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Anomalous chiral lagrangians and e^+e^- annihilation into three pions

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We investigate the reaction $e^+e^- \rightarrow 3\pi$ that can be described by the anomalous part of an effective chiral lagrangian. We calculate the next-to-leading corrections to the process and make an estimate of all-order correction within a specific chiral model of pseudoscalars and vector resonances. Our final results describe the process from threshold to the omega resonance in terms of a single parameter.

Effective chiral lagrangians provide a description of low energy QCD which incorporates the low energy theorems implied by chiral invariance. The lagrangian can be divided into an anomalous and a non-anomalous part. For the latter a systematic study of leading and next-to-leading contributions in the expansion in powers of external momenta has been carried out [1]. The calculation of the next-to-leading order involves the determination of the finite part of counterterms added to cancel the divergences arising in one loop calculations of the non-renormalizable tree level lagrangian [2]. These have been shown to be almost saturated by the tree level, lowest order exchange of vector meson resonances between pseudoscalar states [3]. In this paper we want to discuss to which extent an experimental study of the process $e^+e^- \rightarrow 3\pi$ can constrain the form of the anomalous part of the effective lagrangian at next-to-leading order. High luminosity Φ factories [4] are expected to provide accurate results for this reaction.

The leading order anomalous effective lagrangian is given by the Wess–Zumino term [5]

$$\begin{aligned} L_{WZ} = & -N_c \left(\frac{i}{240\pi^2} \int d\Sigma^{ijklm} \text{tr}(\Sigma \partial_i \Sigma^\dagger \Sigma \partial_j \Sigma^\dagger \Sigma \partial_k \Sigma^\dagger \Sigma \partial_l \Sigma^\dagger \Sigma \partial_m \Sigma^\dagger) \right. \\ & + \frac{e}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} A_\mu \text{tr}[Q(\partial_\nu \Sigma \partial_\alpha \Sigma^\dagger \partial_\beta \Sigma \Sigma^\dagger - \partial_\nu \Sigma^\dagger \partial_\alpha \Sigma \partial_\beta \Sigma^\dagger \Sigma)] \\ & \left. + i \frac{e^2}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} (\partial_\mu A_\nu) A_\alpha \text{tr}(Q^2 \partial_\beta \Sigma \Sigma^\dagger + Q^2 \Sigma^\dagger \partial_\beta \Sigma + \frac{1}{2} Q \Sigma Q \Sigma^\dagger \partial_\beta \Sigma \Sigma^\dagger - \frac{1}{2} Q \Sigma^\dagger Q \Sigma \partial_\beta \Sigma^\dagger \Sigma) \right), \end{aligned} \quad (1)$$

where Σ is related to the standard matrix representation M of the pseudoscalar octet by the relation involving the pion decay constant f , $\Sigma = \exp[(2i/f)M]$. This lagrangian is of the fourth order in the momenta of pseudoscalar particles in contrast with the non-anomalous case which is of second order. The amplitude for the reaction $\gamma^* \rightarrow \pi^+(p_1)\pi^-(p_2)\pi^0(p_0)$ following from eq. (1) is

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$$A = -\frac{ie}{\sqrt{2}\pi^2 f^3} \epsilon^{\mu\nu\alpha\beta} A_\mu p_{1\nu} p_{2\alpha} p_{0\beta}. \quad (2)$$

From charged pion decay we take the constant f equal to 132 MeV; with this value the neutral pion decay into two photons agrees satisfactorily with the experimental result (7.58 against 7.7 ± 0.6 eV). Corrections to eq. (2) come from diagrams in fig. 1.

The divergences in the calculation are tamed by dimensional regularization ($\overline{\text{MS}}$ scheme) and the result reads

$$1 + \frac{1}{16\pi^2 f^2} \left[\frac{1}{3} \lambda (p_{01}^2 + p_{02}^2 + p_{12}^2 + 3k^2) - \frac{1}{3} \left((p_{01}^2 + p_{02}^2 + p_{12}^2) \log \frac{m_\pi^2}{\mu^2} + 3k^2 \log \frac{m_K^2}{\mu^2} \right) \right. \\ \left. + \frac{5}{9} (p_{01}^2 + p_{02}^2 + p_{12}^2 + 3k^2) + \frac{4}{3} [F(m_\pi^2, p_{01}^2) + F(m_\pi^2, p_{02}^2) + F(m_\pi^2, p_{12}^2) + 3F(m_K^2, k^2)] \right], \quad (3)$$

where p_{ij}^2 is the squared invariant mass of i and j pions, $\lambda = 1/\epsilon + 1 + \log 4\pi - \gamma$ and the function F is

$$F(m^2, q^2) = m^2 (1 - \frac{1}{4}x) \sqrt{\frac{x-4}{x}} \log \frac{\sqrt{x} + \sqrt{x-4}}{-\sqrt{x} + \sqrt{x-4}} - 2m^2, \quad x = \frac{q^2}{m^2}. \quad (4)$$

The full result for the amplitude is obtained by multiplying the expression above by the tree level result in eq. (2). The divergent terms are manifestly of sixth order in the external momenta; they are regularized by appropriate counterterms which appear in the effective lagrangian at this order,

$$L_6^c = C_1 \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} \text{tr} [Q(\partial_\alpha \Sigma^\dagger \Sigma + \partial_\alpha \Sigma \Sigma^\dagger)] \text{tr} (\chi \partial_\beta \Sigma^\dagger - \chi^\dagger \partial_\beta \Sigma) \\ + C_2 \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} \text{tr} [(\Sigma Q \partial_\alpha \Sigma^\dagger + Q \Sigma \partial_\alpha \Sigma^\dagger - \partial_\alpha \Sigma Q \Sigma^\dagger + \Sigma \partial_\alpha \Sigma^\dagger Q) \\ \times (\partial_\lambda \partial_\beta \Sigma \partial^\lambda \Sigma^\dagger - \partial_\lambda \Sigma \partial_\beta \partial^\lambda \Sigma^\dagger + \partial_\beta \Sigma \partial^\lambda \Sigma^\dagger - \partial^2 \Sigma \partial_\beta \Sigma^\dagger)] \\ + C_3 \epsilon^{\mu\nu\alpha\beta} \partial^\lambda F_{\beta\lambda} \text{tr} [\partial_\mu \Sigma^\dagger \partial_\nu \Sigma \partial_\alpha \Sigma^\dagger \{Q, \Sigma\}]. \quad (5)$$

The quantities C_1 , C_2 and C_3 contain the counterterms and their finite parts should be fixed by experimental data. Assuming SU(2) flavour symmetry ($m_u = m_d$), the term proportional to C_1 is zero and the behaviour of

Fig. 1. One loop diagrams for the process $\gamma^* \rightarrow 3\pi$.

the e^+e^- cross section depends upon the remaining coefficients and upon the renormalization scale μ . The amplitude including the next-to-leading order is

$$A = -\frac{ie}{\sqrt{2}\pi^2 f^3} \epsilon^{\mu\nu\alpha\beta} A_\mu p_{1\nu} p_{2\alpha} p_{0\beta} \left(1 + C_1^{\pi K}(\mu^2) - \frac{64\pi^2}{3e} C_2^{\pi}(\mu^2) (p_{01}^2 + p_{02}^2 + p_{12}^2) - \frac{16\pi^2}{e} C_3^{\pi}(\mu^2) k^2 \right), \quad (6)$$

where $C_1^{\pi K}(\mu^2)$ is the finite part of the one loop correction in eq. (3) at a given mass scale μ , i.e., with the terms proportional to λ subtracted. To estimate the amount of the correction to the tree level result, we need an estimate of the constants C_1 , C_2 and C_3 at some renormalization mass: this can be obtained by extending the “resonance dominance” observed in the non-anomalous case to our case. We adopt a specific model where resonances are present in the effective lagrangian as fundamental fields and we make the ansatz that the unknown coefficients are dominated at low energy by the contribution obtained by integrating out the resonances. In practice, one considers the effective interactions among pions generated by tree level diagrams in fig. 2, where the resonance propagator has been expanded in powers of pion’s momenta. One obtains the same type of interactions of the next-to-leading order original lagrangian. More specifically, we have used a model where vector meson resonances are the gauge bosons of a “hidden” local symmetry [6]. The corresponding lagrangian is

$$L_\rho^\epsilon = \epsilon^{\mu\nu\alpha\beta} \text{tr} [a_1 a_\mu a_\nu a_\alpha \sigma^\dagger D_\beta \sigma + i g a_2 \sigma^\dagger \rho_{\mu\nu} \sigma \{a_\alpha, \sigma^\dagger D_\beta \sigma\} + i e a_3 F_{\mu\nu} (\xi^\dagger Q \xi + \xi Q \xi^\dagger) \{a_\alpha, \sigma^\dagger D_\beta \sigma\}],$$

which contains the following hermitian operators:

$$i\sigma^\dagger D_\mu \sigma = i\sigma^\dagger \partial_\mu \sigma - g\sigma^\dagger \rho_\mu \sigma + v_\mu, \quad \rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + ig[\rho_\mu, \rho_\nu],$$

$$v_\mu = \frac{1}{2i} [\xi(\partial_\mu + ieQA_\mu)\xi^\dagger + \xi^\dagger(\partial_\mu + ieQA_\mu)\xi], \quad a_\mu = \frac{1}{2i} [\xi(\partial_\mu + ieQA_\mu)\xi^\dagger - \xi^\dagger(\partial_\mu + ieQA_\mu)\xi],$$

where

$$\xi = \exp[(i/f)M], \quad (7)$$

ρ_μ is the standard matrix representation of vector mesons with ideal mixed nonet symmetry and σ is a 3×3 unitary matrix parametrized in terms of a set of unphysical scalars (compensating fields) which can be replaced by the identity matrix in the unitary gauge, generating in this way a mass term for vector mesons and γ - ρ couplings.

The lagrangian in eq. (7) is parametrized in terms of the constants a_1 , a_2 , a_3 , f and g . After integration of the

Fig. 2. Lowest order diagrams with vector meson exchange for the process $\gamma^* \rightarrow 3\pi$.

resonances, the effective lagrangian is of the same form of the one in eq. (5) with the following correspondence between the C 's and a 's:

$$C_1^r = 0, \quad C_2^r = \frac{e(a_2 + 2a_3)}{16m_\rho^2}, \quad C_3^r = \frac{e(a_1 + 4a_2)}{16m_\rho^2}, \quad (8)$$

where the constant g has been eliminated through a relation valid in this model:

$$\sqrt{2}fg = m_\rho. \quad (9)$$

The same constant g can be related, having fixed the value of f , to two different processes: $\omega \rightarrow e^+e^-$ and $\rho \rightarrow \pi\pi$. The values of g obtained by requiring agreement with the experiments are given by

$$g_{\omega \rightarrow e^+e^-} = 4.0, \quad g_{\rho \rightarrow \pi\pi} = 4.2. \quad (10)$$

We get similar values with an average $g = 4.1$; by replacing this value in eq. (9) one obtains a value for $m_\rho \simeq 765$ MeV very close to the experimental value $m_\rho = 768.3$ MeV. For this reason we do adopt this value although the width for the process $\rho \rightarrow e^+e^-$ does then come out a bit too low.

The parameter a_3 is eliminated by fitting the experimental value of the radiative decay $\omega \rightarrow \pi^0\gamma$. This process fixes the combination $a_2 + 2a_3$ up to a sign ambiguity. This can be resolved by considering the decay $\eta \rightarrow \pi^+\pi^-\gamma$ which has been calculated in ref. [7] with a specific choice of the sign. The opposite choice would destroy the better agreement with the experimental value (53 ± 10 eV) of the next-to-leading calculation of the width (47 eV) with respect to the tree level result (27 eV). The correct choice corresponds to the relation

$$a_3 = -3/16\pi^2 - \frac{1}{2}a_2. \quad (11)$$

A last relation is obtained from the omega width into three pions. Only diagrams like (a) and (b) in fig. 2 contribute to it and the experimental result can be obtained for values of a_1 and a_2 lying on the curve in fig. 3. The calculation has been done by taking into account the finite width of the ρ resonance in its propagator. The coefficients a_1 and a_2 are bounded in modulus; the particular case where the ω decay goes entirely through the ρ channel is obtained for $a_1 = 4a_2$.

Fig. 3. The ellipse defines the values of parameters a_1 and a_2 which give the correct width for the $\omega \rightarrow 3\pi$. Dotted, solid and dashed portions of the ellipse denote values of the parameters a_1 and a_2 whose corresponding cross sections are within one, two or more standard deviations from the data in fig. 5, respectively.

Fig. 4. The $\gamma^* \rightarrow 3\pi$ cross section near threshold; the tree level result (dashed line), the one loop correction (solid line) and the all order prediction from a resonance model (dot dashed line). The last two curves correspond to the choice A of the parameters on the ellipse in fig. 3.

The next-to-leading order prediction is now parametrized by a single constant. In fig. 4 we compare for the cross section near threshold the tree level result (dashed line) with the next-to-leading order calculation (solid line) and the all order estimate (dot dashed line) based on the vector resonance model. In the latter case we consider the same diagrams leading to the effective pion lagrangian and we keep the higher order terms which appear from the expansion of propagators. The expression for the total amplitude in this case is given by

$$A = -\frac{ie}{\sqrt{2}\pi^2 f^3} \epsilon^{\mu\nu\alpha\beta} A_\mu p_{1\nu} p_{2\alpha} p_{0\beta} [1 + C_l^{\pi K} + \pi^2(4a_2 - a_1)P_\omega + \frac{4}{3}\pi^2(a_2 - 2a_3)\Sigma_{(ij)}P_\rho^{(ij)} - \frac{8}{3}\pi^2 a_2 \Sigma_{(ij)}P_\omega P_\rho^{(ij)} + \pi^2(a_1 + 8a_3)] ,$$

with

$$P_\omega = \frac{m_\omega^2}{m_\omega^2 - k^2 - im_\omega \Gamma_\omega}, \quad P_\rho^{(ij)} = \frac{m_\rho^2}{m_\rho^2 - p_{(ij)}^2 - im_\rho \Gamma_\rho}. \quad (12)$$

The last term is the subtraction of the resonant term at zero external momenta and is required by the non-renormalization theorem at zero external momenta of the Wess–Zumino term. The plot is obtained from values of a_1 and a_2 corresponding to the point A on the ellips of fig. 3. In fig. 5 we plot the all order cross section up to the omega peak energy where it merges with the expected resonant behaviour. The data, taken from the VEPP-2M Collaboration [8], are compared with the curves corresponding to points A, B and C on the ellipse and slightly favour the first two points. Dotted, solid and dashed portions of the ellipse denote values of the parameters a_1 and a_2 whose corresponding cross sections are within one, two or more standard deviations from the data, respectively. The complete cross section for the cases A, B and C down to threshold is given in table 1.

On the omega peak, any choice of the parameters on the ellipse gives the same cross section, up to differences due to the non-resonant contributions like those from diagram (c) in fig. 2; we also get a few percent systematic underestimate of the cross section due to a corresponding underestimate of the $\omega \rightarrow e^+e^-$ width. In the intermediate energy region, the differences among the curves amplify, although in absolute magnitude they are in the range of picobarns, but at zero energy – which is not the case at threshold due to the non-zero pion mass – all predictions are bound to reduce to the tree level result. once the constraints of non-renormalization at zero

Fig. 5. The $\gamma^* \rightarrow 3\pi$ cross section near the omega peak. The solid, dashed and dot dashed lines correspond to the choice of parameters A, B and C, respectively, on the ellipse in fig. 3.

Table 1

The $\gamma^* \rightarrow 3\pi$ cross section in nanobarns at various energies for the choice of parameters a_1 and a_2 of points A, B and C on the ellipse of fig. 3.

$\sqrt{s}$ (MeV)	A	B	C
450	1.7×10^{-5}	1.1×10^{-5}	5.4×10^{-7}
500	9.3×10^{-4}	5.7×10^{-4}	7.7×10^{-5}
550	8.9×10^{-3}	5.5×10^{-3}	1.5×10^{-3}
600	5.1×10^{-2}	3.3×10^{-2}	1.5×10^{-2}
650	0.25	0.17	0.11
681	0.70	0.52	0.38
691	1.00	0.76	0.59
701	1.47	1.13	0.91
711	2.19	1.74	1.45
725	4.08	3.38	2.93
735	6.80	5.80	5.17
745	12.3	10.9	9.94
755	25.8	23.6	21.9
765	70.3	66.9	63.3
782	1.47×10^3	1.49×10^3	1.45×10^3

energy and of fitting the omega peak are implemented, one could expect a substantial equivalence of the predictions obtained by adopting a different model for vector resonances like the one based on tensor couplings which has been shown in the non-anomalous case to be equivalent to the one considered in this paper.

A detailed investigation of this equivalence for the anomalous case is in progress.

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