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COMPACTIFICATIONS OF THE ELEVEN-DIMENSIONAL SUPERMEMBRANE**E. BERGSHOEFF**¹*Blackett Laboratory, Imperial College, London SW7 2BZ, UK***M.J. DUFF, C.N. POPE***Center for Theoretical Physics, Physics Department, Texas A&M University, College Station, TX 77843-4242, USA*

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We construct new vacua for the eleven-dimensional supermembrane in which spacetime is the product of four-dimensional anti-de Sitter space and a compact seven-dimensional Einstein space, and the membrane is a sphere of non-zero radius in the anti-de Sitter space. In one class of solution the radius is a specific multiple of the anti-de Sitter scale parameter while in a second class the radius is arbitrary. Remarkably, only two of the Freund–Rubin compactifications admit vacua of the latter class: the round seven-sphere with $N=8$ supersymmetry and the $N(0, 1)$ space with $N=3$ supersymmetry.

It is not yet known whether the eleven-dimensional supermembrane [1] yields a consistent quantum theory of gravity, but it is the only supersymmetric extended-object theory, apart from superstrings, that has passed the consistency checks that have so far been applied [2–4]. The existence of the local world-volume fermionic symmetry of the supermembrane action requires that the eleven-dimensional background fields be solutions of the equations of motion of eleven-dimensional supergravity^{#1} [1,5]. Thus the supermembrane has revived an interest in compactifications of $d=11$ supergravity. These were studied in some detail a few years ago [6], but in common with most Kaluza–Klein theories there was a severe problem of vacuum degeneracy. In the case of the supermembrane, however, one must not only choose a background that satisfies the $d=11$ supergravity equations but also one must solve the field equations of the supermembrane itself.

All $d=11$ supergravity backgrounds admit solutions where the membrane is collapsed to a point, but in this paper we shall focus our attention on solutions of the membrane equations where spacetime is the product of four-dimensional anti-de Sitter space and a compact seven-dimensional Einstein space (i.e. a Freund–Rubin solution [7]) and where the membrane adopts a stable, spherical configuration of non-zero radius in anti-de Sitter space. We shall show that only a small subset of the infinity of such Freund–Rubin compactifications admit membrane solutions of this kind. In fact, we have found two distinct types of non-collapsed membrane solution; in one case the radius of the spherical membrane is a specific multiple of the anti-de Sitter scale param-

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^{#1} Strictly speaking, for a particular classical membrane configuration Siegel symmetry requires only that the $d=11$ supergravity equations should hold on the three-dimensional membrane submanifold. However, we are really interested in considering spacetime backgrounds for which *any* configuration of the membrane exhibits Siegel symmetry, and hence we require that the supergravity equations should be satisfied everywhere in the $d=11$ spacetime manifold.

eter, while in the other case the radius of the membrane is arbitrary. As discussed at the end of the paper, the latter solutions are particularly interesting. For example, in the limit of infinite radius, the small fluctuations about these vacua are described by a free-field theory. Remarkably enough, only two of the Freund–Rubin compactifications admit membrane vacua of this sort. For one, the internal space is the round seven-sphere, yielding $N=8$ spacetime supersymmetry [6], whilst for the other, the internal space is the $N(0, 1)$ space which is an $SO(3)$ bundle over CP^2 , yielding $N=3$ spacetime supersymmetry [8,9].

The solutions that we shall describe in this paper were inspired by the configurations that we considered in an earlier paper [10]. In that paper, we adopted an ansatz in which the anti-de Sitter radial coordinate r and the coordinates of the internal space were independent of the membrane coordinates τ, σ and ρ . We found solutions with $r=\infty$ for any Freund–Rubin compactification, but only at the expense of introducing a new coordinate r' where the jacobian of the transformation vanished at $r=\infty$. In this paper we avoid such pathologies, and are able to obtain bona fide solutions by permitting a τ dependence of one of the internal-space coordinates.

Our starting point is the eleven-dimensional supermembrane [1] action

$$S = - \int d\tau d\sigma d\rho (\sqrt{-h} + \frac{1}{3} \epsilon^{ijk} \partial_i Z^A \partial_j Z^\Sigma \partial_k Z^\Omega B_{\Omega\Sigma A}),$$

$$h_{ij} = \Pi_i^a \Pi_j^b \eta_{ab}, \quad i, j = 0, 1, 2, \quad a, b = 0, 1, \dots, 10,$$

$$\Pi_i^a = \partial_i Z^A E_A^a,$$

$$Z^A = (X^M, \Theta^\alpha), \quad M = 0, 1, \dots, 10, \quad \alpha = 1, 2, \dots, 32, \quad (1)$$

where (τ, σ, ρ) are the coordinates on the world-volume with metric h_{ij} , $E_A^a(X^M, \Theta)$ is the supervielbein and $B_{\Omega\Sigma A}(X^M, \Theta)$ is the super three-form potential appropriate to the eleven-dimensional superspace. $X^M(\tau, \sigma, \rho)$ ($M=0, 1, \dots, 10$) are the bosonic coordinates and $\Theta(\tau, \sigma, \rho)$ are the fermionic coordinates of eleven-dimensional superspace.

The action (1) has a local fermionic symmetry

$$E_A^\alpha \delta Z^A = 0, \quad E_A^\alpha \delta Z^A = (1 + \Gamma)^\alpha{}_\beta \kappa^\beta, \quad (2)$$

provided that the eleven-dimensional field equations are satisfied. In eq. (1), the matrix Γ is given by

$$\Gamma = (1/3!) \epsilon^{ijk} E_i^A E_j^B E_k^C \Gamma_{ABC}, \quad (3)$$

and has the important properties $\Gamma^2 = 1$ and $\text{tr } \Gamma = 0$ as a consequence of which $\frac{1}{2}(1 \pm \Gamma)$ are projection operators. The bosonic field equation, for a configuration where the spacetime gravitino $\Psi_M(x)$ and the fermionic coordinates $\Theta(\tau, \sigma, \rho)$ are zero, is

$$\partial_i (\sqrt{-h} h^{ij} \partial_j X^N g_{MN}) - \frac{1}{2} \sqrt{-h} h^{ij} \partial_i X^N \partial_j X^P \partial_M g_{NP} + \frac{1}{3} \epsilon^{ijk} \partial_i X^N \partial_j X^P \partial_k X^Q H_{MNPQ} = 0, \quad (4)$$

where $H_{MNPQ} = 4\partial_{[M} B_{NPQ]}$. The existence of the local fermionic symmetry (2) requires that g_{MN} and H_{MNPQ} satisfy the usual bosonic equations of $d=11$ supergravity [1,5]. As in ref. [10], we shall first consider the $d=11$ supergravity background in which the spacetime is $AdS_4 \times S^7$, i.e., the direct product of a four-dimensional anti-de Sitter space with inverse radius a and coordinates $x^\mu = (t, \theta, \phi, r)$ and a seven-sphere with coordinates y^m ($m=1, \dots, 7$). The four-index field strength is proportional to the Levi-Civita tensor in AdS_4 : $H_{\mu\nu\rho\sigma} = \frac{3}{2} a \sqrt{-g} \epsilon_{\mu\nu\rho\sigma}$, where g is the determinant of the AdS_4 metric $g_{\mu\nu}(x)$. We use a coordinate system in which the AdS_4 metric is given by

$$ds^2 = -(1 + a^2 r^2) dt^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + (1 + a^2 r^2)^{-1} dr^2. \quad (5)$$

On S^7 , we take the standard $SO(8)$ -invariant Einstein metric, satisfying $R_{mn} = \frac{3}{2} a^2 g_{mn}$.

In order to construct our solutions of the field equation (4) for the bosonic membrane coordinates, we regard S^7 as being a $U(1)$ bundle over CP^3 , and write its metric as

$$d\hat{s}^2 = (4/a^2) [(d\psi + 2A)^2 + ds^2(\text{CP}^3)], \quad (6)$$

where $ds^2(\text{CP}^3)$ is the standard Fubini–Study metric on CP^3 , with Einstein metric satisfying $R_{ab} = 8g_{ab}$, ψ is the coordinate on the $U(1)$ fibres, with period $\Delta\psi = 2\pi$, and A is a one-form potential satisfying $dA = J$, where J is the Kähler two-form on CP^3 .

In terms of three complex coordinates q^a ($a = 1, 2, 3$), the metric on CP^3 may be written as

$$ds^2(\text{CP}^3) = (1 + \bar{w}^c w^c)^{-1} d\bar{w}^a dw^a - (1 + \bar{w}^c w^c)^{-2} \bar{w}^a dw^a w^b d\bar{w}^b \quad (7)$$

and A takes the form

$$A = \frac{1}{2}i(1 + \bar{w}^b w^b)(w^a d\bar{w}^a - \bar{w}^a dw^a). \quad (8)$$

To solve (4), we make the following ansatz^{#2}

$$t = (2/\alpha a)\psi = \tau, \quad (9)$$

$$\theta = \sigma, \quad (10)$$

$$\phi = \rho, \quad (11)$$

$$r = \text{constant}, \quad (12)$$

$$w^a = \text{constant}, \quad (13)$$

where α is a constant and w^a are the coordinates of CP^3 . With this ansatz, h_{ij} is given by

$$h_{ij} = \text{diag}(-(1 - \alpha^2 + a^2 r^2), r^2, r^2 \sin^2 \sigma). \quad (14)$$

Substituting (9)–(14) into (4), we find that all components of the equation are satisfied identically except in the r direction. This equation yields

$$-a^2(1 - \alpha^2 + a^2 r^2)^{-1/2} r^3 - 2(1 - \alpha^2 + a^2 r^2)^{1/2} r + 3ar^2 = 0. \quad (15)$$

To solve (15), we distinguish between two cases, namely where $\alpha = 1$ and where $\alpha \neq 1$ (without loss of generality, we shall always take $\alpha \geq 0$). For $\alpha = 1$, the left-hand side of (15) becomes identically zero, and so the equation is satisfied by any value of r . For $\alpha \neq 1$, eq. (15) has the solution

$$r = [4(\alpha^2 - 1)/3a^2]^{1/2} \quad (16)$$

which implies that to obtain a real solution we need $\alpha > 1$.

Because of the relation (9), the value of the constant α is not arbitrary, but is related to the periodicities $\Delta\psi$ and Δt of the coordinates ψ and t . The coordinate ψ on the $U(1)$ fibres of S^7 has the period 2π . More generally, one can factor S^7 by the cyclic group Z_p , corresponding to identifying ψ with period $2\pi/p$. This yields the cyclic lens space $L(1, p)$. Special cases are $L(1, 1) = S^7$ and $L(1, 2) = \text{RP}^7$. If p is greater than 1, the isometry group will be the $SU(4) \times U(1)$ subgroup of $SO(8)$ that is a manifest symmetry of the metric (6). This follows from the fact that under $SU(4) \times U(1)$, the 28 of $SO(8)$ decomposes as $28 \rightarrow 1_0 + 15_0 + 6_1 + 6_{-1}$, and so the 6_1 and 6_{-1} Killing vectors have ψ dependence of the form $\exp(\pm i\psi)$, and are hence eliminated by the identification of ψ and $\psi + 2\pi/p$. A priori, one may assign any period one wishes to the time coordinate t on AdS. However, if we construct the ten Killing vectors ξ_{AB} of $SO(3, 2)$ on AdS ($A, B = 0, 1, 2, 3, 5$), we see that

^{#2} This ansatz differs from that of ref. [10] where ψ was assumed constant, which corresponds, in effect, to setting $\alpha = 0$ in (9). A solution of (15) at $r = \infty$ was obtained but only at the expense of transforming to a new coordinate r' for which the jacobian of the transformation vanishes at $r = \infty$. In this paper we shall avoid such pathological “solutions”.

$$\begin{aligned}
\xi_{05} &= a^{-1} \frac{\partial}{\partial t}, \quad \xi_{12} = \frac{\partial}{\partial \phi}, \quad \xi_{23} + i\xi_{31} = \exp(i\phi) \left(i \frac{\partial}{\partial \theta} - \cot \theta \frac{\partial}{\partial \phi} \right), \\
\xi_{15} + i\xi_{10} &= \exp(-iat) \left[-ia^{-1} \sin \theta \cos \phi \sin \beta \frac{\partial}{\partial t} + \operatorname{cosec} \beta \left(\cos \theta \cos \phi \frac{\partial}{\partial \theta} - \sin \phi \operatorname{cosec} \theta \frac{\partial}{\partial \phi} \right) \right. \\
&\quad \left. + \cos \beta \sin \theta \cos \phi \frac{\partial}{\partial \beta} \right], \\
\xi_{25} + i\xi_{20} &= \exp(-iat) \left[-ia^{-1} \sin \theta \sin \phi \sin \beta \frac{\partial}{\partial t} + \operatorname{cosec} \beta \left(\cos \theta \sin \phi \frac{\partial}{\partial \theta} + \cos \phi \operatorname{cosec} \theta \frac{\partial}{\partial \phi} \right) \right. \\
&\quad \left. + \cos \beta \sin \theta \sin \phi \frac{\partial}{\partial \beta} \right],
\end{aligned} \tag{17}$$

$$\xi_{35} + i\xi_{30} = \exp(-iat) \left(-ia^{-1} \cos \theta \sin \beta \frac{\partial}{\partial t} - \sin \theta \operatorname{cosec} \beta \frac{\partial}{\partial \theta} + \cos \beta \cos \theta \frac{\partial}{\partial \beta} \right), \tag{18}$$

where $ar = \tan \beta$. The $SO(2) \times SO(3)$ Killing vectors (17) are compatible with any periodicity for t , but the remaining six Killing vectors (18) are ill-defined unless the period of t is an integer multiple of $2\pi/a$. Thus maximal spacetime symmetry requires $\Delta t = (2\pi/a)q$. From (9), it therefore follows that

$$\alpha = 2/pq, \tag{19}$$

where the eleven-dimensional background is the product of the q -fold covering of AdS and the lens space $L(1, p)$. The case $\alpha^2 = 1$ can be achieved either by taking the background to be $(\text{AdS}) \times \text{RP}^7$, or to be $(\overline{\text{AdS}}) \times S^7$, where $\overline{\text{AdS}}$ is the *double* covering of AdS. The only way of achieving the case $\alpha > 1$ is to take the background to be $(\text{AdS}) \times S^7$, and hence $\alpha = 2$. From (16), this implies that $r = 2/a$.

Thus, in the case of S^7 , there are two kinds of solution: one with $\alpha = 1$ for which the radius of the membrane is arbitrary and the other with $\alpha = 2$ for which the radius is fixed at $r = 2/a$. What about other choices of M^7 , the seven-dimensional internal Einstein space? For solutions of the type (9), we still require that M_7 be a $U(1)$ bundle over some six-manifold M_6 , with metric

$$ds^2 = (4/a^2) [c^2(d\psi + 2A)^2 + ds^2(M_6)], \tag{20}$$

where c is a constant. In the case where M_6 is Einstein-Kähler (satisfying $R_{ab} = 8g_{ab}$ and $dA = J$ as before), then $c = 1$ and the period of ψ is given by

$$\Delta\psi = \pi k/2p, \tag{21}$$

where k is that non-negative integer such that the first Chern class evaluated on $H_2(M, \mathbb{Z})$ is $\mathbb{Z} \cdot k$, i.e., the integers divisible by k [11]. As before, p is any integer and $p \geq 2$ corresponds to factoring M_7 by $\mathbb{Z}_p$. Hence, from (9),

$$\alpha = k/2pq, \tag{22}$$

where, as before, we are considering a q -fold cover of AdS. In six dimensions one can show that $k \leq 4$ with equality if and only if $M_6 = \text{CP}^3$ [12]. Thus aside from the $k = 4$ cases $M_7 = S^7$ or RP^7 which we have already discussed, the only other way of achieving $\alpha = 1$ is to take $k = 2$, $p = q = 1$. We know of one example of such a space, where $M_6 = \text{SU}(3)/[\text{U}(1) \times \text{U}(1)]$ with its Einstein-Kähler metric [9]. The resulting M_7 is the $N(0, 1)$ space of refs. [8,9].

To see this, we use the description of the $N(0, 1)$ space given in ref. [9], in which it is written as an $SO(3)$ bundle over CP^2 . The metric may be given in the form

$$ds^2 = \lambda^2 (\Sigma^i - A^i)^2 + d\bar{s}^2, \quad (23)$$

where λ is a constant (the squashing parameter), Σ^i denotes a set of three left-invariant one-forms on the group $SO(3)$, $d\bar{s}^2$ is the standard metric on CP^2 with $R_{ab} = 6g_{ab}$, and A^i is a connection on the anti-self-dual spin bundle of CP^2 . In the natural orthonormal basis, the Ricci tensor for the metric (23) takes the diagonal form $(\beta, \beta, \beta, \beta, \gamma, \gamma, \gamma)$, where $\beta = 6 - 6\lambda^2$ and $\gamma = 4\lambda^2 + \frac{1}{2}\lambda^{-2}$. Thus it is Einstein if $\lambda^2 = \frac{1}{2}$ or if $\lambda^2 = \frac{1}{10}$. The case $\lambda^2 = \frac{1}{2}$ can also be described as a $U(1)$ bundle over the Einstein–Kähler metric on $SU(3)/[U(1) \times U(1)]$. Rescaling (23) to obtain Einstein metrics satisfying our normalization condition $R_{mn} = \frac{3}{2}a^2 g_{mn}$, we have

$$d\bar{s}^2 = (4\lambda^2/a^2)(1 - \lambda^2)[(\Sigma^i - A^i)^2 + \lambda^{-2} d\bar{s}^2], \quad (24)$$

where $\lambda^2 = \frac{1}{2}$ or $\lambda^2 = \frac{1}{10}$. Focusing on a $U(1)$ subgroup of the $SO(3)$ fiber group, we note that if Σ^i is written in terms of Euler angles θ', ϕ', ψ' , we have $\Sigma^3 = d\psi' + \cos \theta' d\phi'$, where ψ' has period 2π and Σ^1 and Σ^2 do not involve ψ' . Thus comparing with (20) (with $c=1$), we see that the periodicity of ψ for our $N(0, 1)$ metric is $\Delta\psi = \pi$ if $\lambda^2 = \frac{1}{2}$, and $\Delta\psi = \frac{3}{5}\pi$ if $\lambda^2 = \frac{1}{10}$. Thus from (21), we find that when $\lambda^2 = \frac{1}{2}$ the integer k is equal to 2, and we have a solution of the membrane equations with $\alpha = 1, p = q = 1$.

When $\lambda^2 = \frac{1}{10}$, the metric on $N(0, 1)$ can still be written in the form (20), but now the metric on $M_6 = SU(3)/[U(1) \times U(1)]$ is no longer Einstein–Kähler, and so the discussion in terms of the integer k is no longer applicable. However, we still have the relation between Δt and $\Delta\psi$ implied by eq. (9), which yields the result

$$\alpha = 3/5pq.$$

Thus in this case there is no solution of the membrane equations, since α must be greater than or equal to 1. In the same way, we can analyze the case of the squashed seven-sphere solution of $d=11$ supergravity [13]. This can be written as a $U(1)$ bundle over a homogeneous metric on CP^3 that is neither Einstein nor Kähler (see, for example, ref. [14]). Here, we can also describe the metric as an $SU(2)$ bundle over S^4 , the only relevant difference from the $N(0, 1)$ discussion above is that now the Euler angle ψ' has period 4π rather than 2π , since the fiber group is $SU(2)$ rather than $SO(3)$. Thus for the round S^7 , which is a $U(1)$ bundle over the Einstein–Kähler metric on CP^3 , we have $\Delta\psi = 2\pi$ and hence $k=4$, as we saw earlier, while for the squashed Einstein metric on S^7 we have $\Delta\psi = \frac{6}{5}\pi$. From (9), we therefore find that for the squashed S^7 ,

$$\alpha = 6/5pq, \quad (25)$$

and so there is a solution with $\alpha = \frac{6}{5}, p = q = 1$. From eq. (16), we see that in this case the membrane resides at $r = (2/5a)(\frac{11}{3})^{1/2}$.

Further fixed-radius solutions for which M_6 is not Einstein may also be found. For example, if $M_6 = CP^2 \times S^2$, one can construct an infinite class of Einstein metrics on the spaces $M_7 = M(m, n)$, where m and n are integers characterizing the winding numbers of the $U(1)$ bundles over CP^2 and S^2 [15–17]. Solutions of the membrane equations are possible if $q=1$ and $M_7 = M(1, 0) = S^5 \times S^2$, $M_7 = M(0, 1) = CP^2 \times S^3$ or $M_7 = M(1, 1)$. The corresponding values of α^2 are $\frac{8}{3}, \frac{4}{3}$ and $(2 + 2\beta - \beta^2)/(1 + \beta) \approx 1.297621$ respectively, where β is the real cube root of 2. Other solutions are provided by some isolated examples of the $Q(l, m, n)$ spaces [18], which are $U(1)$ bundles over $S^2 \times S^2 \times S^2$ [19]. None of the $M(m, n)$ or $Q(l, m, n)$ spaces that allow membrane solutions has any spacetime supersymmetry.

It should be emphasized that the reason why we find that only a small number of the Freund–Rubin compactifications can support non-collapsed membrane solutions of the kind we are discussing is because we are demanding that the time coordinate in anti-de Sitter space should have period $2\pi q/a$, in order that it have $SO(3, 2)$ isometry group. If we were prepared to allow an arbitrary period for t , the constant α in the ansatz (9) would be a free parameter, and so we would always be able to choose it to be greater than or equal to unity, as required by (16), for any Freund–Rubin compactification. The price we would pay, as discussed earlier, is that our four-dimensional spacetime would no longer be maximally symmetric, but would instead only have $SO(3) \times SO(2)$ as its isometry group. However, if we were content to accept a spacetime with $SO(3) \times SO(2)$ isometry group,

there would no longer be any reason to single out the anti-de Sitter line element; there are many other metrics, such as Schwarzschild (de Sitter) or Taub-NUT (de Sitter), that also have $SO(3) \times SO(2)$ as isometry group. Thus the same requirement of maximal symmetry that rules out these spacetimes also implies that we should have the period of t equal to $2\pi q/a$ in AdS.

The fact that we are forced at all to identify time periodically in anti-de Sitter space has another important consequence, namely that the lowest-energy eigenvalue E_0 used in the classification of $SO(3, 2)$ representations can only take quantised values. Since the time dependence of the wavefunctions is $\exp(iE_0 t)$, we must have $E_0 = \pi n / \Delta t = na/2q$, where n is an integer (we remind the reader that unitary representations of $SO(3, 2)$ can admit either periodic or anti-periodic bosons and fermions). Consequently, when one looks at the four-dimensional fields arising from the Kaluza-Klein reduction of $d=11$ supergravity, only those Fourier modes whose lowest energies take these quantised values can occur in the fluctuation spectrum. In fact, the lowest-energy values of the Fourier modes for generic Freund-Rubin compactifications do not take such values; for example the E_0 values for the squashed seven-sphere are in general irrational multiples of a [6]. Indeed the only known example for which all the lowest energies do obey the relation $E_0 = na/2q$ is the round seven-sphere, and it seems probable that this is the only space to have this property. In particular the $N(0, 1)$ space that we discussed earlier does not. Thus it seems that only for the round seven-sphere does the Kaluza-Klein theory make sense on periodically identified AdS; for all other internal spaces one must go to the simply connected covering space of AdS, where $-\infty < t < \infty$.

If we now turn to the supermembrane theory, and to the solutions discussed in this paper which *required* that t be periodically identified, we find therefore that only for the case of the round seven-sphere could the Kaluza-Klein modes be part of the supermembrane spectrum. Interestingly enough, the arbitrary radius seven-sphere solutions required either the single cover of AdS and $S^7/Z_2 = \mathbb{RP}^7$ for the internal space or the double cover of AdS and S^7 for the internal space, and for both of these possibilities the bosons are *periodic*. There has been some dispute in the singleton literature as to whether one should take the double cover of AdS in order to avoid anti-periodic bosons. It is curious therefore that the membrane avoids anti-periodic bosons automatically.

We have seen that only a small number of Freund-Rubin compactifications can support non-collapsed spherical membranes. Of these, just two allow $\alpha=1$ solutions, where the radius of the membrane can be chosen arbitrarily. It is curious that these happen to be the two Freund-Rubin compactifications with the largest numbers of spacetime supersymmetries, namely $N=8$ for the round seven-sphere [6] and $N=3$ [8] for the $\lambda^2 = \frac{1}{2}$ $N(0, 1)$ space. There are several reasons why the possibility of the membrane having an arbitrary radius is especially interesting:

(1) Vacuum solutions with a continuous parameter (in this case the membrane radius) are important for a treatment of semi-classical quantisation, since they can provide information about the structure of the theory at the non-perturbative level. In this respect they resemble the rotating membrane solutions of refs. [20–22]. Indeed the solutions in this paper *are* rotating solutions, in the sense that the membrane is rotating around the $U(1)$ fibers in the internal space. This explains how the membrane configuration with non-zero radius can be stable against collapse. Corresponding to each isometry of the $d=11$ spacetime, there is a conserved Noether charge Q , given by

$$Q = - \int d\sigma d\rho (\sqrt{-h} h^{0i} K_M \partial_i X^M - \epsilon^{0ij} \partial_i X^M \partial_j X^N A_{NM}), \quad (26)$$

where K^M is the Killing vector associated with each isometry and $\partial_{[M} A_{NP]} = K^L H_{LMNP}$. The de Sitter energy (the Noether charge for the Killing vector ξ_{05}) for an $\alpha=1$ solution with radius r is given by

$$E = 4\pi r / a^2 \quad (27)$$

and the $U(1)$ charge q (the Noether charge for the Killing vector $\partial/\partial\psi$) is given by

$$q = -8\pi r / a^2. \quad (28)$$

(2) The arbitrary-radius solutions admit an unbroken membrane supersymmetry when the radius tends to infinity. As discussed in ref. [10], the criterion for a spacetime supersymmetry to be preserved by the membrane configuration is that the spacetime Killing spinor should be annihilated by $(1 - \Gamma)$, where Γ is given by eq. (3). It is a remarkable feature of anti-de Sitter space that its Killing spinors become eigenstates of Γ asymptotically, as one moves out to spatial infinity. After making an $11 = 4 + 7$ split, Γ tends to $\gamma \otimes \mathbf{1}$ as r tends to infinity, where $\gamma = \gamma_0 \gamma_1 \gamma_2$ and γ_μ denote the four-dimensional Dirac matrices, and the $d = 11$ supersymmetry parameter ϵ behaves like

$$\epsilon \sim (\epsilon^- r^{-1/2} + \epsilon^+ r^{1/2}) \otimes \eta, \quad (29)$$

where $\epsilon^\pm$ are four-component spinors on the boundary of AdS, satisfying $\gamma \epsilon^\pm = \pm \epsilon^\pm$, and η is a Killing spinor in the internal space [10]. Thus we can obtain a supersymmetric membrane vacuum only when we are allowed to take the radius of the membrane to infinity (the membrane at the end of the universe).

(3) Fluctuations about the membrane at the end of the universe are described by a free field theory. In a semi-classical analysis, one quantises the fluctuations around a classical solution. For the arbitrary-radius solutions in this paper, the fluctuations must be rescaled by appropriate powers of r in order to obtain finite and non-zero kinetic terms as r is taken to infinity. Under these same rescalings, however, the interaction terms are damped down by inverse powers of r , so that in the limit $r \rightarrow \infty$ the fluctuation lagrangian describes a free field theory. This may have important consequences for the renormalisability of the $d = 11$ supermembrane [23,24].

(4) Singletons live on the boundary of anti-de Sitter space [25]. Thus one might expect [10] that the fluctuations described above should fall into the ultra-short singleton supermultiplets of the $N = 8$ supersymmetry discussed in (2), since this is the only way in which a spin-0/spin- $\frac{1}{2}$ system can have $N = 8$ supersymmetry. However, explicit calculation reveals that in contrast to the supersymmetric singleton lagrangian on $S^1 \times S^2$ [23,24] that requires bosonic mass terms but no fermionic mass terms [25], neither the fermion nor boson rescaled fluctuations that we discussed in (3) have mass terms. This means that while the fermion fluctuations are singletons the bosons are not, and so supersymmetry is broken. The resolution of the paradox seems to be that at the same time as performing the rescalings of the fluctuations described in (3), one would also need to rescale the supersymmetry parameter ϵ in order to obtain sensible supersymmetry transformation rules. However, this rescaling then invalidates the conclusions about the vacuum supersymmetry discussed in (2), since the rescaled ϵ is no longer annihilated by $(1 - \Gamma)$ as r tends to infinity. In fact the absence of a mass term for the bosons means that the time dependence of their solutions is of the form $\exp(i\omega t)$ with $\omega = \sqrt{l(l+1)}$ instead of $\omega = \sqrt{l(l+1) + \frac{1}{4}} = l + \frac{1}{2}$, and so given the periodicity condition on t , there are no solutions possible except when $l = 0$. Thus the fluctuations around the solutions described in this paper apparently are comprised of free fermion singletons but no non-trivial bosons. Upon quantisation, when the classical solutions are promoted to quantum creation and annihilation operators acting on the vacuum state of the theory, the fermionic singletons will generate massless and massive states. There will be an infinity of massless states, corresponding to the product of two singleton operators. These states, which will all be bosonic, are a subset of those discussed in ref. [26].

Finally, as already remarked elsewhere [27], the membrane at the end of the universe naturally favours compactifications of eleven-dimensional supergravity to four, rather than any other, dimension.

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