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The cohomology of the Brink–Schwarz superparticle

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We show that the covariantly gauge-fixed action of the ten-dimensional Brink–Schwarz superparticle has a fermionic, nilpotent symmetry whose cohomology is given by the $(8+8)$ -supersymmetric Yang–Mills multiplet.

1. Introduction

Applying the standard Batalin–Vilkovisky quantisation procedure [1] to the Brink–Schwarz action (BS) [2] for a superparticle, one obtains a quadratic gauge-fixed action [3,4] which determines that the hamiltonian is P^2 , and a system of coordinates and momenta. This system forms two representations of $OSp(10|4)$ [5], which leads [6] to a counting of 8 bosonic and 8 fermionic field degrees of freedom. One also obtains a BRST operator Ω , whose cohomology (the states which are annihilated by Ω and where two states are identified which differ by the BRST of another state) leads to a classical system with $N=2$ “twisted” supersymmetry [7]. However, the light-cone quantisation of this BS action leads to an “ $N=1$ ” Yang–Mills supermultiplet in ten dimensions.

We will give in this paper another (covariant) BRST operator based on the same fields, whose cohomology gives also the $N=1$ Yang–Mills supermultiplet in ten dimensions.

The action corresponding to the BS superparticle is given by

$$S_{cl} = P\dot{X} - \theta\dot{\theta} - \frac{1}{2}gP^2. \quad (1)$$

The covariant quantisation using the Batalin–Vilkovisky procedure has been performed in refs. [3,4] with different gauge fixing functions. We will show in ref. [8] that these gauge fixing functions are two distinct members of a one-parameter family of gauge fixing functions, all of which lead to the same gauge fixed action. Also the procedure of ref. [9] leads to the same result (up to the “non-minimal” fields which were not considered there). The common result is a gauge-fixed action which is essentially given by an infinite number of free, quadratic terms for bosonic as well as fermionic fields:

$$S_{g.f.} = P\dot{X} - \frac{1}{2}P^2 + b\dot{c} + \sum_{l=0}^{\infty} \sum_{p=q}^{\infty} \bar{\lambda}^{p,q} \dot{\theta}_{p,q} + \sum_{q=1}^{\infty} \sum_{p=q}^{\infty} \lambda_{p,q} \dot{\bar{\theta}}^{p,q}. \quad (2)$$

We thus have the following field and commutation relations:

$$\begin{aligned} \{c, b\} &= 1, \\ [\theta_{p,q}, \bar{\lambda}^{r,s}] &= \delta_p^r \delta_q^s, \\ [\bar{\theta}^{p,q}, \lambda_{r,s}] &= \delta_r^p \delta_s^q. \end{aligned} \quad (3)$$

The fields on the second line are fermionic for even $p+q$ and bosonic for $p+q$ odd. In the third line it is the reverse: bosonic for $p+q$ even and fermionic for $p+q$ odd. In each case $p \geq q$. In the second line goes from 0 to ∞ while in the third line $q \geq 1$. Fields with

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lower indices have positive chirality for $p+q$ even (an upper spinor index), and negative for $p+q$ odd (a lower spinor index), while the reverse is true for fields with upper indices. The γ -matrices have both their indices up or down and are symmetric. We can then avoid to write the spinor indices. Note that a bar above a spinor has nothing to do with Dirac conjugation, but indicates that the corresponding spinor was an antighost.

The BRST operator $\Omega_{\min}$ which follows from a naive application of the BV formalism on infinite reducible systems is given by

$$\Omega_{\min} = \frac{1}{2}cP^2 + \sum_{p=0}^{\infty} \bar{\lambda}^{p,0} (\not{p}\theta_{p+1,0} - 2\theta_{p+2,0}b) + \sum_{q=1}^{\infty} \sum_{p=1}^{\infty} \bar{\lambda}^{p,q} \lambda_{p,q}. \quad (4)$$

We still show below that this quantisation (this BRST operator) has as cohomology solution not the 8+8 Yang-Mills supermultiplet, but rather exhibits and $N=2$ twisted supersymmetry [7].

At this point one can adopt one of the following two points of view. One strategy is to start from a different classical action (modified BS superparticle) whose covariant quantization avoids some of the issues involved in the quantization of the BS superparticle action. One proposal for such a modified BS superparticle action has been made in ref. [10]. Due to infinite field redefinitions the analysis of the cohomology is obscured and not clear to us. There will be no need for such dangerous redefinitions in our treatment of the problem, which starts from a different BRST operator. Another proposal was made in ref. [11]. There the modified BS action involves an additional fermionic coordinate. A candidate solution of the cohomology is given, but the full analysis of the cohomology is complicated due to the intrinsic non-linearity of the proposal.

Another point of view is to keep the gauge fixed action eq. (2), in view of the fact that they form representations of $\text{OSp}(10|4)$, and thus lead to a right counting of field degrees of freedom for obtaining the ten-dimensional matter multiplets. One can then ask oneself the following question: is it possible to associate to this action a nilpotent BRST operator Ω which can realise explicitly the theorem that the graded dimension of the metaplectic (spinor) representation

of the $\text{OSp}(10|4)$ supergroup is equal to the dimension of the spinor representation of the orthogonal group $\text{SO}(6)$, or $\text{SU}(4)$ which describes light-cone gauge spinors, and is equal to four. This operator is supposed to show that from the infinite set of 16-dimensional Lorentz-covariant spinors of alternating statistics and chirality [forming a metaplectic representation of $\text{OSp}(10|4)$] all fields, except one four-dimensional spinor of $\text{SO}(6)$, can be gauged away.

This operator Ω should correspond to the symmetry of the gauge-fixed action, it should be nilpotent and should serve for defining the physical states according to the usual rules.

It is the purpose of this letter to show that the answer to this question is yes. We will give such a nilpotent fermionic symmetry Ω .

2. Lemma's

The calculation of the cohomology solutions of a BRST operator for a superparticle is facilitated by two lemma's, which we will now discuss, and which were also used in ref. [10].

The first one will enable us to consider only solutions with $P^2=0$, and will allow us to forget about the b - c ghosts.

Lemma 1. For general nilpotent BRST operators of the form

$$\Omega = \Omega_0 + \frac{1}{2}cP^2 - 2bf \quad (5)$$

(Ω_0 and f are independent of x , c and b) the solutions to the cohomology problem of Ω are given by a double copy of the solutions of the cohomology problem of Ω_0 .

First, the nilpotency of Ω determines that

$$\Omega_0^2 = P^2 f. \quad (6)$$

We consider the Fock space built from states $|0, p\rangle_c$ satisfying

$$c|0, p\rangle_c = 0, \quad P|0, p\rangle_c = p|0, p\rangle_c. \quad (7)$$

In this space we consider a general state of the form

$$|\Psi\rangle = (\Psi_0 + 2b\Psi_1)|0, p\rangle_c, \quad (8)$$

where Ψ_0 and Ψ_1 are independent of c and b . One obtains then

$$\Omega|\Psi\rangle = (\Omega_0\Psi_0 + p^2\Psi_1) - 2b(f\Psi_0 + \Omega_0\Psi_1)|0, p\rangle_c. \quad (9)$$

We first consider the case $p^2 \neq 0$. The solution of $\Omega|\Psi\rangle = 0$ is then given by

$$|\Psi\rangle = (\Psi_0 - 2bp^{-2}\Omega_0\Psi_0)|0, p\rangle_c. \quad (10)$$

All these states can be gauged away as they are

$$|\Psi\rangle = \Omega(2bp^{-2}\Psi_0)|0, p\rangle_c. \quad (11)$$

So we conclude already that the physical states have $p^2 = 0$ as one should expect for massless particles.

We now consider the case $p^2 = 0$. In that case the equation $\Omega|\Psi\rangle = 0$ leads to

$$\begin{aligned} \Omega_0\Psi_0|0, p\rangle_c &= 0, \\ (\Omega_0\Psi_1 + f\Psi_0)|0, p\rangle_c &= 0. \end{aligned} \quad (12)$$

So we have to consider first the equation

$$\Omega_0|\chi\rangle = 0 \quad (13)$$

for states $|\chi\rangle$ built from $|0, p\rangle_c$ with $p^2 = 0$ and independent of c and b . If we know these solutions we still have to consider whether there exists a state χ with

$$\Omega_0|\chi\rangle = f|\psi\rangle. \quad (14)$$

There always exists a solution to this equation. It can be constructed by first considering $|\psi(p^2)\rangle$, the extension of the state $|\psi\rangle$ to $p^2 \neq 0$ (if we had first chosen coordinates such that only p^+ was different from zero, then we consider now the extension to $p^- = p^2/2p^+ \neq 0$). This extension is of course not unique, but one can check that the non-uniqueness will give rise only to terms which will be gauged away below. Then $\Omega_0|\psi(p^2)\rangle = p^2|\chi(p^2)\rangle$. Now we have, because of eq. (6) and $p^2 \neq 0$ that $f|\psi(p^2)\rangle = \Omega_0|\chi(p^2)\rangle$ and the limit to $p^2 = 0$ gives the solution of eq. (14). We get thus two solutions of the cohomology equations:

$$|\Psi\rangle = |\psi\rangle - 2b|\chi\rangle, \quad |\Psi\rangle = b|\psi\rangle. \quad (15)$$

They are in fact the same solution, once built on the vacuum $|0\rangle_c$, and the last one is based on the vacuum $b|0\rangle_c = |0\rangle_b$. This doubling is already well known even for the case of the bosonic particle. We still have to show that if

$$|\psi\rangle = \Omega_0|A\rangle \quad (16)$$

we can construct in both cases a state whose full BRST transform cancels eq. (15). For the first case we consider again an extension of eq. (16)

$$|\psi(p^2)\rangle - \Omega_0|A(p^2)\rangle = p^2|\xi(p^2)\rangle, \quad (17)$$

which then leads to

$$|\chi(p^2)\rangle - f|A(p^2)\rangle = \Omega_0|\xi(p^2)\rangle. \quad (18)$$

Its limit to $p^2 = 0$ is sufficient to show that in the first representation in eq. (15), if eq. (16) holds, then

$$|\Psi\rangle = \Omega(|A\rangle + 2b|\xi\rangle). \quad (19)$$

In the second representation we can just obtain it from

$$|\Psi\rangle = \Omega b|A\rangle. \quad (20)$$

This proves the lemma and we can further just consider eq. (13), and its gauge invariance.

The second lemma which we will use, concerns the "trivial quartets". This lemma is a reformulation of a theorem in ref. [12].

Lemma 2. Suppose that we have a fermion θ , its canonical conjugate momentum λ , a boson ϕ and its momentum π . We call this a trivial quartet if the BRST operator contains these fields only in one term where a boson is multiplied by a fermion:

$$\Omega = \theta\pi + \Omega_1. \quad (21)$$

Then the whole system can be omitted and we can continue to consider only the cohomology of Ω_1 .

One can show this in different Fock representations. Let us take here the Fock space built from the vacuum $|0\rangle$ which satisfies

$$\lambda|0\rangle = \pi|0\rangle = 0. \quad (22)$$

We can represent the momenta then by derivatives to the coordinates. A general state is of the form

$$|\Psi\rangle = (\Psi_0 + \theta\Psi_1)|0\rangle, \quad (23)$$

where the Ψ_i are functions of ϕ and other fields not contained in the quartet. We get

$$\Omega|\Psi\rangle = \left[\Omega_1\Psi_0 + \theta\left(\frac{\partial}{\partial\phi}\Psi_0 - \Omega_1\Psi_1\right) \right]|0\rangle. \quad (24)$$

Define now χ such that

$$\frac{\partial \chi}{\partial \phi} = \Psi_1. \quad (25)$$

The BRST invariant states are then

$$|\Psi\rangle = |\Psi_0^{(0)}\rangle + (\Omega_1 \chi + \theta \Psi_1) |0\rangle, \quad (26)$$

where $|\Psi_0^{(0)}\rangle$ is invariant under Ω_1 and does not depend on fields of the quartet, while Ψ_1 is arbitrary. Then we can gauge away the full Ψ_1 and the part of $|\Psi_0^{(0)}\rangle$ which is $\Omega_1 |A\rangle$ by using

$$\Omega(|\Psi_0^{(0)}\rangle + \chi|0\rangle). \quad (27)$$

This shows that only the cohomological solutions of Ω_1 are left, and hence the lemma is proven.

3. The cohomology of the “minimal BRST operator” and twisted $N=2$ supersymmetry

Now we will determine the cohomology of the BRST operator eq. (4). The last term can be omitted and determines that $\theta_{p,q}, \bar{\lambda}^{p,q}, \bar{\theta}^{p,q}, \lambda_{p,q}$ for $q \geq 1$ form trivial quartets. If we then go to the frame where $p^\mu = p^+ \delta^\mu_+$, the relevant part of the BRST operator reduces to

$$\Omega_0 = \sum_{p=0}^{\infty} \bar{\lambda}^{p,0} \gamma^- \theta_{p+1,0} P^+. \quad (28)$$

Splitting the spinors in, e.g.,

$$\bar{\theta} = -\frac{1}{2}(\gamma^+ \gamma^- + \gamma^- \gamma^+) \theta \equiv \gamma^+ \theta^- + \gamma^- \theta^+, \quad (29)$$

this becomes

$$\Omega_0 = \sum_{p=0}^{\infty} \bar{\lambda}^{p,0} \gamma^- \theta_{p+1,0} P^+. \quad (30)$$

So we delete again these fermions and their conjugates $\theta_{p,0}^+$ and $\bar{\lambda}^{p+1,0,+}$ for $p \geq 0$. The solutions of the cohomology are then only $\theta_{0,0}^-$ and its conjugate $\lambda^{0,0,+}$.

Covariantly we could write these solutions as

$$\bar{\lambda}^{0,0} \text{ and } s^{0,0} \equiv \not{P} \theta_{0,0} - 2\theta_{1,0} b, \quad (31)$$

up to gauge transformations determined by

$$[\Omega_{\min}, \bar{\lambda}_{1,0}] = \not{P} \bar{\lambda}^{0,0}, \quad [\Omega_{\min}, 2b\theta_{0,0}] = \not{P} s_{0,0}. \quad (32)$$

The solution contains thus eight fermionic spinor components as creation and eight as annihilation op-

erators. Therefore we could construct from the creation operators 2^7 bosonic and 2^7 fermionic states. These could form a minimal multiplet of $N=2$ supersymmetry in ten dimensions, but as already remarked in ref. [7] it is a twisted $N=2$ supersymmetry as can be seen by diagonalizing the commutation relations

$$\{\bar{\lambda}^{0,0}, s^{0,0}\} = \not{P}, \quad (33)$$

using the variables

$$q^0 = \bar{\lambda}^{0,0} - s^{0,0}, \quad \tilde{q}^0 = \bar{\lambda}^{0,0} + s^{0,0}. \quad (34)$$

We see then that the physical variables form an $N=2$ “twisted” supersymmetry algebra

$$\{q^0, q^0\} = -2\not{P}, \quad \{\tilde{q}^0, \tilde{q}^0\} = +2\not{P}. \quad (35)$$

The representations of such an $N=2$ twisted super algebra cannot contain only states with a positive definite norm. The reason of this is the relative minus sign in eq. (35). In a lightcone frame one finds that P^+ should be both non-negative as well as non-positive. The only consistent value of P^+ is therefore zero.

At first sight it seems a surprise to find an $N=2$ twisted supersymmetry being realized on the physical variables, since the classical BS superparticle action eq. (1) only has an $N=1$ supersymmetry which is given by

$$\delta X^\mu = \epsilon \gamma^\mu \theta, \quad \delta \theta = \epsilon. \quad (36)$$

This supersymmetry corresponds to q^0 after the various redefinitions which have been performed to obtain the action eq. (2). The $\tilde{q}^0$ supersymmetry appears after the gauge fixing.

4. The new BRST operator

In view of the above it is natural to ask oneself the question whether in principle there exists a choice for the BRST transformation rules of the fields that occur in the gauge-fixed action eq. (2) such that its cohomology leads to the same spectrum as one finds in the lightcone gauge. Clearly this BRST operator Ω should break the $N=2$ twisted supersymmetry corresponding to $\Omega_{\min}$ down to an $N=1$ supersymmetry. We constructed this operator essentially by a trial and error procedure. Meanwhile one of us has given an-

other way to understand the result [13]. An essential feature of Ω is that it leads to non-trivial BRST transformation rules of the non-minimal ghost fields. This is natural since the non-minimal fields are essential in the $\text{OSp}(10|4)$ multiplet, which we believe to be the basis of a correct counting of physical degrees of freedom.

We introduce the following notations:

$$\begin{aligned} s^{p,q} &= \not{P}\theta_{p,q} - 2\theta_{p+1,q}b, \\ \bar{s}_{p,q} &= \not{P}\bar{\theta}^{p,q} - 2\bar{\theta}^{p-1,q}b \quad \text{for } p > q, \\ \bar{s}_{p,p} &= \not{P}\bar{\theta}^{p,p}, \\ s'^{p,q} &= s^{p,q} + (\lambda_{p-1,q} + \lambda_{p,q+1})Z_{p-1-1} \quad \text{for } p > q, \\ \bar{s}'_{p,q} &= \bar{s}_{p,q} + (\bar{\lambda}^{p+1,q} + \bar{\lambda}^{p,q-1})Z_{p-q+1} \quad \text{for } p > q, \\ \bar{s}'_{p,p} &= \bar{s}_{p,p} + (\bar{\lambda}^{p+1,p} + \bar{\lambda}^{p,p-1}) - 2b(\theta_{p,p} - \theta_{p-1,p-1}), \\ Z_n &= (-)^{n(n-1)/2}. \end{aligned} \quad (37)$$

(Remember that $\lambda_{p,0}=0$.) All the primed quantities defined above, will be BRST invariant. Finally we introduce also the generalisations of eq. (34)

$$q^p = \bar{\lambda}^{p,p} - s^{p,p}, \quad \tilde{q}^p = \bar{\lambda}^{p,p} + s^{p,p}, \quad (38)$$

which satisfy the infinite $N=2$ twisted supersymmetry commutation relations

$$\{q^p, q^q\} = -2\not{P}\delta_{p,q}, \quad \{\tilde{q}^p, \tilde{q}^q\} = +2\not{P}\delta_{p,q}. \quad (39)$$

We further split these in q^0 , and for $p \geq 0$

$$F^p = q^{p+1} + \tilde{q}^p, \quad \tilde{F}^p = q^{p+1} - \tilde{q}^p. \quad (40)$$

Our proposal for Ω is the following:

$$\begin{aligned} \Omega &= \frac{1}{2}cP^2 + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} (\bar{\lambda}^{p,q}s^{p+1,q} + \lambda_{p+2,q+1}\bar{s}'_{p+1,q+1}) \\ &+ \sum_{p=0}^{\infty} \lambda_{p+1,p+1}F^p. \end{aligned} \quad (41)$$

Note that all terms in Ω are consistent with the ghost number assignments in table 1 in the sense that the action has zero ghost number and the BRST operator has ghost number 1. One can also check that the operator Ω is nilpotent.

The investigation of the cohomology goes as in the previous section. First we simplify the BRST operator, by using $p^2=0$ and $b=c=0$ as implied by the first lemma. Due to the equation

Table 1
Ghost numbers.

c	1
b	-1
$\theta_{p,q}$	$p-q$
$\bar{\lambda}^{p,q}$	$-p+q$
$\bar{\theta}^{p,q}$	$q-p-1$
$\lambda_{p,q}$	$-q+p+1$

$$\begin{aligned} &\sum_{p=q}^{\infty} \sum_{q=0}^{\infty} \lambda_{p+2,q+1} \bar{s}'_{p+1,q+1} + \sum_{p=0}^{\infty} \lambda_{p+1,p+1} F^{p,-} \\ &= \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \bar{\lambda}^{p,q,-} (s'^{p+1,q,+} - s^{p+1,q,+}), \end{aligned} \quad (42)$$

the BRST operator simplifies to

$$\begin{aligned} \Omega_0 &= 2 \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} (\bar{\lambda}^{p,q,-} s'^{p+1,q,+} + \lambda_{p+2,q+1} \bar{s}'_{p+1,q+1}) \\ &+ 2 \sum_{p=0}^{\infty} \lambda_{p+1,p+1} F^{p,+}. \end{aligned} \quad (43)$$

The crucial step in analysing the BRST cohomology is now the change of variables. We started from the + and - parts of $\theta_{p,q}$, $\bar{\lambda}^{p,q}$ for $p \geq q$ and $q \geq 0$, and $\bar{\theta}^{p,q}$, $\lambda_{p,q}$ for $p \geq q$ and $q \geq 1$. In those canonically conjugated pairs we now have that, e.g., the canonical conjugate of $\theta_{p,q}^+$ is $\bar{\lambda}^{p,q,-}$, etc.

The introduction of s and $\bar{s}$ in eq. (37) implies the change of variables from $\theta_{p,q}^-$ (for $p > q$, $q \geq 0$) and $\bar{\theta}^{p,q,-}$ (for $p \geq q$, $q \geq 1$) to

$$\begin{aligned} \theta_{p,q}^- &\equiv \theta_{p,q}^- + \frac{1}{2P^+} \gamma^- (\lambda_{p-1,q,+}^+ + \lambda_{p,q+1,+}^+) Z_{p-q-1} \\ &= \frac{1}{2P^+} \gamma^- s'^{p,q,+}, \\ \bar{\theta}^{p,1,-} &\equiv \bar{\theta}^{p,q,-} + \frac{1}{2P^+} \gamma^- (\bar{\lambda}^{p+1,q,+} + \bar{\lambda}^{p,q-1,+}) Z_{p-q+1} \\ &= \frac{1}{2P^+} \gamma^- \bar{s}'_{p,q}^+. \end{aligned} \quad (44)$$

These definitions keep these sets of variables (anti)commuting. Further we can go from $\theta_{p,p}^-$ and $\bar{\lambda}^{p,p,+}$ to $q^{p,+}$ and $\tilde{q}^{p,+}$. Finally we combine $\tilde{q}^{p,+}$ and $q^{p+1,+}$ to the canonical conjugates $F^{p,+}$ and $\tilde{F}^{p,+}$, leaving $q^{0,+}$ untouched. (These last steps were also performed in ref. [10]). We have thus as indepen-

dent variables the canonically conjugated pairs ($p \geq q$, $q \geq 0$ except when indicated otherwise)

$$\begin{aligned} \theta_{p,q}^+, \quad \bar{\lambda}^{p,q,-}, \\ \bar{\theta}^{p,q,+}, \quad \lambda_{p,q}^- \quad \text{for } q \geq 1, \\ \theta_{p,q}'^-, \quad \bar{\lambda}^{p,q,+} \quad \text{for } p > q, \\ \bar{\theta}'^{p,q,-}, \quad \lambda_{p,q}^+ \quad \text{for } q \geq 1, \\ F^{p,+}, \quad \bar{F}^{p,+} \quad \text{for } p \geq 0, \\ q^{0,+}. \end{aligned} \quad (45)$$

Looking at eq. (43) we immediately recognise now all the quartets which can be discarded. We therefore conclude that the only physical variable is $q^{0,+}$ which satisfies the $N=1$ supersymmetry algebra

$$\{q^{0,+}, q^{0,+}\} = -P^+ \gamma^+, \quad (46)$$

and we have thus four fermionic creation and four annihilation operators. The smallest representation of this superalgebra is given by the $8+8$ Yang-Mills supermultiplet and hence we obtain the correct physical spectrum.

Covariantly, we can say that the BRST invariant is q^0 as defined in eq. (34) up to gauge transformations defined by

$$[\Omega, \bar{\lambda}^{1,0} - 2b\theta_{0,0}] = \not{P} q^0. \quad (47)$$

5. Remarks

One would like to set up a covariant quantization scheme which, when applied to a superparticle action, would not only lead to the gauge-fixed action eq. (2) but also to the new BRST operator Ω given in eq. (41). This issue will be discussed in a separate paper [13].

It turns out that there exists a classical action, describing the superparticle, which is different from the BS action. However, the light-cone gauge and the covariant gauge of the new action coincide with those of the BS action. In fact, the BS action is a partially gauge-fixed new action, in which from all gauge symmetries of the original action only the so-called κ -symmetry is still not gauge-fixed.

The new theory [13] contains only first-class constraints. The quantization of it leads in a straightforward way to the gauge-fixed action, given in eq. (2), and to the BRST-operator Ω , given in eq. (41) of the present paper, where the existence of this operator was discovered.

It would be interesting to see whether a similar proposal for the BRST operator might also lead to the correct spectrum in the case of superstrings.

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