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THE SPECTRUM OF SPINNING SUPERPARTICLES

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We construct the action and transformation rules of a massless spinning superparticle in D dimensions which has N worldline as well as M target space supersymmetries. The spectrum of physical states for different values of N, M and D is presented. We also discuss the action, symmetries and spectrum of massive spinning superparticles.

It is well known that the one-dimensional action for a massless spin-0 particle in D dimensions can be supersymmetrized in two different ways. One way is to introduce a local one-dimensional worldline supersymmetry [1,2]. The thus obtained supersymmetric model is called the “spinning particle” and describes, upon quantization, a massless spin-½ particle in D dimensions. Supersymmetric actions with N (N arbitrary) local worldline supersymmetries have also been considered [3-5]. The authors of ref. [4] called the corresponding model the N-extended spinning particle. Upon quantization, the model describes a massless spin-N/2 particle in D dimensions. In order to obtain this result it is necessary to gauge the O(N) symmetry that rotates the different supersymmetries into each other, i.e. to consider gauged N-extended worldline supersymmetry.

The other way to supersymmetrize the action for a spin-0 particle is to introduce a rigid D-dimensional target space supersymmetry. In this way one obtains the “superparticle” action [6] which, upon quantization, describes a massless supermultiplet in D dimensions. It turns out that these superparticle actions are invariant under an additional local fermionic so-called “kappa-symmetry” [7]. Actions with M (M arbitrary) rigid target space supersymmetries have also been considered [8] and they describe supermultiplets which form representations of the M-extended super-Poincaré algebra.

Instead of introducing either worldline or target space supersymmetry separately it is also possible to introduce these two supersymmetries at the same time. In ref. [9] an action was constructed with one local worldline supersymmetry and one local 4-dimensional target space supersymmetry. The model is called the spinning superparticle and describes, upon quantization, a 4-dimensional scalar and vector multiplet. A similar action, but with 10-dimensional instead of 4-dimensional target space supersymmetry was considered in ref. [10] and was shown to describe two 10-dimensional gravitini multiplets.

It is the purpose of this paper to give a unified treatment of the above cases considered. To be precise, we will present the action and transformation rules of a massless spinning superparticle in D dimensions which has N local worldline and M rigid target space supersymmetries. This model will be called the massless (N, M, D) spinning superparticle and we will give the spectrum of physical states for different values of N, M and D. At the end of this letter we will also discuss the action, transformation

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rules and spectrum of the massive \((N, M, D)\) spinning superparticle.

The dynamics of the \((N, M, D)\) spinning superparticle can be described in terms of two sets of fields. The first set are the coordinates of the spinning superparticle: \(\{X^\mu(\tau), \Psi^\mu_{\mu}(\tau), \theta^m(\tau)\}\). The parameter \(\tau\) is an evolution parameter. The \(X^\mu(\mu = 1, \ldots, D - 1)\) are commuting variables, whereas the \(\Psi^\mu_{\mu}\) (\(i = 1, \ldots, N\)) and \(\theta^m(m = 1, \ldots, M)\) are anticommuting variables. The \(\mu\) and \(\alpha\) are vector and spinor indices respectively of the spacetime Lorentz group \(SO(1, D - 1)\). The \(\alpha\) index is given by \(\alpha = 1, \ldots, 4\xi\), where \(\xi\) is the real dimension of the spinor representation of \(SO(D - 1, 1)\). The \(i\) index is a vector index of \(O(N)\). The \(W'(\theta^m)\) are the \(N\)-worldline (\(M\)-target space) supersymmetric partners of \(X\). The second set of variables are the one-dimensional \(N\)-extended supergravity gauge fields: \(\{V(\tau), \chi'(\tau), f^a(\tau) = -f^a(\tau)\}\). The \(V\) is the einbein, the \(\chi\) are the \(N\) gravitini and the \(f^a\) are the \(O(N)\) gauge fields. In terms of the variables introduced above the action of the massless \((N, M, D)\) spinning superparticle is given by

\[
S = \frac{1}{2} \int d\tau \left( V P^\mu P^\nu \eta_{\mu\nu} - \Psi^\mu \mathcal{D}_0 \Psi_{\mu} \right).
\]

Here \(\eta_{\mu\nu}\) is the Minkowski spacetime metric, \(\eta_{\mu\nu} = \text{diag}( -, +, +, +, +, +, +, +)\), and \(P_{\mu}\) and \(\mathcal{D}_0\) are defined by

\[
P_{\mu} \equiv V^{-1} \left( \dot{X}_\mu + i \theta^m \mu_{\mu} \theta^m - \chi^\nu \Psi^\mu_{\nu} \right),
\]

\[
\mathcal{D}_0 \Psi^\mu = \dot{\Psi}^\mu - f^a \mu_{\mu}.
\]

The \(\gamma^\nu\) are the \(\gamma\)-matrices of the \(SO(1, D - 1)\) Lorentz group. The properties of these \(\gamma\)-matrices and of the spacetime spinor \(\theta\) depend on the value of \(D\) and are given in refs. [11,12].

The action (1) is invariant under five types of gauge transformations: worldline diffeomorphism invariance with parameter \(\xi(\tau)\), \(N\)-extended worldline supersymmetry with parameter \(\epsilon'(\tau)\), \(O(N)\) invariance with parameter \(\epsilon'_{\mu}(\tau)\), local \(M\)-extended \(\kappa\)-invariance with parameter \(\kappa^m(\tau)\) and new bosonic gauge invariances with parameter \(\lambda^m\). The \(\xi\) and \(\lambda\) (\(\epsilon\) and \(\kappa\)) are commuting (anti-commuting) parameters. Note that \(\kappa\) is a spacetime spinor and worldsheet scalar.

The infinitesimal worldline reparametrizations are (we omit from now on the spacetime spinor index)

\[
\delta X^\mu = \xi X^\mu, \quad \delta V = (\xi V)', \quad \delta \Psi^\mu_{\mu} = \xi \Psi^\mu_{\mu}, \quad \delta \chi^\nu = (\xi \chi^\nu)', \quad \delta \theta^m = \xi \theta^m, \quad \delta f^a = (\xi f^a)'.
\]

The worldline supersymmetries are

\[
\delta X^\mu = \epsilon' \Psi^\mu_{\nu}, \quad \delta V = 2 \epsilon' \chi^\nu, \quad \delta \Psi^\mu_{\mu} = P_{\mu} \epsilon', \quad \delta \chi^\nu = \mathcal{D}_0 \epsilon'.
\]

The \(O(N)\) gauge transformations are

\[
\delta \Psi^\mu_{\nu} = \mathcal{D}_0 \Psi^\mu_{\nu}, \quad \delta \chi^\nu = b^\nu \chi^\nu, \quad \delta f^a = \mathcal{D}_0 b^a.
\]

The \(\kappa\)-transformations are

\[
\delta X^\mu = i \theta^m \mu_{\mu} \delta \theta^m, \quad \delta V = -4 \theta^m \kappa^m, \quad \delta \theta^m = i \gamma^\mu \kappa^m P_{\mu}.
\]

The bosonic \(\lambda\) invariances are

\[
\delta X^\mu = i \theta^m \gamma^\mu \delta \theta^m, \quad \delta \theta^m = \lambda^m \delta \theta^m.
\]

The parameter \(\lambda^m\) satisfies

\[
C \gamma^\nu \lambda^m = (C \gamma^\nu \lambda^m)^T,
\]

where \(T\) indicates a transposition in the spinor indices. E.g. in \(D=4\) the general solution of eq. (9) is given by \(\lambda^m = \lambda^m(\theta^m, \theta^m)\). Note that in general this \(\lambda\)-invariance is not an independent symmetry. For instance the \(\lambda^m(\theta^m)\) transformations in \(D=4\) can be obtained as a field-dependent \(\kappa\)-transformation with parameter given by

\[
\kappa^m = -i \lambda^m(\theta^m) \gamma_{\nu} P_{\nu} F^m.
\]

For completeness we also list here the on-shell \(\kappa\) and \(\epsilon\) commutator algebra:

\[
[\delta(\epsilon_2), \delta(\epsilon_1)] = \delta_{\epsilon_2 \epsilon_1} (\delta(\epsilon_2)) + \delta(\epsilon_2) \epsilon_1 + \delta(\epsilon_1) \epsilon_2 + \text{equations of motion},
\]

The specific value of \(\xi\) for different values of \(D\) is given in table 1. 

---

\[\text{Table 1}\]

\begin{tabular}{|c|c|}
\hline
\(D\) & \(\xi\) value \\
\hline
4 & 1 \\
5 & 2 \\
6 & 3 \\
\hline
\end{tabular}
\[ \{ \delta (\kappa_1), \delta (\kappa_2) \} = \delta_{\kappa_1, \kappa_2}(\xi) + \delta (\epsilon '= -\xi') \]
\[ + \delta (b_{\mu} = -\xi'_{\mu}) + \delta (\lambda = -\xi) \]
+ equations of motion,
\[ \{ \delta (\kappa), \delta (\epsilon) \} = 0 + \text{equations of motion}, \]
(11 cont’d)

where \( \chi = 2V^{-1} \epsilon'_i \), \( \xi' = -4iV^{-1} \kappa'_{\mu} \kappa'_{\nu} P^\mu \) and \( \lambda \) is the trace of \( \lambda_{mn} : \lambda \equiv (1/M) \lambda_{mn} \delta_{mn} \).

Besides local gauge invariances the action (1) also has the following rigid target space symmetries: translations \( P^\mu \) with parameter \( a^\mu \), Lorentz rotations \( M^\mu_\nu \) with parameter \( A^\mu_\nu = -A^\nu_\mu \), dilatations \( D \) with parameter \( \alpha \), and supersymmetry transformations \( Q^m_\mu (m = 1, \ldots, M) \) which are spacetime spinors. The translations, Lorentz rotations and dilatations are
\[ \delta X^\mu = a^\mu + A^\mu_\nu V^\nu + \alpha X^\mu, \quad \delta V = 2\alpha V, \]
\[ \delta \Psi^\mu_\nu = A^\mu_\nu \Psi^\nu, \quad \delta \chi' = \alpha \chi', \]
\[ \delta \theta^m = \frac{1}{4} A^m_\nu \epsilon^\nu_{\mu} \theta^\mu + \frac{1}{2} \alpha \theta^m, \]
(12)
while the supersymmetries are given by
\[ \delta X^\mu = i\epsilon^\mu_{\nu \rho} \theta^\nu, \quad \delta \theta^m = \epsilon^m. \]
(13)

The local and global transformations given so far are valid for any value of \((N, M, D)\). For special values of \((N, M, D)\) the action (1) possesses additional (super) conformal invariances. First of all one can show that the \( N \)-extended spinning particle \((M=0)\) is invariant under special conformal transformations \( K^\mu \) with parameter \( c^\mu \) for all values of \( N \) and \( D \). These special conformal rotations are given by [13–15]
\[ \delta X^\mu = 2c^\cdot XX^\mu - X^2 c^\mu, \quad \delta V = 4c^\cdot VX, \]
\[ \delta \Psi^\mu_\nu = 2c^\cdot \Psi X^\mu - 2c^\cdot X^\cdot \Psi^\nu, \quad \delta \chi' = 2c^\cdot X\chi' + 2Vc^\cdot \Psi, \]
\[ \delta \theta^m = 4X^\cdot c^\cdot \theta^m. \]
(14)

On the other hand, it is suggestive to think that the \( M \)-extended superparticle \((N=0)\) is superconformal invariant for those values of \( M \) and \( D \) for which there exists a corresponding \( M \)-extended superconformal algebra in \( D \) dimensions. These superconformal algebras have been classified and are given by [16]
\[ D=2, \quad \text{OSp}(M|2), \]
\[ D=3, \quad \text{OSp}(M|4), \]
\[ D=4, \quad \text{SU}(2, 2|M), \]
\[ D=5, M=1, \quad F_4, \]
\[ D=6, M=1, \quad \text{OSp}(6, 2|2). \]
(15 cont’d)

The superconformal transformation rules of the \((0, M, 3)\) and \((0, M, 4)\)-extended superparticle have been given in ref. [8]. It would be interesting to see whether the \((0, M, 2)\), \((0, 1, 5)\) and \((0, 1, 6)\)-extended superparticles are also superconformal invariant. Summarizing, we can say that the spinning particle action \((M=0)\) is conformal invariant for all values of \( N \) and \( D \) and the superparticle action \((N=0)\) is superconformal invariant for special values of \( M \) and \( D \). We will present arguments below indicating that the spinning superparticle action \((N, M, \neq 0)\) is not superconformal invariant for any value of \((N, M, D)\) with \( D \geq 4 \).

We now present the canonical quantization of the spinning superparticle. A covariant quantization for special values of \((N, M, D)\) can be achieved (see e.g. ref. [9] for the \((0, 1, 4)\)- and \((1, 1, 4)\)-models, ref. [13] for the \((1, 0, 4)\)-models, refs. [2,18] for the \((2, 0, 4)\)- and refs. [3–5] for the general \((N, 0, 4)\)-model) but is problematic for general values of \( N, M \) and \( D \). For our purposes, however, it is enough to discuss the canonical quantization in the lightcone gauge. We define the lightcone gauge by
\[ X^{\mu}_0 = 0, \quad V = 1, \]
\[ \Psi^{\mu*}_0 = 0, \quad \chi' = 0, \]
\[ \gamma^+ \theta^m = 0, \quad f^0 = 0, \]
(16)
where \( W^0 = (1/\sqrt{2})(W^0 \pm W^{d-1}) \) for any \( d \)-vector \( W^\mu \) and \( X^{\mu}_0, \Psi^{\mu*}_0 \) are the constant \((\tau\)-independent) modes of \( X^{\mu}, \Psi^{\mu*} \). This lightcone gauge fixes all gauge symmetries except for the \( \tau \)-independent \( O(N) \)-rotations with parameter \( b^\mu_0 \). In the lightcone gauge (16) one can solve the field equations and one is left with the following set of physical variables:
\[ (X^{\mu}_0, p^+), \quad \Psi^{\mu*}_0 (X^{\mu}_0, p^+), \quad S^{m}. \]
(17)

\[ ^{a4} \text{For some recent progress in the covariant quantization of superparticles see, however, ref. [19].} \]
\[ ^{a5} \text{We ignore here boundary conditions. See refs. [20, 13, 21] for a proper treatment of the boundary terms.} \]
Here \( X_\alpha^\gamma, X_\gamma^\alpha, \Psi_{\mu l}^\gamma \) \((l = 1, \ldots, D-2)\) are the time-independent modes of \( X^-, X^+ \), \( \Psi_{\mu l}^\gamma \); \( p^+, p^\gamma \) are the time-independent modes of \( \dot{X}^+, \dot{X}^- \) and \( S^m \) is defined by
\[
S^m = \frac{1}{\sqrt{2}} \gamma^b \gamma^c \gamma^{\gamma'-\gamma} \theta^m. \tag{18}
\]

Using standard methods one can derive the rigid target-space transformations of the physical variables in the lightcone gauge. We only give here the translations \( P \) and the supersymmetry transformations \( Q \). They are given by the rigid target space transformations (12), (13) plus compensating field-dependent gauge transformations. The specific form of the transformation rules is uniquely determined by the requirement that they leave the lightcone gauge (16) invariant. They are given by
\[
\delta X_\alpha^\gamma = a^\alpha - a^{\gamma'} \gamma^\alpha + \frac{1}{2} a^{\gamma'} \gamma^\gamma \gamma^\alpha, \quad \delta X_\gamma^\alpha = a^\gamma - a^{\alpha'} \gamma^\gamma + \frac{1}{2} a^{\alpha'} \gamma^\gamma \gamma^\gamma, \tag{19}
\]

and
\[
\delta X_\alpha^\gamma = \frac{2i}{\sqrt{2}} \gamma^b \gamma^c \gamma^{\gamma'-\gamma} \theta^m, \quad \delta X_\gamma^\alpha = -i \frac{p^\gamma}{(\sqrt{2} p^+)^3} \gamma^b \gamma^c \gamma^\gamma + \frac{i}{\sqrt{2} p^+} \gamma^b \gamma^c \gamma^{\gamma'-\gamma} \theta^m, \quad \delta S^m = \frac{1}{2} \frac{p^\gamma}{\sqrt{2} p^+} \gamma^b \gamma^c \gamma^\gamma + \gamma^b \gamma^c \gamma^{\gamma'-\gamma} \theta^m, \tag{20}
\]

respectively. The supersymmetry parameters \( \alpha^m \) and \( \beta^m \) are defined by \( \alpha^m = \frac{1}{2} \gamma + \gamma^\gamma \epsilon^m \) and \( \beta^m = \frac{1}{2} \gamma + \gamma^\gamma - \epsilon^m \).

Using Dirac’s quantization procedure one can derive the Dirac brackets of the physical variables and the expressions for the \( P \) and \( Q \) Noether charges. They are given by
\[
\{X_\alpha^\gamma, p^+\} = 1, \quad \{S^m, S^m\} = 4i \delta^{mn} \gamma^+ + 2i \gamma^+ \epsilon^m + \gamma^+ \epsilon^m, \tag{21}
\]
and
\[
P^+ = p^+, \quad Q^\alpha^\gamma = -\frac{2i}{\sqrt{2}} \gamma^b \gamma^c \gamma^{\gamma'-\gamma} \theta^m, \quad P^\gamma = p^\gamma, \quad Q^\gamma^\alpha = -2i \sqrt{p^+} \gamma^\gamma \theta^m, \quad P^- = \frac{1}{2} \gamma^\gamma \gamma^\gamma \gamma^m, \tag{22}
\]

respectively. Here \( Q^\alpha^\gamma = \frac{1}{2} \gamma + \gamma^\gamma \theta^m \) and \( Q^\gamma^\alpha = \frac{1}{2} \gamma - \gamma^\gamma \theta^m \). Furthermore the global \( O(N) \) invariance leads to the constraint
\[
\Psi_{\mu l}^\gamma = 0. \tag{23}
\]

The \( P \) and \( Q \) Noether charges satisfy the following \( M \)-extended super-Poincaré algebra in \( D \) dimensions:
\[
\{Q^\alpha^\gamma, Q^\beta^\delta\} = -\frac{1}{i} p^\gamma \theta^m, \quad \{Q^\alpha^\gamma, \tilde{Q}^\beta^\delta\} = \frac{1}{i} p^\gamma \theta^m + \delta_{\gamma\delta} \theta^m, \quad \{Q^\alpha^\gamma, \tilde{Q}^\gamma^\alpha\} = \frac{1}{i} p^\gamma \theta^m - \delta_{\gamma\delta} \theta^m. \tag{24}
\]

We are now in a position to calculate the spectrum of physical states of the \((N, M, D)\) spinning super-particle. It is clear from the Dirac brackets (21), that the spectrum of the spinning super-particle is given by the direct product of the spectra of the \((N, 0, D)\) spinning particle and the \((0, M, D)\) superparticle. Therefore we first discuss these two special cases. Then we can proceed to the general case.

(a) \((N, 0, D)\). For \(M=0\) there is no target-space supersymmetry and the particle states span representations of the bosonic \(D\)-dimensional Poincaré algebra or its conformal extension. If \(N=0\) as well, we obtain the simple scalar theory in \(D\) dimensions, described completely by the coordinates \(X^\mu\) and the associated momenta, thus reducing eq. (21) to the Heisenberg subalgebra generated by \((X^\mu, p^\mu, p^\gamma)\).

The spectrum consists of a single spinless particle in \(D\) dimensions. If \(N \geq 1\) the Heisenberg algebra is supplemented by the Clifford algebra of the \(\Psi^\mu\), which can be represented by gamma-matrices \(\gamma^\mu\) of the transverse Lorentz group. For \(N=1\) this is the only modification and the groundstate of the theory is a spinor of the transverse Lorentz group. However, for \(N \geq 2\) we must take into account the extra constraint of eq. (23), which leaves no solution for odd \(D\), whilst for even \(D\) the spectrum is given by \(SO(D-2)\) representations characterized by Young tableaux consisting of \((D-2)/2\) rows and \(N/2\) columns.\(^{55}\) According to ref. \cite{22} these are precisely the representations of the \(D\)-dimensional light-cone conformal group \(SO(D-2, 2)\).\(^{66}\) Actually, there are always two such representations for any \(N \geq 1\), which

\(^{55}\) A column of width one-half represents a spinor index.

\(^{66}\) For \(N=2\) the spectrum contains additional representations \([4, 15]\) but these will not be considered here.
generalizing the $D=4$ case correspond to multispinors with only dotted or undotted indices, respectively. For even $N$ these representations describe two bosonic states of opposite duality, for odd $N$ two tensor-spinors of opposite duality-chirality. As a consequence, the spectrum of the $(N, 0, D)$ spinning particle is reducible for any even $D$ and $N \geq 1$, with both representations describing conformal states, whilst for odd $D$ only the cases $N = (0, 1)$ exist and describe conformal representations as well.

(b) $(0, M, D)$. Next we turn to the $M$-extended superparticle models, for which $N = 0$. They are characterized by the reduced form of the algebra (21) obtained by ignoring the $\Psi$-operators and the constraint (23). We choose a fixed light-like $D$-momentum

\[ p^+ = \omega, \quad p^- = p' = 0. \]

In this frame $Q_\mu \sim S_\mu$ and $[Q_-, Q_-] \neq 0$, whereas $Q_+$ anti-commutes with itself and with $Q_-$. Therefore $Q_+$ generates only zero-norm states which can be ignored. For the creation of physical states we only need to consider the action of the $2\zeta M$ real components of $Q_\mu$ on a Clifford vacuum $|\Omega\rangle$. These components of $Q_-$ transform under the transverse Lorentz group $\text{SO}(D-2)$ and the automorphism group $\mathcal{A}$ of the super-Poincaré algebra, and generate a Clifford algebra $\mathcal{C}_{M,D}$ of real dimension $2^{\mathcal{C}M}$. In table 1 we have listed the $\text{SO}(D-2) \times \mathcal{A}$ group for $D = 2, \ldots, 11$, and for $M = 1$ the particular multiplet of real dimension $2\zeta$ to which $Q_-$ (or $S$) belongs. For general $M \geq 1$ the components of $Q_\mu$ belong to $M$ such multiplets. In order to obtain the spectrum of the physical states of the $(0, M, D)$ superparticle one must decompose the $2^{\mathcal{C}M}$-dimensional representation of $\mathcal{C}_{M,D}$ into $\text{SO}(D-2) \times \mathcal{A}$ representations. This can be done for general $M$ and $D$ (see e.g. ref. [12]). In table 1 we have indicated the multiplet structure of $M = 1$.

For specific values of $M$ and $D$, for which there exists a superconformal algebra $\mathcal{S}_7$, the $(0, M, D)$ spectrum consists of superconformal representations only. For instance, for $M = 1$ we see from table 1, that the spectrum in $D = 2, \ldots, 6$ contains only scalar multiplets, i.e. scalars and spinors, which belong to conformal representations. However, for $D = 7, \ldots, 11$ the spectrum contains Yang–Mills or gravity multiplets which are not conformal representations. Summarizing, the spinning particle spectrum $(M = 0)$ is conformally invariant for all values of $N$ and $D$, but the superparticle spectrum $(N = 0)$ is conformally invariant only for special values of $M$ and $D$.

(c) $(N, M, D), (N, M) \geq 1$. Finally we turn to the spinning superparticle, the models with both $(N, M) \neq 0$. We observe that the spectrum is not conformally invariant for any $D \geq 4$. For even $D$ this is im-

\[ \text{Cf. the discussion surrounding eq. (15).} \]

Table 1

<table>
<thead>
<tr>
<th>$D$</th>
<th>$\text{SO}(D-2) \times \mathcal{A}$</th>
<th>rep $S$</th>
<th>Spectrum $(0, 1, D)$</th>
<th>Spectrum $(1, 1, D)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\text{SO}(2) \times U(1)$</td>
<td>$(\frac{1}{2}, 1) + (-\frac{1}{2}, -1)$</td>
<td>$1 + 1$ scalar</td>
<td>$1 + 1$ scalar</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>$1$</td>
<td>$1 + 1$ scalar</td>
<td>$1 + 1$ scalar</td>
</tr>
<tr>
<td>4</td>
<td>$\text{SO}(3) \times \text{USp}(2)$</td>
<td>$(2, 2)$</td>
<td>$2 + 2$ scalar</td>
<td>$2 + 2$ scalar, $2 + 2$ YM</td>
</tr>
<tr>
<td>5</td>
<td>$\text{SO}(4) \times \text{USp}(2)$</td>
<td>$(2, 1; 2)$</td>
<td>$4 + 4$ scalar</td>
<td>$4 + 4$ YM</td>
</tr>
<tr>
<td>6</td>
<td>$\text{SO}(5) \times \text{USp}(2)$</td>
<td>$(4, 2)$</td>
<td>$8 + 8$ YM</td>
<td>$64 + 64$ gravitino</td>
</tr>
<tr>
<td>7</td>
<td>$\text{SO}(6) \times U(1)$</td>
<td>$(4, 1) + (\bar{4}, -1)$</td>
<td>$8 + 8$ YM</td>
<td>$64 + 64$ gravitino</td>
</tr>
<tr>
<td>8</td>
<td>$\text{SO}(7)$</td>
<td>$8$</td>
<td>$8 + 8$ YM</td>
<td>$64 + 64$ gravitino</td>
</tr>
<tr>
<td>9</td>
<td>$\text{SO}(8)$</td>
<td>$8^+$</td>
<td>$8 + 8$ YM</td>
<td>$64 + 64$ gravitino</td>
</tr>
<tr>
<td>10</td>
<td>$\text{SO}(9)$</td>
<td>16</td>
<td>$128 + 128$ gravity</td>
<td>$64 + 64$ gravitino$^*$</td>
</tr>
</tbody>
</table>

$^*$ Cf. the discussion surrounding eq. (15).
mediately related to the fact that the spinning particle spectrum is reducible, i.e., contains one state with only “undotted” spinor indices \( \alpha \) and one state with only “dotted” indices \( \bar{\alpha} \). Therefore, even if the superparticle spectrum is conformal (made up of states with only dotted or undotted indices), the spinning superparticle will always contain one multiplet with mixed indices \( \alpha \bar{\alpha} \). For instance, from table 1 we see that the \( (1, 1, 4) \) spinning-superparticle spectrum contains a conformal \( 2 + 2 \) Yang-Mills multiplet, but also a non-conformal \( 2 + 2 \) scalar multiplet [9]. Similarly, the \( (1, 1, 6) \) spinning superparticle spectrum contains a conformal \( 4 + 4 \) tensor multiplet and a non-conformal \( 4 + 4 \) Yang–Mills multiplet.

For odd \( D \) we observe, that the only allowed values of \( N \) are \( (0, 1) \), and that the only dimensions in which the simple superconformal representations of ref. [14] exists are \( D = 3 \) for arbitrary \( M \), and \( D = 5 \) for \( M = 1 \). The \( D = 5 \) case can be excluded, since table 1 shows the spectrum to be a non-conformal \( 4 + 4 \) Yang–Mills multiplet. This confirms our statement about the non-existence of conformal spinning superparticles in \( D \geq 4 \). We are left with only the \( D = 2 \) and \( D = 3 \) cases, for which conformal invariance is impossible if there are spin \( s \geq 1 \) states, thus eliminating \( M > 2 \) for \( D = 3 \) and \( M > 4 \) for \( D = 2 \). For example, from table 1 we deduce the existence of conformal \( 1 + 1 \) scalar multiplets for the \( (1, 1, 2) \) and \( (1, 1, 3) \) models.

For \( D \geq 4 \) an irreducible spectrum of conformal states for the spinning particle can be achieved by imposing a duality-chirality constraint [15]. It will be interesting to see what the corresponding modification of the spinning superparticle is. Probably, this modified theory will be superconformal for all values of \( M \) and \( D \) for which the superconformal algebra of ref. [14] exists.

We now also consider the massive \((N, M, D)\) spinning superparticle. The massive \((1, 0, D)\) spinning particle has already been considered in ref. [2] and the general massive \((N, 0, D)\) spinning particle has been considered in refs. [3, 15]. Recently, also the spectrum of the \((1, 1, 4)\) massive spinning superparticle has been calculated [23]. We here treat the general case. The action for the massive \((N, M, D)\) spinning superparticle is given by

\[
S = S_0 + S_m, \tag{26}
\]

where \( S_0 \) is the action for the massless \((N, M, D)\) spinning superparticle given in eq. (1) and \( S_m \) is given by \((m)\) is a mass parameter).

\[
S_m = \frac{1}{2} \int d\tau \left( V m^2 + \Psi_5 \partial_\tau \Psi_5 + 2m \chi' \Psi_5 - 2m \tilde{\theta} \tilde{\eta} m \right). \tag{27}
\]

The action (27) makes only sense for those values of \( D \) for which \( \tilde{\theta} \tilde{\eta} m \) is not a total derivative, e.g., \( D = 9 \). If it is a total derivative, one should take \( M = 2 \) and replace \( \tilde{\theta} \tilde{\eta} m \delta_{mn} \) by \( \tilde{\theta} \tilde{\eta} m \epsilon_{mn} \) [23]. The massive spinning superparticle action is invariant under the same symmetries as the massless spinning superparticle except for the conformal symmetries (dilatations \( \lambda \) and special conformal boosts \( K' \)). However, the transformation rules now involve extra terms containing the mass parameter. In the case of the supersymmetry and the “\( \kappa \)” transformations these extra terms are given by

\[
\delta_m \chi' = i \tilde{\theta} \tilde{\eta} m \delta_m \theta' m, \quad \delta_m \Psi_5 = m \chi', \quad \delta_m \theta' m = m \epsilon_{mn} \tag{28}
\]

In the lightcone gauge (16) one obtains the following mass corrections to the transformations given in eqs. (19) and (20):

\[
\delta_m \chi' = \frac{m}{(\sqrt{p^+})^2} \alpha^m S'^m + \frac{1}{2} a^+ \frac{m^2}{(p^+)^2}, \quad \delta_m S'^m = - \frac{i}{2} \sqrt{p^+} \gamma^+ \alpha^m. \tag{29}
\]

The corresponding Noether charges are given by

\[
P = \frac{1}{2p^+} \left( p' p' + m^2 \right), \quad Q^m = \frac{\tilde{Q}^m}{\sqrt{p^+}} = \frac{1}{(i p^\gamma_5 + m)} S'^m. \tag{30}
\]

The \( P \) and \( Q \) Noether charges now obey a supersymmetry algebra involving a central charge \( Z \):

\[
\{ Q^m, \tilde{Q}^n \} = \frac{1}{4} (i p^\gamma_5 - m) \gamma^+ \gamma^+ \delta^{mn}. \tag{31}
\]

We are now able to analyze the spectrum of the massive \((N, M, D)\) spinning superparticle. As for the massless spinning superparticle one finds that the spectrum is given by the direct product of the massive \((N, 0, D)\) spinning particle spectrum and the massive \((0, M, D)\) superparticle spectrum. The spec-
trum of the general massive \((N, 0, D)\) spinning particle can be found in refs. [3, 15]. The spectrum of the massive \((0, M, D)\) superparticle is given by the smallest massive supermultiplet of the \(M\)-extended \(D\)-dimensional super-Poincaré algebra with a central charge given by (31). For \(D = 4, ..., 9\) these multiplets have been given in ref. [12]. As an example let us consider the \((1, 2, 4)\) massive spinning superparticle. The spectrum is given by the direct product of the massive \((1, 0, 4)\) spinning particle spectrum and the massive \((0, 2, 4)\) superparticle spectrum:

\[
(2, 1) \otimes [(1, 2) + (2, 1)] = (2, 2) + (1, 1) + (3, 1),
\]

where the first entry corresponds to the transverse \(SO(3)\) group and the second entry to the \(SU(2)\) automorphism group. We thus confirm the result of ref. [23] that the massive \((1, 2, 4)\) spinning superparticle spectrum is given by a massive gauge hypermultiplet [24] which consists of a complex scalar and vector and two Dirac spinors.

One way to generalize the results of this paper is to consider the massless or massive \((N, M, D)\) spinning superparticle in a curved background. For the \((N, 0, D)\) spinning particle this has already been done in ref. [4]. It would be interesting to see what restrictions on the background one obtains for general values of \(N, M\) and \(D\). It would also be of interest to investigate the covariant quantization of the spinning superparticle along the lines of ref. [18].

Finally, it would be interesting to see which of the spinning superparticle models considered in this paper can be generalized to so-called spinning superstring models, i.e., two-dimensional models which have both local worldsheet as well as rigid spacetime supersymmetry. Such models have already been considered in ref. [25].

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References

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