text{tr}(A^2) + \frac{1}{2} [\text{tr}(A)]^2 \right\}. \quad (5)$$

Elimination of the metric g_{ij} in (5) leads to a qua-

dratic matrix equation for which one solution is $A = 1$ [the other is non-(3-dimensional) Lorentz covariant]. Substituting for A in (5) yields I_{D} . The action (5) is just one of a class with the same properties. The generic action has the form

$$I = \int d^2 \xi \sqrt{-g} \left(\sum_{p=0}^{\infty} \sum_{\{n_k\}} (\text{tr } A)^{n_1} \dots \times (\text{tr } A^k)^{n_k} \dots (\text{tr } A^p)^{n_p} C_p(n_1 \dots n_p) \right), \quad (6)$$

where the integers $\{n_k\}$ are restricted by $\sum_{k=1}^p k n_k = p$, for each p , and the $C_p(n_1, \dots, n_p)$ are constants. For generic values of these constants the equation for g_{ij} admits the solution $A \propto 1$ and the action (6) then reduces to I_{D} . A particular example in this class is

$$I = -\frac{1}{2} \int d^3 \xi \sqrt{-g} [\text{tr}(A) + \det A] \quad (7)$$

a nice feature of which is that the solutions are $A = \pm 1, 0$ (i.e. all 3-dimensional Lorentz covariant).

The class of actions (6) can be extended by rescaling the metric by an arbitrary scalar ϕ (which is equivalent to rescaling A by ϕ^{-1}) and then eliminating ϕ by a field equation. One obtains in this way a conformally invariant action that is obviously equivalent to the original one. Applying this procedure to (4), for example, one obtains

$$I_{\text{IV}} = - \int d^3 \xi \sqrt{-g} (\text{tr } A)^{3/2}, \quad (8)$$

which has been suggested in ref. [7].

The fact that many of the models of the class considered here allow solutions of the g_{ij} equation that do *not* lead to the Dirac membrane action (in addition to those that do) may be considered a defect. However, to the extent that these models *are* considered equivalent to I_{D} (by excluding unphysical solutions of the g_{ij} equation) the theorem to be given below will apply.

3. Reduction to the Howe–Tucker action

We shall call an action a spinning membrane action if (i) it possesses a linearly realized local world-volume supersymmetry, (ii) on elimination of all bosonic auxiliary fields, and then setting the fermions to zero the action reduces to I_{D} . Since any ac-

tion can be expanded in powers of fermion fields, collectively denoted by ψ , it then follows that, on elimination of auxiliary fields, the spinning membrane action takes the form

$$I_1 = I_D(X^\mu) + I_F[X^\mu, \psi], \quad (9)$$

where I_F is at least quadratic in Fermi fields ψ .

Now, the action

$$I_2 = I_{HT}[X^\mu, g_{ij}] + I_F[X^\mu, \psi] \quad (10)$$

with the auxiliary field g_{ij} appearing *only* in I_{HT} , is equivalent to (9) in the following sense: (i) the elimination of the auxiliary field g_{ij} in I_2 yields I_1 , (ii) any symmetry of I_1 is also a symmetry of I_2 , i.e. we can always assign a transformation to g_{ij} in such a way that any symmetry of I_1 is preserved. This is well known, but for the sceptic we provide the proof here. The action I_1 can be written as

$$I_1 = I_2[X^\mu, g_{ij}(X), \psi], \quad (11)$$

where $g_{ij}(X)$ is the result for g_{ij} obtained by solving the equation $\delta I_2 / \delta g_{ij} = 0$. Hence

$$\begin{aligned} \delta I_1 &= \delta X^\mu \left(\frac{\delta I_2}{\delta X^\mu} + \frac{\delta g_{ij}}{\delta X^\mu} \frac{\delta I_2}{\delta g_{ij}} \right) + \delta \psi \frac{\delta I_2}{\delta \psi} \\ &= \left(\delta X^\mu \frac{\delta I_2}{\delta X^\mu} + \delta \psi \frac{\delta I_2}{\delta \psi} \right)_{g_{ij}=g_{ij}(X)}, \end{aligned} \quad (12)$$

since $\delta I_2[X^\mu, g_{ij}(X), \psi] / \delta g_{ij}$ is *identically* zero. The variation of I_2 is

$$\delta I_2 = \left(\delta X^\mu \frac{\delta I_2}{\delta X^\mu} + \delta \psi \frac{\delta I_2}{\delta \psi} \right) + \delta g_{ij} \frac{\delta I_2}{\delta g_{ij}}. \quad (13)$$

If I_1 has a symmetry such that $\delta I_1 = 0$, it follows from (12) that the term in round brackets in (13) vanishes when $g_{ij} = g_{ij}(X)$, i.e. when $\delta I_2 / \delta g_{ij} = 0$. It is therefore proportional to $\delta I_2 / \delta g_{ij}$. Since δg_{ij} multiplies $\delta I_2 / \delta g_{ij}$ we can always choose it such that $\delta I_2 = 0$.

We have now established by construction of the action (10) the following theorem.

Theorem. If there exists *any* spinning membrane action (as defined above) there must exist a linearly realized locally supersymmetric (spinning membrane) extension of I_{HT} .

The utility of this theorem is that it obviates the necessity of considering actions such as those of refs.

[6,7] when investigating the possibility of a spinning membrane action. We need only investigate one such action in the entire class, the simplest being I_{HT} .

4. Supersymmetrization of the Howe–Tucker action

We shall investigate the supersymmetrization of I_{HT} by means of the $d=3$ supergravity tensor calculus [9]. The field X^μ now appears in a *matter* multiplet $(X^\mu, \lambda^\mu, F^\mu)$ the equation $\delta I_{HT} / \delta X^\mu = 0$ up to fermions, which is the correct equation for X^μ . One can also show that $F^\mu = \xi^\mu$, for constant d -vector ξ^μ , does *not* yield the correct equation (although it would be allowed by the flat d -space tensor calculus). It is therefore tempting to suppose that the F^μ equation of motion *must* be $F^\mu = 0$, but to prove this we would have to show that no other possible equation for F^μ would yield the required equation for X^μ . Even if a constant d -vector ξ^μ does not appear in the equation for F^μ it does not follow that the solution of this equation will not introduce it, and the most general possible equation for F^μ is difficult to analyze. To proceed we shall therefore make the *assumption* of d -dimensional Lorentz covariance, i.e. that a constant d -vector ξ^μ cannot appear. (We believe that it should be possible to dispense with this assumption and we shall present evidence for this shortly, but we shall not pursue this here.) In this case, the elimination of the F^μ field leads to terms in the action that are at least quadratic in the matter fields X^μ, λ^μ . This means that the supersymmetrization of the cosmological term in I_{HT} can be considered independently of the matter fields, which we can henceforth suppose to be absent.

The (off-shell) $d=3$ supergravity multiplet consists of (e_i^a, ψ_i, S) , where e_i^a is the dreibein, ψ_i is the gravitino, and S an auxiliary field. Their (Poincaré) supersymmetry transformation laws are [9]

$$\delta e_i^a = \bar{\epsilon} \gamma^a \psi_i, \quad \delta \psi_i = 2(D_i + \frac{1}{4} \gamma_i S) \epsilon,$$

$$\delta S = \bar{\epsilon} (-\frac{1}{2} S \gamma^\mu \psi + \gamma^\mu D_i \psi_j). \quad (14)$$

The multiplets at our disposal for the construction of invariant actions are found as the components of the Ricci-scalar superfield W and the γ -traceless Ricci-tensor vector-spinor superfield Ω' [10]. In $Sl(2, \mathbb{R})$ spinor notation the latter is written as $\Omega_{\alpha\beta\gamma}$ with total

symmetry in the three spinor indices. With all fermions set to zero the components of these superfields are

$$W \sim (S; 0; -\frac{3}{4}S^2 + \frac{1}{2}R), \quad (15)$$

$$\Omega_{\alpha\beta\gamma} \sim (0; R_{ij} - \frac{1}{3}\eta_{ij}R, \partial_\mu S; 0). \quad (16)$$

The $\Omega_{\alpha\beta\gamma}$ superfield satisfies the identity

$$\nabla^\alpha \Omega_{\alpha\beta\gamma} = i\nabla_{\beta\gamma} W \quad (17)$$

[where the superspace covariant derivatives $\nabla_A = (\nabla_\alpha, \nabla_i)$ satisfy $\nabla_{(\alpha} \nabla_{\beta)} = i\nabla_{\alpha\beta} \equiv (i\gamma^{0i})_{\alpha\beta} \nabla_i$ and $\nabla_\alpha^\beta \equiv \epsilon^{\beta\gamma} \nabla_{\alpha\gamma}$]. Both W and $\Omega_{\alpha\beta\gamma}$ contain the S auxiliary field, but from them one can construct a superfield $C_{\alpha\beta\gamma}$ (also symmetric in three spinor indices) which is S -independent, and which we shall call the "Cotton-tensor superfield". At the linearized level it is given by

$$C_{\alpha\beta\gamma} = D^\delta D_{\delta(\alpha} \Omega_{\beta\gamma)} + 12i\partial_{(\alpha} \partial_{\beta\gamma)} S - \frac{3}{2}i\partial_{(\alpha\beta} D_{\gamma)} W \quad (18)$$

[where $D_A = (D_\alpha, \partial_i)$ are the flat superspace covariant derivatives]. It satisfies

$$D^\alpha C_{\alpha\beta\gamma} = 0 \quad (19)$$

as a consequence of (17). With all fermions set to zero its components are

$$C_{\alpha\beta\gamma} \sim (0; \epsilon^{ijk} \partial_j (R_{ik}^l - \frac{1}{4}\delta_k^l R) \equiv C^{il}, 0; 0), \quad (20)$$

where C^{il} is the symmetric, traceless and covariantly conserved Cotton tensor [11].

We must exclude from the bosonic action terms containing $(\partial S)^2$ as these will necessarily lead to a differential rather than an algebraic equation of motion for S . In this case S will cease to be auxiliary and the bosonic action will not ultimately reduce to just $\int d^3\xi \sqrt{-g}$. This restricts invariants constructed from W alone to have the superspace form $\int d^2z E f(W)$, where f is an arbitrary function (and E is the superspace density [10]). Using the tensor calculus results of ref. [9] we see that this yields the lagrangian

$$\mathcal{L} = \sqrt{-g} [f'(S) (-\frac{3}{4}S^2 + \frac{1}{2}R) + S f(S)]. \quad (21)$$

We shall return to the analysis of this lagrangian shortly. We now want to explain why other invariants constructed using the Ω -superfield need not be considered. Such invariants will generically contain terms with factors of $(\partial S)^2$, and we can reject these for the reason given previously, but it is possible to construct invariants that do not contain any $(\partial S)^2$ terms

(note that terms *linear* in ∂S cannot arise); an example is $\int d^3\xi E C_{\alpha\beta\gamma} C^{\alpha\beta\gamma}$. These invariants all contain terms with undifferentiated factors of S (or for which integration by parts can be performed to produce such factors), but they also have the property that each term contains at least one ∇R factor. Integrating by parts will only throw the derivative onto another R factor (or onto S). Now, we are going to require shortly that the elimination of S from (21) does not produce a $\sqrt{-g}R$ term. The inclusion of terms of the form $\sqrt{-g} \nabla R S^n R^m$, etc., cannot change the coefficient of the $\sqrt{-g}R$ term. We shall see below that this coefficient cannot vanish, so that we need not go on to consider the coefficients of terms containing ∇R . Consequently we need not consider invariants constructed from $\Omega_{\alpha\beta\gamma}$.

At this juncture we should mention that there is *one* invariant that *cannot* be constructed exclusively from the superfields W and $\Omega_{\alpha\beta\gamma}$ because the integrand is not local Lorentz invariant. This is the 3-dimensional conformal supergravity action, which is a Chern-Simons term for the 3-dimensional superconformal group $OSp(1/4)$ [12,13]. The equation of motion of 3-dimensional conformal supergravity is just the vanishing of the Cotton-tensor superfield. This invariant is the only conformally invariant one and therefore the only one that is S -independent. It contains derivatives of the metric g_{ij} , however, and so is excluded by our requirements for an acceptable spinning membrane action. As an aside we may also remark that the Cotton tensor C^{ij} is nothing but the group curvature $R_{ij}^a(K)$ associated with the proper conformal generator K^a of the conformal group, subject to the constraint that the group curvatures $R_{ij}^{ab}(M)$ and $R_{ij}^a(P)$ of the Poincaré subgroup vanish [the remaining curvature $R_{ij}(D)$ associated with the dilatation generator D then vanishes by a Bianchi identity].

We now return to the analysis of the lagrangian (21). For $f=1$ we obtain $\sqrt{-g}S$ and for $f(S)=S$ we obtain $\frac{1}{2}\sqrt{-g}(R + \frac{1}{2}S^2)$. Both of these terms must appear in the action in order that S has a solution of the form $S=1+\dots$, which is needed in order to obtain the $\sqrt{-g}$ term. These two terms are not sufficient, however, because they also lead to a $\sqrt{-g}R$ term. The important question is whether we can find a function f such that on eliminating S to obtain $S(R)$ the lagrangian $\mathcal{L}(S(R), R)$ is just $\sqrt{-g}$. We shall

now show that this is not possible. Requiring both that the S -equation of motion hold and that $\mathcal{L}(S(R), R)$ be independent of R is equivalent to requiring

$$\frac{\delta \mathcal{L}}{\delta S}(S, R) = 0, \quad \frac{\delta \mathcal{L}}{\delta R}(S, R) = 0. \quad (22)$$

From (19) we see that this requires that S satisfy

$$f'(S) = 0, \quad f''(S) \left(-\frac{3}{4}S^2 + \frac{1}{2}R \right) + f(S) = 0. \quad (23)$$

From the first of these equations it follows that $S = S_0$, a constant, but the second equation is then

$$\frac{1}{2}f''(S_0)R + [f(S_0) - \frac{3}{4}S_0^2 f''(S_0)] = 0. \quad (24)$$

We cannot require $R = \text{constant}$ because this would constitute a constraint on g_{ij} . Therefore (22) implies the two equations

$$f''(S_0) = 0, \quad f(S_0) = 0. \quad (25)$$

Recalling that S_0 is defined by $f'(S_0) \equiv 0$ we see from (19) that $\mathcal{L}(S_0, R) \equiv 0$.

To summarize, we conclude that, within the tensor calculus (and assuming d -dimensional Lorentz invariance) the bosonic Howe–Tucker lagrangian does not admit of a spinning membrane extension.

5. Conclusions

While it is generally accepted that all supersymmetric invariants of $N=1$ supersymmetry can be constructed by means of a tensor calculus (or equivalently superfields) we do not know of any proof of this. One way to proceed without using a tensor calculus is the laborious order by order Noether construction. In this case we would start from an action of the form

$$\int d^3\xi e [4m^2 + im(a_1 \bar{\psi}_i \gamma^{ij} \psi_j + a_2 \bar{\psi}' \psi_i) + b_1 \bar{\psi}_i \gamma^{ijk} D_j \psi_k + b_2 \bar{\psi} \cdot \gamma D \cdot \psi + b_3 \bar{\psi}' \mathcal{D} \psi_i + \dots] \quad (26)$$

(as for the construction of $(\text{adS})_3$ supergravity theories [14]), where m is a mass parameter introduced to give every term in the action the same dimension. Using the transformation rules suggested by the tensor calculus, but with S now an arbitrary function of R , we soon discover that (without the addition of an eR term) such an action cannot be locally supersym-

metric unless terms of the form $m^{-1} \partial_i \bar{\psi} \partial^i \psi$ are included. These higher derivative terms, which all involve inverse powers of m allow the cancellation of variations of (26) but lead to further variations that can be cancelled only by the addition of yet higher derivative terms. In this way one appears to be constructing an invariant that involves an infinite sum of terms of higher and higher powers of m^{-1} . We do not know whether such a construction could ultimately be successful; if so, it would presumably be an invariant outside the scope of the tensor calculus.

If we assume that such higher derivative fermion terms cannot appear in the action one can easily investigate the possibility that d -dimensional Lorentz invariant fails. Introducing new terms in (24) of the type $\bar{\psi} \cdot \gamma \lambda^{\mu} \xi_{\mu}$ one again discovers that supersymmetrization of the $m^2 e$ term is not possible without the introduction of an eR term.

We have investigated the possibility of a spinning membrane propagating in flat spacetime. It is conceivable that such an action exists only in particular curved spacetimes. We can mention here a model of $N=8$, $d=2$ conformal supergravity in which the target space curvature is related to the curvature of the world-sheet [15]. We should mention that Ho and Hosotani have proposed [16] a type of membrane field theory equation by following Dirac's derivation of the Dirac equation. The Dirac equation also follows from quantization of the spinning particle (and an analogous result holds for the spinning string), so one would expect that an equation of the type proposed in ref. [16] should also follow from quantization of a spinning membrane action. As we have shown in this paper, however, the construction of such an action cannot be achieved by standard methods and will require some radically new idea, if it is possible at all.

Finally we point out that a choice of "physical" gauge for the supermembrane leads to gauge fixed membrane actions that possess $N=1, 2, 4$ or 8 rigid world-volume supersymmetry according to whether one starts from the $d=4$ -, 5 -, 7 - or 11 -dimensional model [3] (and the action and transformation laws have recently been worked out explicitly for the $d=4$ supermembrane [17]). These models do not possess local world-volume supersymmetry however, and so cannot be considered as spinning membranes. For the Green–Schwarz superstring the analogous gauge fixed

action can be “gauge-unfixed” to yield the spinning string. Thus for the 10-dimensional spinning/superstring there is a choice of two possible Lorentz covariant formulations. The results of this paper for the membrane might be construed as evidence that the *spacetime* Lorentz covariant formulation is the more fundamental one.

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