Chapter 6

Elastic and inelastic scattering in superconductor – two-dimensional electron gas – superconductor junctions with nearly perfect interfaces.

Superconductor (Nb) – two-dimensional electron gas (InAs) – superconductor (Nb) junctions with nearly perfect transparent interfaces are studied experimentally. The high interface quality is the result of a novel cleaning technique. The differential resistance as a function of the applied voltage bias, for different junction lengths $L$, is shown to depend both on the elastic and inelastic scattering rates. We find that for energies close to the Fermi energy the inelastic scattering length $\ell_{in}$ is in the range of 50 to 100 $\mu$m. In the regime where inelastic scattering can be ignored, we find that the reduction of the zero voltage resistance compared to the normal state resistance is approximately equal to the Sharvin resistance, in good agreement with numerical calculations.


6.1 Introduction.

Scattering in the normal region, as opposed to scattering at the interface, of normal-superconductor (NS) structures, has only recently gained interest. Several authors [1, 2, 3, 4, 5, 6] have studied the influence of elastic scattering in disordered conductor—superconductor structures on the Andreev reflection process, also taking into account phase coherent effects. Van der Post et al. [6] investigated the influence of elastic scattering in the InAs channel, ignoring phase coherence, on the subharmonic gap structure due to multiple Andreev reflections [7] in Nb-InAs-Nb junctions. In this chapter we study the influence of both elastic and inelastic scattering in long Nb-InAs-Nb junctions, where phase coherent effects are negligible.

The two-terminal normal conductance $G_N$ of a disordered two-dimensional conductor $N$ is given by the Sharvin conductance, multiplied by the mode averaged transmission probability $\langle T \rangle$ [8, 9],

$$G_N = \frac{2e^2 k_F W}{\hbar \pi} \langle T \rangle,$$

where $W$ is the width of the conductor, and $k_F$ the Fermi momentum. When the conductor is connected to a superconductor $S$ at one side, Andreev reflection [10] will transfer incoming electrons into holes. For a perfect transparent interface, $T_{NS} = 1$, the Andreev reflection probability $P_A$ equals 1 for energies $|E| \leq \Delta$, the superconducting energy gap. Suppose we have a NS contact with a perfectly clean interface and disorder in $N$ causing elastic scattering with a transmission probability $\langle T \rangle$, ignoring phase coherence. Then $G_{NS}$ is given by

$$G_{NS} = \frac{4e^2 k_F W}{\hbar \pi} \langle T \rangle^2 \sum_{n=0}^{\infty} (1 - \langle T \rangle)^{2n} = \frac{4e^2 k_F W}{\hbar \pi} \frac{\langle T \rangle}{2 - \langle T \rangle},$$

where the summation accounts for the particles that are elastically reflected back to the NS interface, and the prefactor $(4e^2/\hbar)$ results from the current doubling in the Andreev reflection process.

The transmission probability $\langle T \rangle$ is limited by momentum relaxation. In both Eq.(6.1) and Eq.(6.2) the current is carried by particles, at the Fermi level, that are
Figure 6.1: Schematic sample layout, two superconductors are placed on top of a 2DEG, width $W$, at a mutual distance $L$. The superconducting quantum well (SQW) present underneath the Nb contacts acts as a superconductor $S$. In the normal region, $0 < x < L$, a non-equilibrium distribution of particles is present.

already in thermal equilibrium, hence elastic and inelastic scattering play the same role. When two superconductors are placed at either side, electrons are injected with an energy $|E| \geq \Delta$, and as a result there is no initial thermal equilibrium in the normal region N. Therefore we also need to take into account energy relaxation, which makes it important to discriminate between the elastic and inelastic scattering rates $P_e$ and $P_i$. The current in the SNS junction is given by two nonequilibrium distribution functions, $f_{||}(E)$ and $f_{\perp}(E)$ (see Fig. 6.1), representing populations of
Elastic and inelastic scattering.

Electrons traveling to right or to the left respectively.

\[ I = \frac{1}{eR_0} \int_{-\infty}^{\infty} \partial E \left[ f_-(E) - f_+(E) \right], \quad (6.3) \]

where the distribution functions are determined by Andreev reflection at the NS interfaces at \( x = 0 \) and \( L \) and elastic and inelastic scattering in the N region [7]. Although in principle an analytical expression for \( G_{SNS} \) is not straightforward, in the absence of inelastic scattering, \( P_i = 0 \), it can be shown that for \( V \to 0 \) the conductance \( G_{SNS} \) is given by [11]

\[ G_{SNS} = \frac{2e^2}{h} \frac{k_W \langle T \rangle}{\pi} \frac{1}{1 - \langle T \rangle}, \quad (6.4) \]

As is clear from Eq.(6.4), the conductance \( G_{SNS} \) diverges for transparency \( \langle T \rangle \to 1 \). Using Eqs.(6.1) and (6.4) we can calculate the total reduction of the resistance at zero applied voltage bias

\[ R_N - \frac{\partial V}{\partial I}(V = 0) = \frac{1}{G_N} - \frac{1}{G_{SNS}} = \frac{h}{2e^2} \frac{\pi}{k_W} \langle T \rangle \quad (= R_{\text{Sharvin}}). \quad (6.5) \]

The reduction is thus exactly the Sharvin resistance, independent of the transmission probability \( \langle T \rangle \) of the normal region.

### 6.2 Experimental setup and results.

To investigate the concepts introduced above, a SNS junction with as perfect as possible interfaces, \( T_{NS} = 1 \), is required. An ideal system in which one could create a structure with nearly perfect Andreev reflection at the NS interface is the geometry discussed in Chapter 3 [12]. When a superconductor is placed on top of a 2DEG an energy gap \( \Delta_{\text{eff}} \) appears in the excitation spectrum of the 2DEG. In Chapter 3 [12] it was shown that, in certain limits, the wave functions in the so-called superconducting quantum well (SQW) are similar to those of a 2D superconductor, and the 2DEG underneath the superconductor S thus acts as a superconductor S'
Experimental setup and results. 97

with an energy gap $\Delta_{\text{eff}}$. The resulting SQW-2DEG-SQW junction acts as a S'NS' junction with nearly perfect interfaces, where the Andreev reflection occurs at the induced energy gap $\Delta_{\text{eff}}$ [13], (see Fig. 6.1).

We used samples based on an InAs quantum well with superconducting Nb contacts. To ensure that the interface is homogeneously transparent, the InAs surface needs to be cleaned, to remove the natural oxide layer, prior to the metal deposition. This can not be done successfully using in situ Ar ion milling, since this drastically changes the mobility and electron density in the 2DEG, as discussed in Chapter 4 [14]. To remove the oxide from the air-exposed InAs surface we used a diluted hydrogen fluoride (HF) solution (1:600), which etches the oxide, and at the same time seems to passivate the surface to prevent it from reoxidation. Afterwards the samples are immediately transferred and loaded into a UHV deposition chamber. Each sample consists of 4 junctions with different lengths $L$ ranging from 1.4 to 4.6 $\mu$m, which allows us to measure one series of junctions with exactly the same electronic parameters. A schematic picture of the junctions is given in Fig. 6.1. After the Nb has been put on top a channel of width $W = 15$ $\mu$m is etched in the InAs to eliminate parallel conductance from the 2DEG outside the junction. Using the electron density $n_S = 2.8 \times 10^{16}$ m$^{-2}$ and the mobility $\mu_e = 0.5$ m$^2$/Vs, measured in a Hall bar geometry, we can estimate $R_{\text{Sharvin}} \approx 6.4$ $\Omega$, and $\ell \approx 0.14$ $\mu$m.

Measurements are performed, using a standard 4-point ac lock-in technique, in a He-4 cryostat, which can be pumped to reach a temperature of 1.3 K. Measured differential resistances as a function of applied voltage bias are given in Fig. 6.2 (top). For all junctions a strong decrease around zero voltage bias is found. The decrease in resistance is interpreted as resulting from multiple Andreev reflections. [7] The dip sets in at about 2.8 meV, twice the superconducting energy gap $\Delta$ of Nb, and the total decrease at zero voltage bias is approximately 9.5 $\Omega$. The Andreev reflection occurs at the NS' interface, see Fig. 6.1, where the induced energy gap of $S'$ is $\Delta_{\text{eff}}$. According to the calculations from Chapter 3 [12] $\Delta_{\text{eff}}$ in the SQW
Elastic and inelastic scattering.

Figure 6.2: Differential resistance as a function of applied voltage bias for different junction lengths $L$ (top). Curves for longer $L$ are displaced by a negative offset of 30 and 60 Ω, for a better comparison. For each length two curves are shown, one measured at 1.3 K (lower one), and one at 4.2 K, which are only different at low voltage bias. The bottom panel shows the normal state resistance $G_\text{N}^{-1}$ (filled circles) and the zero voltage resistance $G_{\text{SNS}}^{-1}$ (open circles), measured at 4.2 K, showing a linear dependence of $\partial V / \partial I$ versus $L$.

is already close to $\Delta$ for an interface transparency $T_{\text{SIN}} \geq 0.5$, consistent with the width of the dip.

The bottom panel of Fig. 6.2 shows the normal state resistance $R_N$, and the
resistance at zero voltage bias $\partial V/\partial I|_{V=0}$, as a function of the junction length $L$. Contrary to the prediction of De Jong, $[9]$ $R_N$ does not extrapolate to $R_N = R_{\text{Sharvin}}$ at $L = 0$, but to a finite value $R_N < R_{\text{Sharvin}}$. At present we do not have an explanation for this $[15]$. The resistance at zero voltage bias $\partial V/\partial I|_{V=0}$ vs. $L$ has nearly the same slope as $R_N$, but is shifted downwards by an amount somewhat larger than $R_{\text{Sharvin}}$ as predicted by Eq.(6.5). A possible explanation for the larger reduction of the zero voltage resistance is that $R_{\text{Sharvin}}$ has increased, either because the electron density $n_S$ has decreased, or because the effective width $W_{\text{eff}}$ is smaller than the width $W$ of the defined channel. For larger junction lengths $L$ we see a slightly smaller decrease of the resistance around $V = 0$. This is the result of an increased inelastic scattering rate, and will be discussed below.

6.3 Numerical simulations.

The differential resistance $\partial V/\partial I$ as a function of the applied voltage can be analyzed using the model proposed by Octavio et al. $[16]$ At a voltage bias $V < 2\Delta/en$ particles have to traverse the junction at least $n + 1$ times before they can enter one of the superconductors. During these $n + 1$ crossings the particles are Andreev reflected $n$ times. To account for scattering in N we add the scattering rates $P_e$ and $P_i$ in the equations for the distribution functions. $[17]$ Close to the NS contacts at $x = 0$ and $x = L$ we have

\begin{align}
f_-(E, L) &= (1 - P_e - P_i)f_-(E - eV, 0) + P_e f_-(E, L) \\
&\quad + P_i f_0(E - eV/2), \tag{6.6}
\end{align}

\begin{align}
f_-(E, 0) &= (1 - P_e - P_i)f_-(E + eV, L) + P_e f_-(E, 0) \\
&\quad + P_i f_0(E + eV/2), \tag{6.7}
\end{align}

where $f_-(E, x)$ is the distribution function of electrons at position $x$, moving either to the left or to the right, see Fig. 6.1, and $f_0(E)$ the Fermi-Dirac distribution. With
Eqs.(6.6) and (6.7), and the simplification $f_\to(E, 0) = 1 - f_\to(-E, L)$ as proposed by Flensberg et al. [18], we can calculate the current as given by Eq.(6.3), but now including also elastic and inelastic scattering in the normal region. Calculated curves are given in Fig. 6.3, where the elastic scattering rates $P_e = 1 - R_{\text{Sharvin}}/R_N$ are chosen to fit the experimental values of Fig. 6.2. For the case that $P_i = 0$ we may write $P_e = 1 - \langle T \rangle$. Extrapolation of the calculated curves to $eV/\Delta = 0$ gives $(\partial V/\partial I)/(1/R_N) = P_e = 1 - \langle T \rangle$, in agreement with Eq.(6.4). Although in principle the calculated curves in Fig. 6.3 should fit the experimental data from Fig. 6.2, the sharp feature at $V = 2\Delta/e$, and also the less pronounced features at $V = 2\Delta/en$, visible in the calculations, are not observed experimentally. We believe that this could partly be due to the nature of our superconductors. The observation of subharmonic gap structures at $V = 2\Delta/en$ is the result of multiple Andreev reflections, [7] and the singularity of the BCS DOS at $\Delta$. Although the Andreev reflection is perfect for $|E| \leq \Delta_{\text{eff}}$ at the 2DEG-SQW interface, due to the exact form of the wave functions in the SQW, the density of states (DOS) will not have the sharp singularity at $\Delta_{\text{eff}}$, known from BCS, but may be rounded.

When we assume that $P_i$ is independent on energy, we can get a reasonably good fit of the data at 1.3 K by choosing $P_i \approx 0.015 \ldots 0.05$, where $P_i$ depends on the junction length $L$. From the inelastic scattering rates we can estimate an inelastic scattering length $\ell_{\text{in}}$ in the range of 50 to 100 $\mu$m. The measured differential resistance above a critical voltage $V_c$, depending on the junction length $L$, is similar for 1.3 and 4.2 K, implying that even at 4.2 K $\ell_{\text{in}} \gg L$. At a voltage bias $V < V_c$, however, the two curves measured at 1.3 and 4.2 K differ, indicating that $P_i$ does depend on temperature. At low voltage bias the total distance particles have to travel, $\ell_{\text{path}} \geq nL \approx (2\Delta/eV)L$, becomes very long, and the critical voltage $V_c$ approximately corresponds to the point where $\ell_{\text{path}} \geq \ell_{\text{in}}(4.2 \text{~K})$. 
Figure 6.3: Numerical calculations of the differential resistance $\partial V/\partial I$ as a function of voltage bias $V$ for an SNS junction with perfect Andreev reflection at the SN interfaces, and finite elastic scattering probability $P_e = 0.787$ (top panel), 0.874 (middle panel) and 0.913 (bottom panel), calculated for different values of the inelastic scattering probability $P_i = 0, 0.01, 0.025$ and 0.05. The values of $P_e$ are chosen to match those of the experimental systems.

6.4 Conclusions.

We have measured the differential resistance of $S'$-2DEG-$S'$ junctions with nearly perfect Andreev reflection at the $S'$-2DEG interfaces, as a function of the junction
length \( L \). The role of the superconductor \( S' \) is played by a superconducting quantum well (SQW) that is present in the InAs underneath the Nb contacts. A large reduction of the differential resistance at zero voltage bias is observed, which is almost independent of \( L \). Based on multiple Andreev reflections the total reduction of the resistance is predicted to be exactly the Sharvin resistance, independent of the transmission \( \langle T \rangle \) of the normal region, in good agreement with the experiment. The small deviations observed in longer junctions we believe to be the result of inelastic scattering. This is confirmed by numerical calculations, including elastic and inelastic scattering probabilities in the normal region.

**References**


